Pitch vs. Frequency:

Pitch = human ear’s *perception* of frequency of a sound vibration

Low pitch ⇔ low frequency of vibration/oscillation

High pitch ⇔ high frequency of vibration/oscillation

Q: Is the relation between {perceived} pitch vs. frequency linear? *e.g.* a straight line $y = mx + b$ relation? A: No… See figure below:

Define frequency “units” associated with *subjective* pitch = *mels* ⇔ analogous to *Hz.*
The Audible Frequency Range of Human Hearing (when young):

\[20 \text{ Hz} < f < 20 \text{ KHz} \quad (\approx 3 \text{ orders of magnitude})\]

As we grow older, the range of frequencies that we can hear decreases (both high and low frequencies – mostly on the high frequency end…)

Frequency ranges of musical instruments typically \(\sim 100 \text{ Hz} \) to \(\sim \text{ few KHz}\)

e.g. guitar
Low E = 82 Hz
High E = 330 Hz

Piano highest note is \(\sim 4200 \text{ Hz}\)
Very little above \(\sim 10 \text{ KHz}\) (squeals & scrapes)

The human ear needs to be able to perceive a sound for minimum length of time \(\Delta t\). In order to determine a pitch – i.e. pure/single-frequency tone – the minimum duration time \(\Delta t\) of the pure tone depends on its frequency:

\[
\begin{array}{ll}
\text{For } f \sim 100 \text{ Hz (} \tau \sim 10 \text{ msec): } & t_{\text{min}} \sim 40 \text{ msec (} \sim 4 \text{ cycles)} \\
\text{For } f \sim 1000 \text{ Hz (} \tau \leq 1 \text{ msec): } & t_{\text{min}} \sim 13 \text{ msec (} \sim 13 \text{ cycles)}
\end{array}
\]

The minimum duration time \(\Delta t\) for human perception of a pitch is certainly in part due our ear & brain processing, but for low frequencies especially, minimum time duration is also due to the uncertainty principle \(\Delta f \Delta t = 1\), which tells us that a pure tone/single-frequency sine wave signal of finite duration \(\Delta t\) in fact has a finite frequency spread \(\Delta f\) ! Only as the time duration \(\Delta t \to \infty\) does \(\Delta f \to 0\).
This can be seen by taking the Fourier transform of a finite-length \( \{ \text{time duration } \Delta t_o \} \) pure-tone/single frequency \( \{ f = f_o \} \) time domain sinusoidal signal \( p(t) = p_o \sin \omega_o t \) to the frequency domain \( p(f) \):

If \( p(t) = p_o \sin \omega_o t = p_o \sin 2\pi f_o t \) for \( |t| \leq \frac{1}{2} \Delta t_o \), and: \( p(t) = 0 \) for \( |t| > \frac{1}{2} \Delta t_o \), and defining the \{rectangular\} window function \( w(t) = 1 \) \((= 0)\) for \( |t| \leq \frac{1}{2} \Delta t_o \) \((|t| > \frac{1}{2} \Delta t_o)\), respectively, then:

\[
p(f) = \int_{t=-\infty}^{t=0} p(t) \cdot \sin \omega o t \, dt = P_o \int_{t=-\infty}^{t=0} w(t) \cdot \sin \omega_o t \cdot \sin \omega o t \, dt = p_o \int_{t=-\frac{1}{2} \Delta t_o}^{t=0} \sin \omega_o t \cdot \sin \omega o t \, dt
\]

Using the trigonometric identity \( \sin A \sin B = \frac{1}{2} [ \cos(A - B) - \cos(A + B) ] \):

\[
p(f) = \frac{1}{2} P_o \int_{t=-\frac{1}{2} \Delta t_o}^{t=0} \left[ \cos(\omega - \omega_o) t - \cos(\omega + \omega_o) t \right] dt
\]

The sinc function \( \text{sinc}(x) = \frac{\sin x}{x} \{ \text{n.b. sinc}(0) = 1 \} \), hence we can write \( p(f) \) as:

\[
p(f) = \frac{1}{2} P_o \Delta t_o \left\{ \text{sinc}\left[\frac{1}{2} (\omega - \omega_o) \Delta t_o\right] - \text{sinc}\left[\frac{1}{2} (\omega + \omega_o) \Delta t_o\right] \right\}
\]

The power spectral density functions \( S_{pp}(f) \propto |p(f)|^2 \) (a frequency domain quantity) for infinite length and finite length sine-wave signals are shown below:
The sinc function \( \text{sinc}[\frac{1}{2}(\omega - \omega_0)\Delta t_o] = \sin[\frac{1}{2}(\omega - \omega_0)\Delta t_o]/\frac{1}{2}(\omega - \omega_0)\Delta t_o \) for sine-wave signals of \{short\} time duration \( \Delta t_o = \tau_o, 2\tau_o, 3\tau_o, 4\tau_o \) where \( \tau_o = 1/f_o \) and the corresponding \# of cycles of oscillation \( N_c \equiv \Delta t_o/\tau_o = 1, 2, 3, 4 \) are shown in the figure below. Note that the width \( \Delta f_o \) of the main peak (at \( f = f_o \)) depends \textbf{inversely} on the time duration \( \Delta t_o \) of the signal, due to the \textbf{uncertainty principle} \( \Delta f \Delta t = 1 \).
Human perception of pitch also depends {~ weakly} on the **loudness** of the sound.

* Effect arises due to non-linearities in the $f$ & $I$ response of the human ear.
* Pitch (perceived $f$) **changes** as loudness increases – see graph below...
* Effect exists only for pure/simple tones (!!!)
* Complex tones show **no** perceived pitch changes with loudness! (why??)

![Graph showing pitch shift (%) vs. $L_p$ (dB) for different frequencies.]

Two ears of same person may **NOT** perceive sound of a given frequency as having the same pitch!!! = DIPLACUSIS – happens **only** for diseased, and/or injured ears.

For **normal** musical purposes, frequency and pitch are synonymous (usually)  

**n.b.** applies **only to periodic** sounds.

Sound **pulses** are made up of a **continuum** of frequencies, sound **pulses** are thus **anharmonic** and hence have **no** characteristic frequency and/or pitch.
The human ear can discriminate changes in sound intensity levels/sound pressure levels/loudnesses of \( \text{JND} = \Delta L = |L_1 - L_2| \sim 1/2 \text{ dB} \); Our ability to do so also depends on frequency and sound pressure level/loudness:

\[
\text{A JND} \sim 1/2 \text{ dB change in sound intensity level corresponds to a fractional change in sound intensity of } \Delta I / I \sim 12\%. \text{ Thus, due to the } \sim \text{ logarithmic response of the human ear, it is not terribly sensitive to changes in the loudness of sounds.}
\]
The typical human ear can discern changes in pitch/frequency at the $\Delta f \sim 3$ Hz level in the frequency range $30 \text{ Hz} \leq f \leq 1000$ Hz. Again, has frequency dependence:

Note that:

At very low frequencies: $\Delta f / f \simeq 3/30 = 10\% (= 2$ semitones),

Whereas at higher frequencies: $\Delta f / f \simeq 3/1000 = 0.3\% (= 0.1$ semitones)

A good musician can discern frequency changes significantly smaller than this – e.g. above $f \geq 500$ Hz: $\approx 0.03$ semitone (i.e. $\Delta f / f \simeq 1/1000 = 0.1\%$)!!!

∴ The human ear/brain is capable of detecting small changes in frequency!!!
The human ear/brain is capable of perceiving a **fundamental** even when no fundamental is actually present!!! This is the so-called **missing fundamental effect**.

This effect is {again} a consequence of the non-linear response in/inside the human ear itself, and/or a non-linear response(s) in the human brain’s **processing** of frequency information – whenever e.g. a **quadratic** non-linear response exists (in any system), if two signals \( A \) and \( B \) with frequencies \( f_A \) and \( f_B \) are input to that system, then sum and difference frequencies \( (f_A + f_B) \) and \( |f_A - f_B| \) are produced! Thus, e.g. a 2\(^{nd}\) harmonic \( 2f_i \) and a 3\(^{rd}\) harmonic \( 3f_i \) can produce a “missing” fundamental from the difference frequency, \( |3f_i - 2f_i| = f_i \)!!! For further details on distortion, read Physics 406 Lecture Notes on “Theory of Distortion I & II”.

For some musical instruments – e.g. the trumpet, the oboe and/or the bassoon – the 2\(^{nd}\) (or even 3\(^{rd}\) and higher) harmonics can actually have a **larger** amplitude than that of the fundamental, however we perceive/hear the “note” that is played on the trumpet (and/or oboe, bassoon) as that of the fundamental!!!

The harmonic spectra – **aka** power spectral density functions \( S_{pp}(f) \) vs. \( f \) and associated {time-averaged} relative phase harmonic phasor plots are shown below – e.g. for the steadily-played notes \( A_4 \) (440.0 Hz) played on the oboe, and \( F_2 \) (87.3 Hz) played on the bassoon:

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![Harmonic Spectra and Phasor Diagrams](image-url)
Note that the vertical axes of $S_{pp}(f)$ vs. $f$ are displayed on a **logarithmic** scale.
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