PHY460: Condensed Matter Physics T Th 03:30 PM- 04:50 PM
Room: 144 Loomis Laboratory

Preliminary Examination

- Solve for the eigenstates and energies of the $1d$ Hamiltonian
$$H = \frac{p^2}{2m} \tag{1}$$
for a system of length L with periodic boundary conditions. That is, solve the Schrodinger equation with these boundary conditions.

- Solve for the eigenstates and energies of the two lowest energy states for the Hamiltonian in Eq. 1 assuming that there is an infinite potential barrier at $x = 0$ and $x = L$.

- What is a boson? What is a fermion? What is the Pauli exclusion principle?

- The generator of spatial translations in 1d is the momentum operator $\hat{p}$ which, in the coordinate representation is written
$$\hat{p} = -i\hbar\frac{d}{dx}. \tag{2}$$
What is the general relation between the translation generator $\hat{p}$, momentum conservation, and a generic Hamiltonian H? Assuming periodic boundary conditions, does the Hamiltonian in Eq. 1 conserve momentum? Why or why not?

- If we have an electromagnetic vector potential $\vec{A}$ how can we determine the magnetic field? What is a gauge transformation of the vector potential? Can we perform a gauge transformation that will change the value of the magnetic field?

- Do/look-up the integral
$$I(x) = \int_{-\infty}^{\infty} e^{ikx}dk. \tag{3}$$
Note that the answer should depend on x.

- Do the integral
$$\int_{-\infty}^{\infty} \delta(x)dx \tag{4}$$
for the Dirac δ-function.

- Plot the function
$$\int_{-\infty}^{x} \delta(x'-2)dx' \tag{5}$$
as a function of x.

- Draw a circle and a square. Describe the rotational symmetries of each shape. Which one has a continuous rotation symmetry? Which one has a discrete rotation symmetry? For the shape with the discrete symmetry list all the possible rotations which leave the shape invariant.

- Using the Einstein summation convention for summing over repeated vector/tensor indices write out the explicit forms for the following (assuming all indices i, j can take the values $1, 2, 3$).
$$\begin{gathered} a_i b_i \\ a_i M_{ij} b_j \\ M_{1i} a_i \end{gathered} \tag{6}$$

- Dirac Bracket notation: Assume we have a Hamiltonian H whose eigenstates $|\phi_i\rangle$ form a complete set of orthonormal states. Formally expand a generic states $|\psi\rangle, |\theta\rangle$ in terms of the complete basis and calculate $\langle\theta|\psi\rangle$, simplifying the result as much as possible. Assume each of the states $|\phi_i\rangle$ has an energy eigenvalue E_i, write the Hamiltonian in terms of the operators $|\phi_i\rangle\langle\phi_i|$.