PHY460: Problem Set #4 Due: Friday 11/3 by 5pm in Yellow PHY460 homework box between Loomis and MRL on 2nd Floor

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1. Consider the Krönig-Penney potential given by

$$U(x) = Va \sum_{s=0}^{s=N} \delta(x - sa) \tag{1}$$

where $N = \frac{L}{a}$, L being the system length.

   (a) For this potential, in the limit $\frac{mVa^2}{2\hbar^2} \ll 1$, find at $k = 0$ the energy of the lowest energy band.

   (b) For the same problem, find the band gap at $k = \frac{\pi}{a}$.

2. Consider a square lattice in two dimensions with the crystal potential

$$U(x, y) = -4U \cos\left(\frac{2\pi x}{a}\right) \cos\left(\frac{2\pi y}{a}\right) \tag{2}$$

Apply the central equation to find approximately the energy gap at the corner point $(\frac{\pi}{a}, \frac{\pi}{a})$ of the Brillouin zone. It will suffice to solve a 2×2 determinental equation.

3. Consider a plane hkl in a crystal lattice.

   (a) Prove that the reciprocal lattice vector $\mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3$ is perpendicular to this plane.

   (b) Prove that the distance between two adjacent parallel planes of the lattice is given by

$$d(hkl) = \frac{2\pi}{|\mathbf{G}|} \tag{3}$$

   (c) Show for a simple cubic lattice with side 'a' that

$$d^2 = \frac{a^2}{h^2 + k^2 + l^2} \tag{4}$$

4. Consider a line of atoms $ABAB \ldots AB$, with an $A-B$ bond length of $\frac{1}{2}a$. The form factors are given by f_A, f_B for atoms A, B respectively. The incident beam of X-rays is perpendicular to the line of atoms.

   (a) Show that the interference condition is $n\lambda = a\cos\theta$, where θ is the angle between the diffracted bream and the line of atoms.

   (b) Show that the intensity of the diffracted bream is proportional to $|f_A - f_B|^2$ for n odd, and to $|f_A + f_B|^2$ for n even.

   (c) Explain what happens if $f_A = f_B$.

5. Suppose we have a semi-conductor crystal with a direct energy gap E_g. Assume that the visible spectrum contains wavelengths from $\lambda = 390 - 750$ nm. What is the minimum value for E_g (in units of electron volts) such that the crystal is completely transparent for all visible light?