IN OUR RECENT CLASSES, WE HAVE BEEN DISCUSSING MAGNETISM AND AT THE END THE TOPIC OF MAGNETIC ORDER. IN PARTICULAR, WE SAW THAT WHEN INTERACTIONS ARE INCLUDED, A COLLECTION OF SPINS WILL SPONTANEOUSLY BREAK SYMMETRY OF THE HAMILTONIAN BELOW SOME CRITICAL TEMPERATURE AND SET UP A LOWER ENERGY MANY-BODY STATE.

I HAVE STATED BEFORE, BUT WANT TO EMPHASIZE THAT THIS BEHAVIOR IS GENERAL AND THAT MAGNETISM IN THIS SENSE SHOULD BE THOUGHT OF AS A PROTOTYPICAL EXAMPLE OF WHAT HAPPENS IN A MANY-BODY SYSTEM CONTAINING INTERACTIONS. ALMOST ALWAYS, THE BEHAVIOR IS THE SAME:

\[
\text{MANY BODY SYSTEM + INTERACTIONS} \rightarrow \text{LOW TEMPERATURE ORDER}
\]

\[
\text{THESE ORDERED STATES:} \quad \bullet \text{SET IN BELOW A CRITICAL ONSET TEMPERATURE}
\]

\[
\text{○ OCCUR SUDDENLY}
\]

\[
\text{○ BREAK SOME LOCAL SYMMETRY}
\]

\[
\text{○ ALTER PHYSICAL PROPERTIES OF THE MATERIAL.}
\]

WE HAVE DISCUSSED SPIN ORDER, BUT THERE IS ALSO LATTICE ORDER, CHARGE ORDER, ORBITAL ORDER, AND MANY OTHER ORDERS ASSOCIATED WITH DIFFERENT INTERACTIONS.
In our final lectures, we are going to consider one other ordered state of particular significance: superconductivity.

Superconductors have come up before when we were discussing band theory, and at the time I only mentioned it as "something crazy" which could negate the validity of Fermi liquid theory and use of the independent electron approximation.

Now, I am telling you that superconductivity is an ordered state of matter that arises in metals due to attractive electron-electron interactions below some critical temperature, $T_c$.

We feel we have an understanding of SC, at least for classical materials (i.e. the original elemental SC. This is a Quantum effect!), but the definition is experimental. A material is a superconductor if it shows two effects below $T_c$.

1. Perfect conductivity ($\rho = 0$)
2. Perfect diamagnetism ($\mathbf{B} = 0$ inside material)

We will see that there are also other typical signatures (such as an energy gap), but any material with the above properties is a SC.
BRIEF HISTORY OF SUPERCONDUCTIVITY

1908 - KAMMERLINGH ONNES FIRST LIQUIFIES $^4$He
IN LEIDEN, HOLLAND, OPENING UP NEW HORIZONS
IN LOW TEMPERATURE RESEARCH

1911 - ONNES (ALMOST IMMEDIATELY) DISCOVERS
PERFECT CONDUCTIVITY IN $^3$He AT LOW TEMPS
(FOLLOWED BY Pb IN 1913 AND MANY OTHERS)

1933 - MEISSNER AND OCHSNER FELD DISCOVERED
SUPERCONDUCTORS EXPEL MAGNETIC FLUX

1935 - FRITZ AND HEINZ LONDON ASSOCIATE
FLUX EXPULSION WITH LOCAL
SUPER CURRENTS INTRODUCE "PENETRATION DEPTH"

1950 - DEVELOPMENT OF GINZBURG-LANDAU
EQUATIONS, WHICH EXPLAINED MACROSCOPIC FEATURES
OF SC. BY ASSUMING A Q.M WAVE FUNCTION

1950 - DISCOVERY OF ISOTOPE EFFECT

1952 - PREDICTION OF VORTICES BY ABRNIKOSOV
$\Rightarrow$ TYPE I; TYPE II SC.
1957 - Bardeen, Cooper & Schrieffer develop microscopic theory of SC in terms of pairs of electrons

1959 - Gorkov shows BCS → G-L theory

1962 - Prediction of Josephson effect, relating current and voltage to quantum phase in superconducting junctions

1979 - Discovery of "unconventional" heavy fermion superconductors (Steglich)

1986 - Discovery of "high-temperature" cuprate superconductors (Bednorz and Müller)

2008 - Discovery of iron-based SC

Today, superconductivity is a major topic of research, as researchers attempt to understand origins of unconventional SC with higher Tc's in the context of BCS theory. Hope: dissipationless electricity and floating trains industrially, superconductors are used most commonly to make solenoid magnets, but also as E.M. filters, detectors, Q.M. computational elements, among other things.
EXPERIMENTAL OVERVIEW

ZERO RESISTIVITY: AS SOON AS PHYSICISTS WERE ABLE TO REACH TEMPERATURES AT THE KELVIN SCALE, THEY SOUGHT TO ANSWER QUESTIONS ABOUT THE NATURE OF CONDUCTIVITY IN METALS.

TWO MAIN PICTURES:

- SOURCES OF SCATTERING "FREEZE OUT"
- ELECTRON MOTION FREEZES OUT (NOTE: PRE-QUANTUM)

WHAT OCCURS FOUND IN Hg WAS THIS

From which he concluded an electrical short.

After convincing himself it was real, it became clear that there was a phase transition at T=4.2K and he had discovered a new state of matter.
NOTES - (1) WHEN \( T < T_c \), \( \rho(T) \) DID NOT JUST GET SMALL. \( \rho \) WAS ZERO WITHIN MEASUREMENT LIMITS. ONNES ATTEMPTED TO MEASURE SMALL \( \rho \) BY PUTTING CURRENT INTO A LOOP AND WATCHING THE INDUCED FIELD DECAY. HE GAVE UP AFTER A YEAR!

MODERN ESTIMATES FROM SIMILAR EXPERIMENTS HAVE PLACED DECAY TIME \( T > 100,000 \) YEARS.

(2) ONCE DISCOVERED, IT TURNED OUT THAT THIS STATE OF MATTER WASN'T RARE. SUPERCONDUCTIVITY HAS BEEN VERIFIED IN NO FEWER THAN 27 ELEMENTS AT AMBIENT TEMPERATURES AND MORE UNDER PRESSURES. THERE ARE LITERALLY THOUSANDS OF SUPERCONDUCTING ALLOYS. NOTABLE EXAMPLES ARE:

- \( \text{Pb} \): \( T_c = 7.2 \) K
- \( \text{Al} \): \( T_c = 1.1 \) K
- \( \text{Si} \): \( T_c = 8.7 \) K FOR \( P > 165 \) kbar

COMMON IN HFS MACHINES → \( \text{Nb}_3\text{Sn} \): \( T_c = 18 \) K
- \( \text{YBa}_2\text{Cu}_3\text{O}_{6.9} \): \( T_c = 90 \) K

DESPITE ITS UBQIUTY, FUNDAMENTAL UNDERSTANDING ELUDED PHYSICISTS FOR 50 YEARS!
Perfect Diamagnetism: The "Meissner-Ochsenfeld Effect"

If a material is in the presence of a magnetic field, spontaneous currents will flow on the surface to shield the interior → perfect diamagnetic response.

The invoking of currents often leads one to associate the Meissner effect with perfect conductivity. THIS IS NOT THE CASE!!

Recall

\[ \vec{E} = \frac{\rho}{\epsilon_0} \]

So for any \( \rho < \infty \), \( \rho = 0 \Rightarrow \vec{E} = 0 \)

\[ \nabla \times \vec{E} = 0 \]

\[ \frac{dB}{dt} = 0 \]

Perfect conductors resist change in field (Lenz's law), and thus would try to "lock-in" any field. Perfect diamagnetism has the opposite effect.
**Critical Fields:** As perfect diamagnetism is associated with currents, it is perhaps not surprising that superconductors cannot screen infinite fields. If applied fields are large enough that the energy associated with currents exceeds the energy gain of the SC state, the state collapses and the material returns to its original metallic state.

**Phase Diagram**

For original superconductors (now known as "type-I", as will be explained momentarily), $T_c$'s are in range 1-10K and $H_c$ ~100G. Experimentally, one could see this in a magnetisation measurement: $\mathbf{\bar{B}} = \mu_0 (\mathbf{H} + \mathbf{M}) = 0$, $\mathbf{M} = -\mathbf{H}$.

**Type-I**

$T < T_c$
Type-II Superconductivity

Since I labelled some S.C. "Type-I", it is probably not surprising that there exist "Type-II" superconductors. The magnetization measurements for these materials look like:

The Meissner effect is seen to be incomplete. Above some $H_{c1} < H_c$, the SC ceases to be a perfect diamagnet and enters a "mixed phase" where the field penetrate the material within fixed regions with quantized flux ("vortices"). This mixed phase persist to some $H_{c2} > H_c$, where the material again becomes metallic. Most superconductors are Type-II, and these materials are most useful as well, as typically $H_c > 100x H_c \sim \text{several Tesla}$ (e.g. MRI scans are typically performed in fields of $3T$).
The superconducting gap: it is not part of the definition but it was an important clue when it was seen that superconductors seem to have an energy gap between the ground state and free carrier levels.

![Graph showing energy levels in metal and superconductor]

This is seen as a suppression of electronic contribution to heat capacity.

![Graph showing heat capacity vs temperature in metal and superconductor]

And also in terms of electromagnetic absorption, where absorption is seen to be zero unless

\[ E_x > 2 \Delta \]

The "2" was unexpected originally, but now understood as an indication that light needs to excite pairs of \( e^- \).

For conventional S.C.

\[ 2 \Delta (T=0) = 8.526 \left( \frac{eV}{K} \right) \sim 10^{-4} \text{eV} \]
The presence of a gap should astound and confuse you. Previously, we associated gaps with insulating behaviour. Insulating gaps arise in the independent electron picture as a result of weak coupling to the lattice. Here, it arises from e^-e^- interactions and represents a complete deviation from our original picture.

A couple other quick properties of note:

**Impurity effects**: Superconductivity is nearly independent of the level of non-magnetic impurities, but extremely sensitive to presence of local magnetic moments. (1 part Fe per $10^3$ atoms sufficient to destroy SC in Mo, Kittel (Ch2))

**Isotope effect**: Superconducting $T_c$ is seen to depend on the isotopic mass of lattice cations. (e.g. $T_c = 4.185K$ for $M = 191.5$ and $T_c = 4.146K$ for $M = 203.4$ amu.)

More generally

$$M^\alpha T_c = \text{constant, with } \alpha \sim \frac{1}{2}$$

This tells you lattice vibrations are somehow important and was an important guiding factor for Bardeen, Cooper and Schrieffer.