The Point Acoustic Monopole

In this example, we show plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial sound intensity, energy density, etc. vs. radial observer/listener position and also vs. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The time-domain and frequency-domain expressions for the complex over-pressure and (radial) particle velocity associated with a point acoustic monopole radiating sound at a single frequency \( \omega = 2\pi f \) are:

\[
\tilde{p}(r,t) = \frac{B_o}{r} e^{i(\omega t-kr)} \quad \text{i.e.} \quad \tilde{p}(r,\omega) = \frac{B_o}{r} e^{-i\omega r} \\
\tilde{u}_r(r,t) = \frac{1}{z_o} \frac{B_o}{r} \left[ 1 - \frac{i}{kr} \right] e^{i(\omega t-kr)} \quad \text{i.e.} \quad \tilde{u}_r(r,\omega) = \frac{1}{z_o} \frac{B_o}{r} \left[ 1 - \frac{i}{kr} \right] e^{-i\omega r}
\]

where the characteristic longitudinal specific acoustic impedance of “free-air” (aka the “great wide open”) is: \( z_o \equiv \rho_o c_o = 415 \, \Omega_{Rayls} \) with phase speed: \( c_o = \omega / k = 344 \, m/s \) @ NTP.

The frequency-domain expressions for the complex radial specific acoustic impedance, complex radial sound intensity and complex radial acoustic energy flow velocity associated with a point acoustic monopole radiating sound at a single frequency are:

\[
\tilde{z}_r(r,\omega) \equiv \frac{\tilde{p}(r,\omega)}{\tilde{u}_r(r,\omega)} = z_o \frac{1}{1 - i/kr} = z_o \frac{1 + i/kr}{1 + (1/kr)^2}
\]

and:

\[
\tilde{I}_r(r,\omega) \equiv \frac{1}{2} \tilde{p}(r,\omega) \cdot \tilde{u}_r^*(r,\omega) = \frac{1}{2} \frac{B_o^2}{z_o} \left[ 1 + i/kr \right]
\]

and:

\[
\tilde{v}_r^E(r,\omega) = \frac{1}{\rho_o} \tilde{z}_r(r,\omega) = \frac{z_o}{\rho_o} \frac{1 + i/kr}{1 + (1/kr)^2} = \frac{c_o}{\rho_o} \frac{1 + i/kr}{1 + (1/kr)^2} = c_o \frac{1 + i/kr}{1 + (1/kr)^2}
\]

We coded up the above formulas using MATLAB to make plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial sound intensity, energy density, etc. vs. radial observer/listener position and also vs. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The first set of plots (Figures 1-6) shows these complex quantities as a function of radial position for the following parameter values: \( f = 300 \, Hz \), \( B_o = 1.0 \, RMS \, Pa-m \).

The second set of plots (Figures 7-12) shows these same complex quantities as a function of frequency for the following parameter values: \( r = 1.0 \, m \), \( B_o = 1.0 \, RMS \, Pa-m \).
Figure 1. Complex over-pressure $vs.$ radial distance from point acoustic monopole.

Figure 2. Complex radial particle velocity $vs.$ radial distance from point acoustic monopole.
Figure 3. Complex specific acoustic impedance vs. radial distance from point acoustic monopole.

Figure 4. Complex radial acoustic intensity vs. radial distance from point acoustic monopole.
Figure 5. Complex radial acoustic energy flow velocity vs. radial distance from point monopole.

Figure 6. 3-D plots of the complex plane vs. radial distance for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.
Figure 7. Complex over-pressure $vs.$ frequency for a point acoustic monopole.

Figure 8. Complex radial particle velocity $vs.$ frequency for a point acoustic monopole.
Figure 9. Complex specific acoustic impedance vs. frequency for a point acoustic monopole.

Figure 10. Complex radial acoustic intensity vs. frequency for a point acoustic monopole.
Figure 11. Complex radial acoustic energy flow velocity vs. frequency for a point monopole.

Figure 12. 3-D plots of the complex plane vs. frequency for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.