N-Slit Interference in 1-Dimension – Simplest Theory

In this example, we show plots of the sound intensity vs. angle and observer/listener position on a screen for the simplest theory of N-slit interference – where the N sound sources, all in phase with each other, are assumed to be comprised of infinitely long, parallel slits of infinitely narrow width – i.e. infinitesimally thin slits, each separated from each other by a distance \( d \). The observer/listener is located far from the sound sources, a perpendicular distance \( L \) \( (m) \) away, and such that the conditions \( d \ll L \). and \( \lambda \ll L \) both hold simultaneously, where \( \lambda \) \( (m) \) is the wavelength of the sound – this is the so-called “far-field” limit.

The expression for 1-D \( N \)-slit interference in this simplest theory is given by (see P406POM Lecture Notes P406POM_Lect3_Part2):

\[
I_{\text{tot}}(\theta) = I_o \frac{\sin^2 \left( \frac{N \delta(\theta)}{2} \right)}{\sin^2 \left( \delta(\theta)/2 \right)} \left( \text{Watts/m}^2 \right) \quad \text{and} \quad \text{SIL}(\theta) = 10 \log_{10} \left( \frac{I_{\text{tot}}(\theta)}{I_{\text{ref}}} \right) \left( \text{dB} \right)
\]

where \( I_o \) \( (\text{Watts/m}^2) \) is the sound intensity associated with a single slit, the phase \( \delta(\theta) = k \Delta L(\theta) \) \( (\text{radians}) \), \( k = 2\pi/\lambda \) \( (\text{radians/m}) \) is the wavenumber and \( \Delta L(\theta) \) \( (m) \) is the angle-dependent path length difference between adjacent sound sources to the observer/listener, located far away from the sound sources. \( I_{\text{ref}} = 10^{-12} \) \( (\text{Watts/m}^2) \) is the reference for the sound intensity level (SIL).

Minima – i.e. intensity zeroes (complete destructive interference) occur when the numerator factor \( N \delta = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi, n = \pm 1, \pm 2, \pm 3, \ldots \) except when the denominator factor simultaneously has \( \delta = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots = n\pi, n = \pm 1, \pm 2, \pm 3, \ldots \) then have \textit{global maxima} of the intensity, where \( I_{\text{tot}} = N^2 I_o \).

From simple trigonometry, it is easy to show that the path length difference \( \Delta L(\theta) = d \sin \theta \), where \( \theta \) is the angle the observer/listener makes with respect to the normal, or forward axis of the array of \( N \) slits.

The corresponding location of the observer’s position \( y_{\text{screen}} \) on a screen located a perpendicular distance \( L \) away from the \( N \) sound sources is: \( y_{\text{screen}} = L \tan \theta \), or conversely: \( \theta = \tan^{-1} \left( y_{\text{screen}}/L \right) \).

We coded up the above formulas using MATLAB to make plots of \( I_{\text{tot}} \) vs. \( \theta \) and \( I_{\text{tot}} \) vs. \( y_{\text{screen}} \) e.g. for \( N = 2, 5 \) and 10 slits, with the following parameter values: \( I_o = 1 \text{ Watt/m}^2 \), slit separation distance(s) of \( d = 1 \text{ m} \), observer/listener distance (at \( \theta = 0 \)) of \( L = 10 \text{ m} \), the speed of propagation in free air/great-wide open: \( v_{\text{air}} = 343 \text{ m/s} \) and frequency of \( f = 1000 \text{ Hz} \), thus \( \lambda = v_{\text{air}}/f = 0.345 \text{ m} \).

In the following figures, note that the angular width of the maxima decreases as the number of slits \( N \) increases. Note also that the number of maxima/minima increases linearly with increasing frequency \( f \), since the phase difference increases linearly with frequency:

\[
\delta(\theta) = k \Delta L(\theta) = (2\pi/\lambda) \Delta L(\theta) = (2\pi f/v_{\text{air}}) \Delta L(\theta) = (2\pi f/v_{\text{air}}) d \sin \theta.
\]
Plots of $I_{\text{tot}}$ vs. $\theta$ for $N = 2$ slits:
Plots of $I_{tot}$ vs. $y_{screen}$ for $N = 2$ slits:
Plots of $SIL$ vs. $\theta$ and $SIL$ vs. $y_{screen}$ for $N = 2$ slits:
Plots of $I_{tot}$ vs. $\theta$ for $N = 5$ slits:
Plots of $I_{tot}$ vs. $\theta$ and $I_{tot}$ vs. $y_{screen}$ for $N = 5$ slits:
Plots of $SIL$ vs. $\theta$ and $SIL$ vs. $y_{\text{screen}}$ for $N = 5$ slits:
Plots of $I_{tot}$ vs. $\theta$ for $N = 10$ slits:
Plots of $I_{\text{tot}}$ vs. $y_{\text{screen}}$ for $N = 10$ slits:
Plots of $SIL$ vs. $\theta$ and $SIL$ vs. $y_{\text{screen}}$ for $N = 10$ slits:
Polar plot of $SIL(\theta)$ vs. $\theta$ for $N = 2$ slits:

Polar plot of $SIL(\theta)$ vs. $\theta$ for $N = 5$ slits:
Polar plot of $SIL(\theta)$ vs. $\theta$ for $N = 10$ slits:
Listing of the MATLAB code:

```matlab
%---------------------------------------------------------------
% 1D_Nslit_Intf_Smpl_Thy.m
% 1D N-slit interference - simplest theory - far-field/plane-wave approx!
% sound waves assumed to be propagating in free air/great wide-open!
% each "slit" treated simplistically as point source - no spatial extent!
%---------------------------------------------------------------
% Written by Prof. Steven Errede  Last Updated: Feb. 7, 2011 10:55 hr
%---------------------------------------------------------------
close all;
clear all;
single thtr(1800);
single thtd(1800);
single Itot1(1800);
single SIL1(1800);
single yscr(2000);
single Itot2(2000);
single SIL2(2000);

% Specify # of slits (n.b. Nslits = 1 => no interference):
Nslits = 2; % 2; 5; 10;
% Specify other needed parameters:
Io = 1.0; % intensity from single slit (Watts/m^2)
Ir = 1.0*10^-12; % reference sound intensity (Watts/m^2)
Vair = 343.0; % speed of propagation of sound - free air (m/s)
freq = 1000.0; % frequency (Hz or cps)
lambda = Vair/freq; % wavelength (m)
Lobs = 10.0; % perp. distance observer from slits (m) n.b. lambda << Lobs
Dsrc = 1.0; % transv. distance between slits (m) n.b. Dsrc << Lobs
%===================================
% Calculate Itot, SIL vs. theta:
%===================================
Thetad = -90.0; % angle theta of observer in degrees
dTheta = 0.1; % step angle in degrees
for i = 1:1800;
    thtd(i) = Thetad; % angle theta of observer in degrees
    Thetar = (pi/180.0)*Thetad; % angle theta of observer in radians
    thtr(i) = Thetar;
    delta = (2.0*pi/lambda)*Dsrc*sin(Thetar); % resultant Nslit phase diff (radians)
    Itot1(i) = Io*(sin(Nslits*delta/2.0)/sin(delta/2.0))^2; % total intensity (Watts/m^2)
    SIL1(i) = 10.0*log10(Itot1(i)/Ir); % Sound Intensity Level (dB)
    Thetad = Thetad + dTheta; % increment angle for next calculation
end
%===================================
% Calculate Itot, SIL vs. yscreen:
%===================================
y = -50.00; % starting position on screen (m)
dy = 0.05; % step size on screen (m);
for i = 1:2000;
    yscr(i) = y; % position of observer on perp. screen (m)
    Thetar = atan(y/Lobs); % angle theta of observer in radians
    delta = (2.0*pi/lambda)*Dsrc*sin(Thetar); % resultant Nslit phase diff (radians)
    Itot2(i) = Io*(sin(Nslits*delta/2.0)/sin(delta/2.0))^2; % total intensity (Watts/m^2)
    SIL2(i) = 10.0*log10(Itot2(i)/Ir); % Sound Intensity Level (dB)
end
```
\texttt{y = y + dy; \ % increment screen position for next calculation}

\texttt{end}

\texttt{figure(01);}
\texttt{plot(thtd,Itot1,'b');}
\texttt{grid on;}
\texttt{xlabel('theta (degrees)');}
\texttt{ylabel('Intensity (Watts/m^2)');}
\texttt{title('Intensity vs. theta');}

\texttt{figure(02);}
\texttt{semilogy(thtd,Itot1,'b');}
\texttt{grid on;}
\texttt{xlabel('theta (degrees)');}
\texttt{ylabel('Intensity (Watts/m^2)');}
\texttt{title('Log10 Intensity vs. theta');}

\texttt{figure(03);}
\texttt{plot(thtd,SIL1,'b');}
\texttt{grid on;}
\texttt{xlabel('theta (degrees)');}
\texttt{ylabel('SIL (dB)');}
\texttt{title('SIL vs. theta');}

\texttt{figure(04);}
\texttt{polar(thtr,SIL1,'b');}
\texttt{grid on;}
\texttt{xlabel('theta (degrees)');}
\texttt{ylabel('SIL (dB)');}
\texttt{title('Polar plot of SIL vs. theta');}

\texttt{figure(11);}
\texttt{plot(yscr,Itot2,'b');}
\texttt{grid on;}
\texttt{xlabel('Yscreen (m)');}
\texttt{ylabel('Intensity (Watts/m^2)');}
\texttt{title('Intensity vs. Yscreen');}

\texttt{figure(12);}
\texttt{semilogy(yscr,Itot2,'b');}
\texttt{grid on;}
\texttt{xlabel('Yscreen (m)');}
\texttt{ylabel('Intensity (Watts/m^2)');}
\texttt{title('Log10 Intensity vs. Yscreen');}

\texttt{figure(13);}
\texttt{plot(yscr,SIL2,'b');}
\texttt{grid on;}
\texttt{xlabel('Yscreen (m)');}
\texttt{ylabel('SIL (dB)');}
\texttt{title('SIL vs. Yscreen');}

\texttt{%==========================================================================}
\texttt{beep;}
\texttt{fprintf('
 1-D Nslit interference simple thy calculation completed !!! \n')}
\texttt{%==========================================================================}