The Point Acoustic Monopole

In this example, we show plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial sound intensity, energy density, etc. vs. radial observer/listener position and also vs. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The time-domain and frequency-domain expressions for the complex over-pressure and (radial) particle velocity associated with a point acoustic monopole radiating sound at a single frequency $\omega = 2\pi f$ are:

$$\tilde{p}(r, t) = \frac{B_o}{r} e^{i(\omega - kr)} = \tilde{p}(r, \omega) e^{i\omega t} \quad \text{i.e.} \quad \tilde{p}(r, \omega) = \frac{B_o}{r} e^{-ikr}$$

$$\tilde{u}_r(r, t) = \frac{1}{z_o} \frac{B_o}{r} \left[ 1 - \frac{i}{kr} \right] e^{i(\omega - kr)} = \tilde{u}_r(r, \omega) e^{i\omega t} \quad \text{i.e.} \quad \tilde{u}_r(r, \omega) = \frac{1}{z_o} \frac{B_o}{r} \left[ 1 - \frac{i}{kr} \right] e^{-ikr}$$

where the characteristic longitudinal specific acoustic impedance of “free-air” (aka the “great wide open”) is: $z_o \equiv \rho_o c_o = 415 \Omega_a \text{ (Rayls)}$ with phase speed: $c_o = \omega / k = 344 \text{ m/s} @ \text{NTP}$.

The frequency-domain expressions for the complex radial specific acoustic impedance and radial sound intensity associated with a point acoustic monopole radiating sound at a single frequency are:

$$\tilde{z}_r(r, \omega) \equiv \frac{\tilde{p}(r, t)}{\tilde{u}_r(r, t)} = \frac{\tilde{p}(r, \omega) e^{i\omega t}}{\tilde{u}_r(r, \omega) e^{i\omega t}} = \tilde{z}_r(r, \omega)$$

$$= z_o \frac{1}{1 - i/kr} = z_o \left[ \frac{1 + i/kr}{1 + (1/kr)^2} \right]$$

and:

$$\tilde{I}_r(r, \omega) \equiv \frac{1}{2} \tilde{p}(r, \omega) \cdot \tilde{u}_r^*(r, \omega) = \frac{1}{2} \frac{1}{z_o} \frac{B_o^2}{r^2} \left[ 1 + \frac{i}{kr} \right]$$

We coded up the above formulas using MATLAB to make plots of the complex over-pressure, radial particle velocity, radial specific acoustic impedance, radial acoustic intensity, energy density, etc. vs. radial observer/listener position and also vs. frequency for fixed radial position associated with an isotropic sound source – the point acoustic monopole.

The first set of plots (Figures 1-9) shows these complex quantities as a function of radial position for the following parameter values: $f = 300 \text{ Hz}, \ B_o = 1.0 \text{ RMS Pa-m}$.

The second set of plots (Figures 10-16) shows these same complex quantities as a function of frequency for the following parameter values: $r = 1.0 \text{ m}, \ B_o = 1.0 \text{ RMS Pa-m}$. 
Figure 1. Complex over-pressure vs. radial distance for a point acoustic monopole.

Figure 2. Complex radial particle velocity vs. radial distance for a point acoustic monopole.
Figure 3. Complex specific acoustic impedance vs. radial distance for a point acoustic monopole.

Figure 4. Complex radial acoustic intensity vs. radial distance for a point acoustic monopole.
Figure 5. 3-D plots of the complex plane vs. radial distance for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.
Figure 6. Potential, kinetic and total acoustic energy density vs. radial distance for a point acoustic monopole.

Figure 7. Potential, kinetic and total acoustic energy density fractions vs. radial distance for a point acoustic monopole.
Figure 8. Propagating, non-propagating and total acoustic energy density vs. radial distance for a point acoustic monopole.

Figure 9. Propagating, non-propagating and total acoustic energy density fractions vs. radial distance for a point acoustic monopole.
Figure 10. Complex over-pressure vs. frequency for a point acoustic monopole.

Figure 11. Complex radial particle velocity vs. frequency for a point acoustic monopole.
Figure 12. Complex specific acoustic impedance vs. frequency for a point acoustic monopole.

Figure 13. Complex radial acoustic intensity vs. frequency for a point acoustic monopole.
Figure 14. 3-D plots of the complex plane vs. frequency for complex over-pressure, particle velocity, radial acoustic specific impedance and acoustic intensity for a point acoustic monopole.
Figure 15. Potential, kinetic and total acoustic energy density vs. frequency for a point acoustic monopole.

Figure 16. Potential, kinetic and total acoustic energy density fractions vs. frequency for a point acoustic monopole.
Figure 17. Propagating, non-propagating and total acoustic energy density vs. frequency for a point acoustic monopole.

Figure 18. Propagating, non-propagating and total acoustic energy density fractions vs. frequency for a point acoustic monopole.