The Tiny Muon versus the Standard Model

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Physics 403

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BNL E821 Muon g-2 Collaboration

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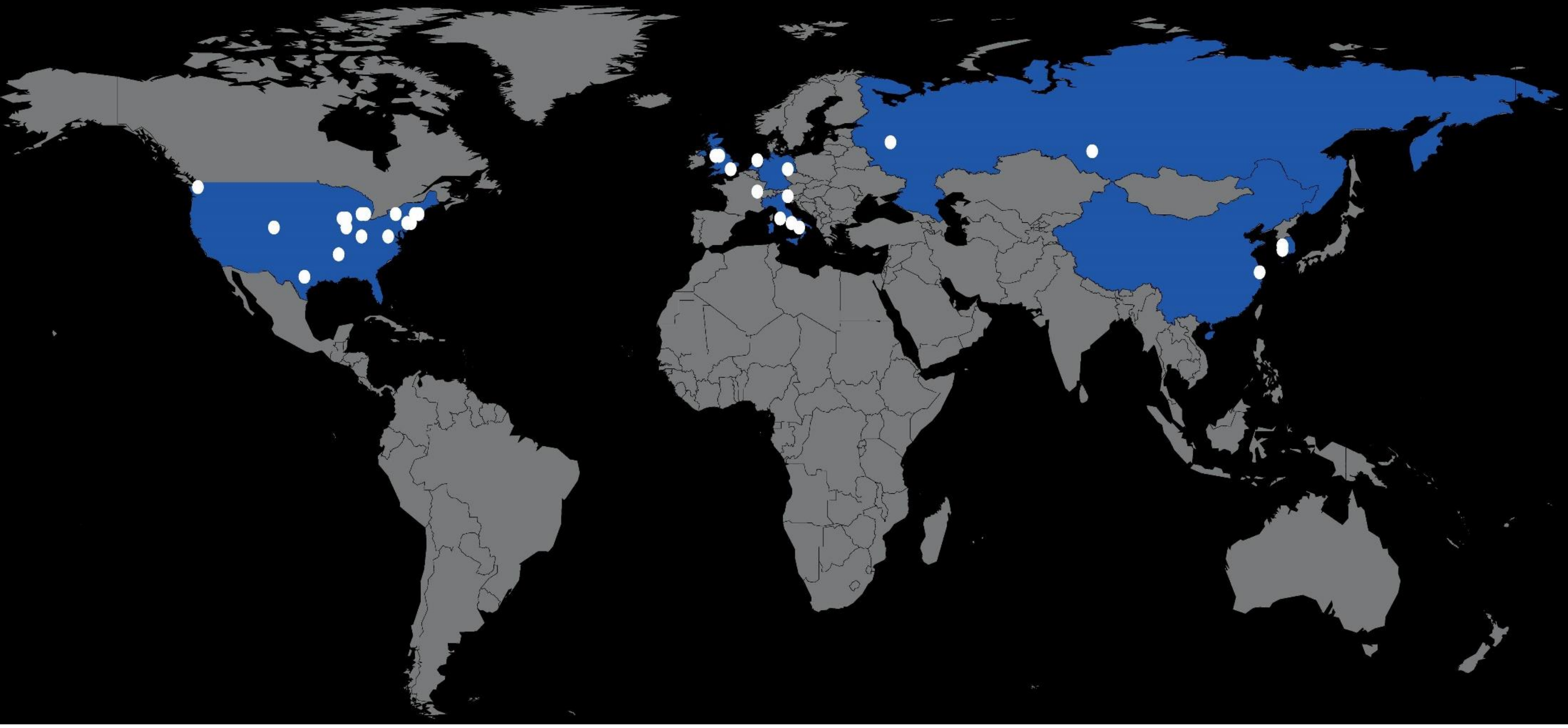
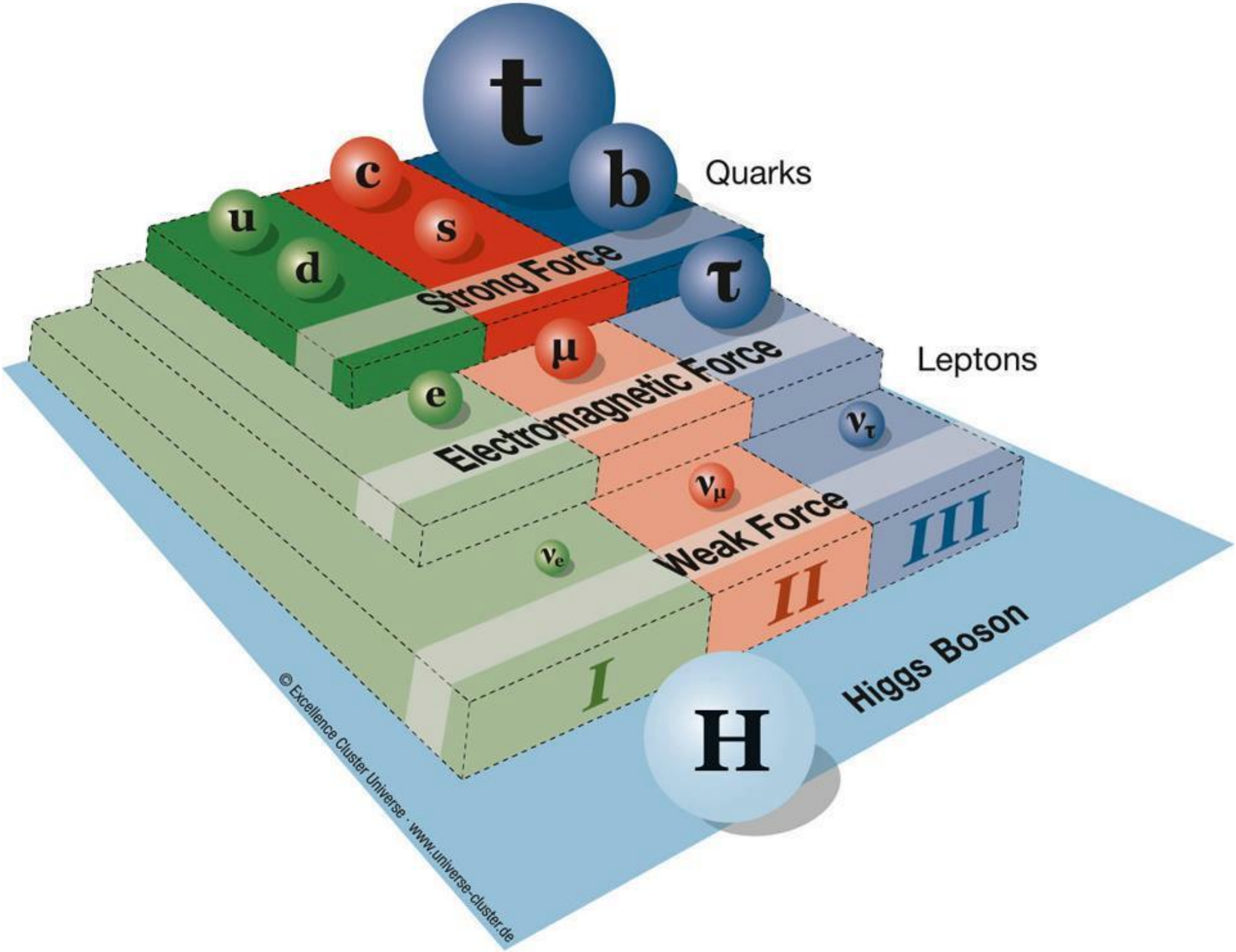
Muon g-2 Collaboration

8 Countries, 35 Institutions, 185 Collaborators



Standard Model of Particle Physics



Components of the Standard Model of Particle Physics

Quarks $\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$

our focus

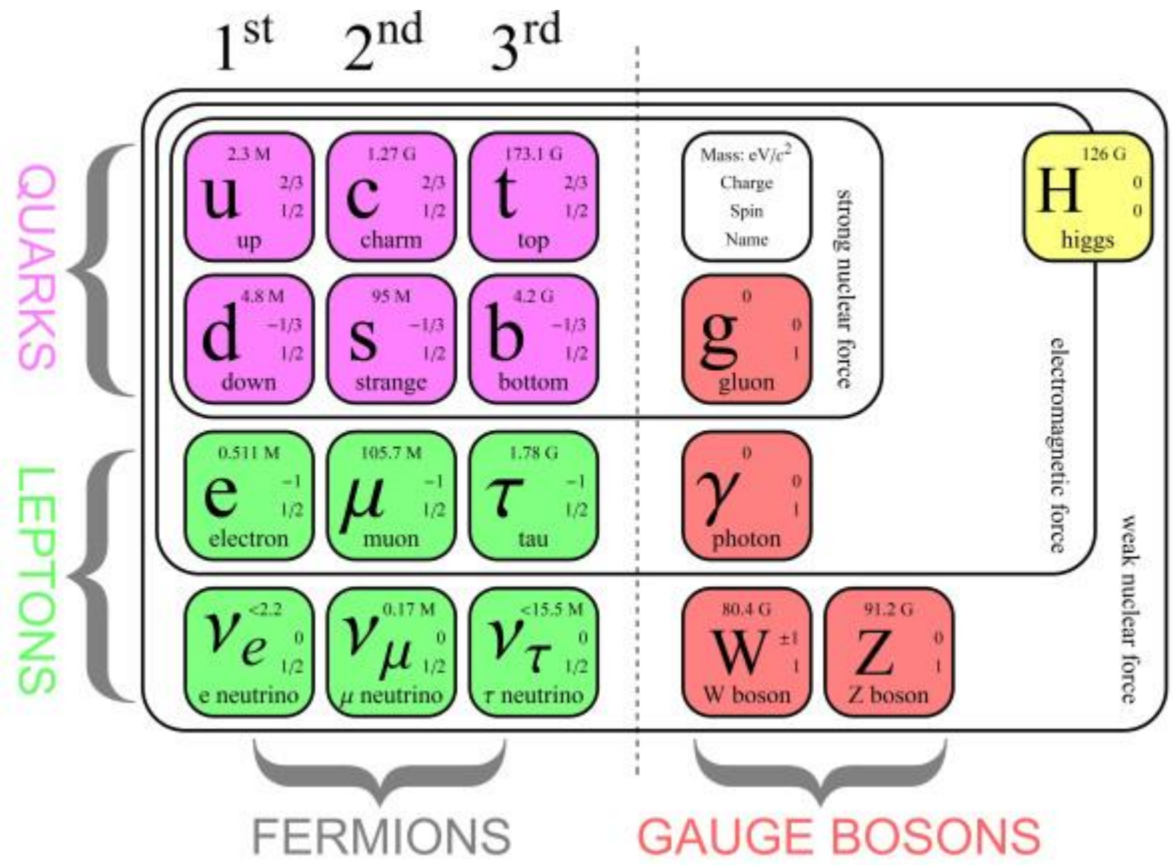
Leptons $\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_e \end{pmatrix} \begin{pmatrix} \tau \\ \nu_e \end{pmatrix}$

Gauge Bosons $\gamma \quad \begin{pmatrix} W^{\pm} \\ Z^{o} \end{pmatrix} \quad \begin{pmatrix} g \\ \cdot \\ g \end{pmatrix}$

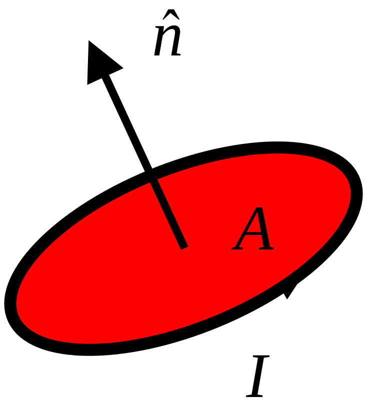
Muon properties

207 × electron mass
"point" particle
decays in 2.2 μs
charged
intrinsic "spin"

Standard Model of Particle Physics

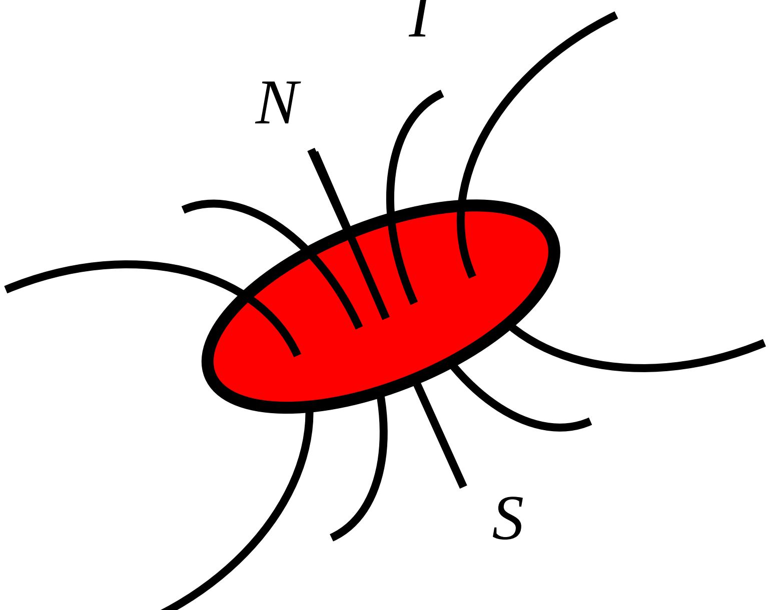


magnetic moment of a current loop



in classical electricity and magnetism
current loop has a magnetic dipole moment

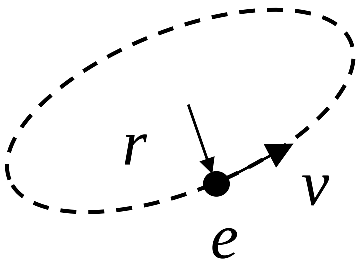
$$\vec{\mu} = I\,A\hat{n}$$



magnetic dipole creates
a magnetic field

current from orbital motion of a charge

magnetic moment proportional
to orbital angular momentum



$$\mu = e\frac{v}{2\pi R}\pi r^2 = \frac{e}{2M}Mvr$$

$$\mu = I\,A$$

$$\vec{L} = M\vec{r}\times\vec{v}$$

$$I = e\frac{v}{2\pi r} \qquad A = \pi r^2$$

$$\vec{\mu} = g_\ell\,\frac{e}{2M}\vec{L} \qquad g_\ell = 1$$

g_ℓ constanct of proportionality is
gyromagnetic ratio or g-factor

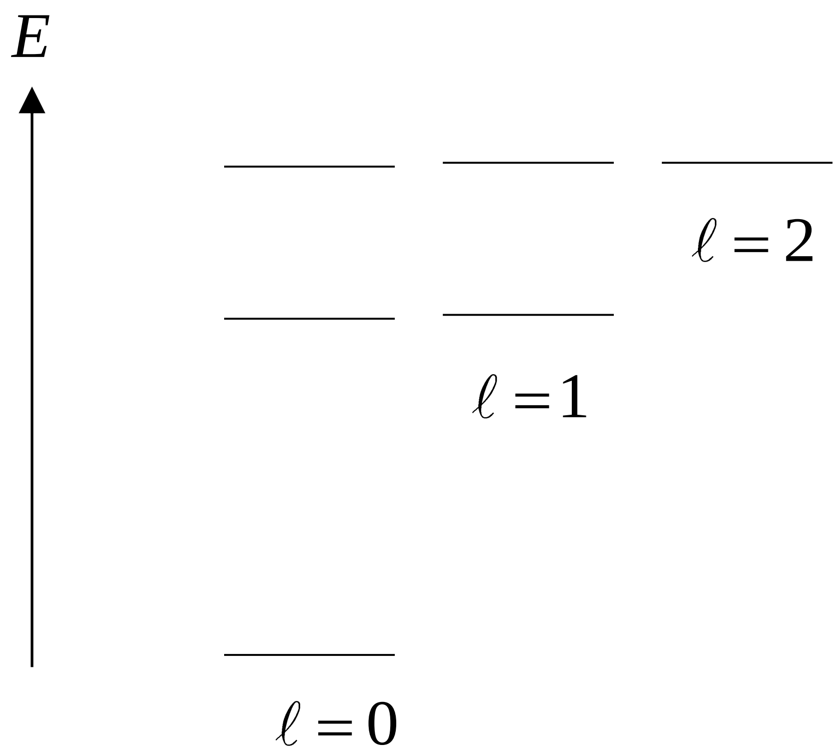
quantization of angular momentum

from atomic structure-- quantum numbers

orbital angular momentum $\ell = 0, 1, 2, \ldots$

azimuthal angular momentum $m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell$

integral quantum numbers



unit of angular momentum

Planck's constant

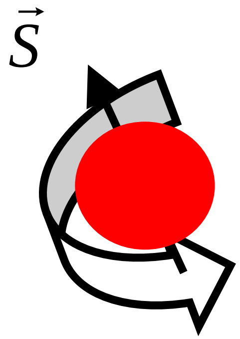
$$\hbar$$

unit of magnetic moment

Bohr magneton

$$\frac{e\hbar}{2M}$$

spin angular momentum and spin magnetic moment



$$s = 1/2$$
$$m_s = \pm 1/2$$

intrinsic spin quantum number is half-integral

$$\vec{\mu} = g_s \frac{e}{2M} \vec{S} \qquad g_s = 2$$

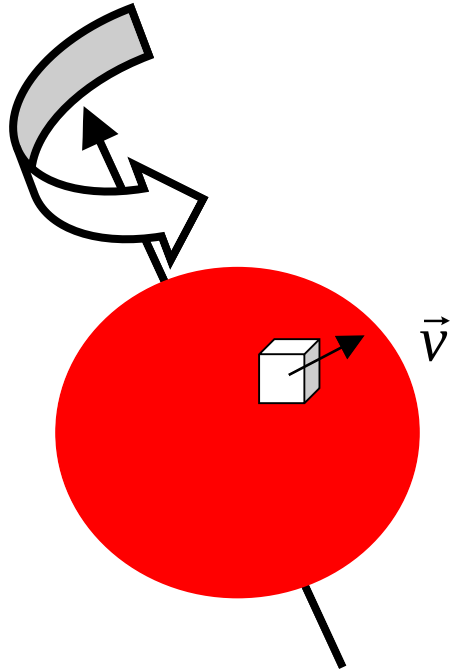
spin g-factor is 2

$$\mu = 2 \frac{e}{2M} \frac{1}{2} \hbar = \frac{e\hbar}{2M}$$

magnetic moment is one Bohr magneton

spin g-factor is beyond classical physics

arbitrary matter/charge distribution gives g = 1



$$\vec{L} = \int \rho_{matter}(\vec{r})\left[\vec{r}\times\vec{v}(\vec{r})\right] dV$$

$$\vec{\mu} = \int \left[\vec{r}\times\vec{J}(\vec{r})\right] dV$$

$$\vec{J}(\vec{r}) = \rho_{charge}(\vec{r})\,\vec{v}(\vec{r})$$

$$\rho_{charge}(\vec{r}) = \frac{e}{M}\rho_{matter}(\vec{r})$$

$$\vec{\mu} = \frac{e}{2M}\vec{L} \quad \Rightarrow \quad g = 1$$

spin g-factor is quantum mechanical

Schrodinger equation

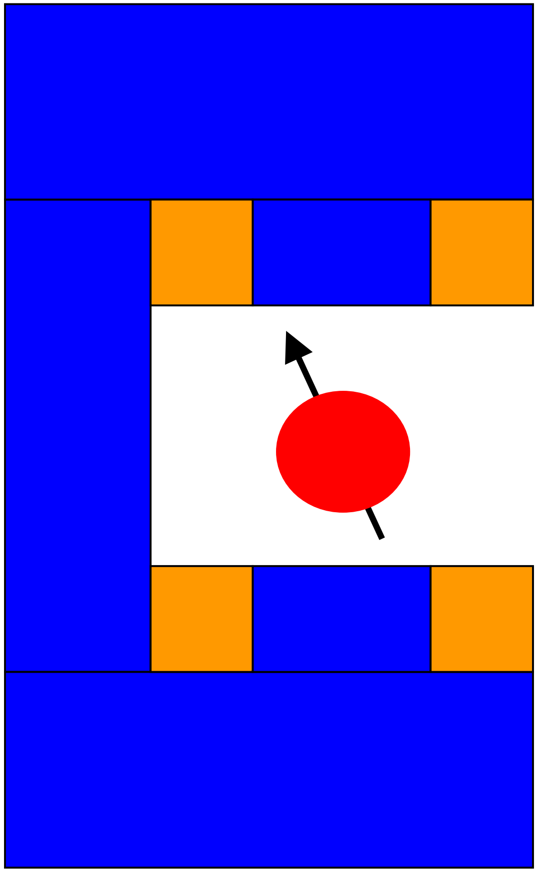
Dirac equation

Field theory

point particle
with spin 1/2
in E and B fields
has g = 2

(these results are not elementary)

magnetic moment in a magnetic field



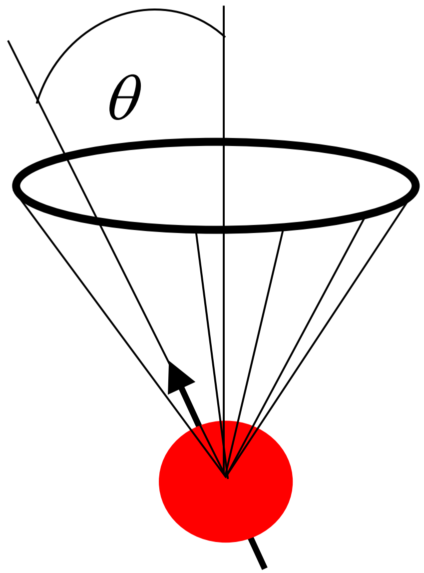
potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$

torque

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

magnetic moment precesses in magnetic field



$$\vec{\tau} = \vec{\mu} \times \vec{B} = \frac{d}{dt}\vec{S}$$

$$\left|\vec{\mu} \times \vec{B}\right| = \left|\vec{\mu}\right|\left|\vec{B}\right|\sin\theta = g_s \frac{e}{2M}\left|\vec{S}\right|\left|\vec{B}\right|\sin\theta$$

$$\left|\frac{d}{dt}\vec{S}\right| = \omega_s \sin\theta\left|\vec{S}\right|$$

$$\omega_s = g_s \frac{e}{2M} B$$

precession (angular) frequency proportional to B and g_s

measure B and ω_s to determine g_s

magnetic moments and g-factors of selected particles

$$\mu = g_s \frac{e}{2M} S = g_s \frac{e\hbar}{2M} \frac{S}{\hbar}$$

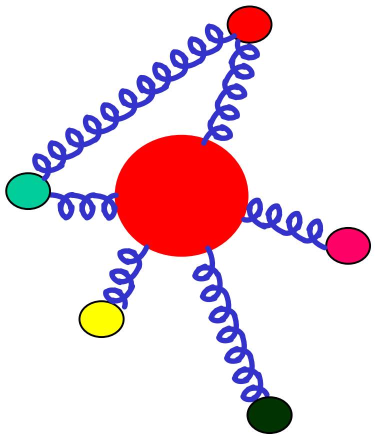
electron	$g_e = 2\times 1.001$	"point" particles
muon	$g_\mu = 2\times 1.001$	
proton	$g_p = 2\times 2.79$	composite particles
neutron	$g_n = 2\times -1.91$	

g-factor anomaly

anomaly defined $$a \equiv \frac{g-2}{2}$$

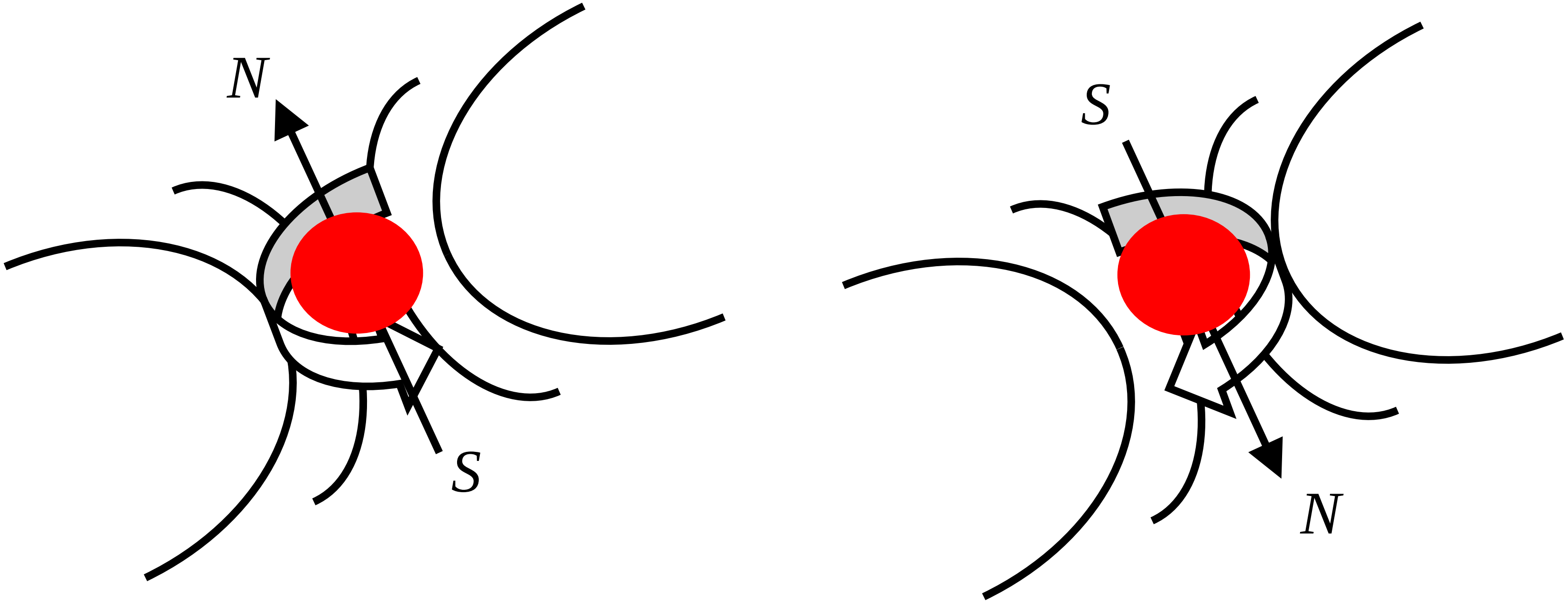
anomaly explained

all particles are
surrounded by
virtual particles
and fields

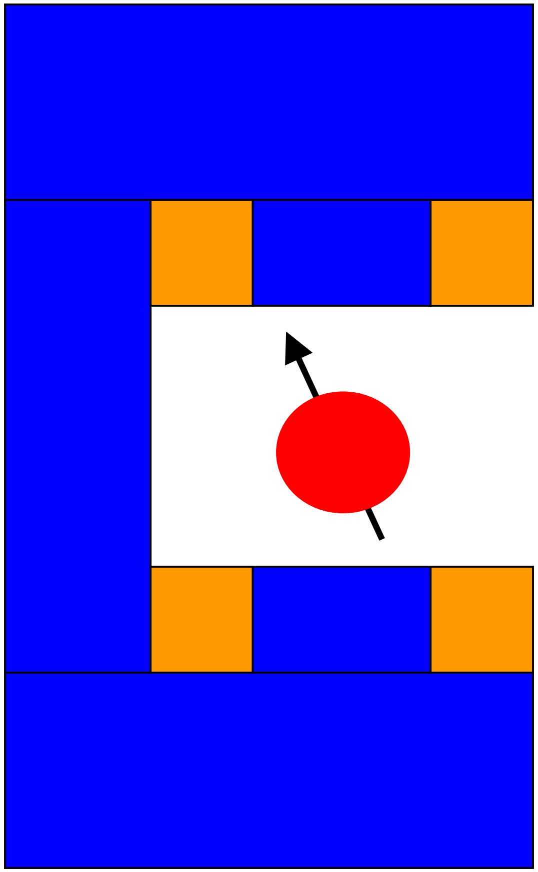


(this figure is a cartoon)

spin direction is quantized

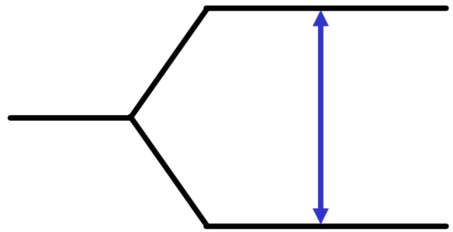


g - 2 > 0



potential energy

$$U = -\vec{\mu} \cdot \vec{B}$$



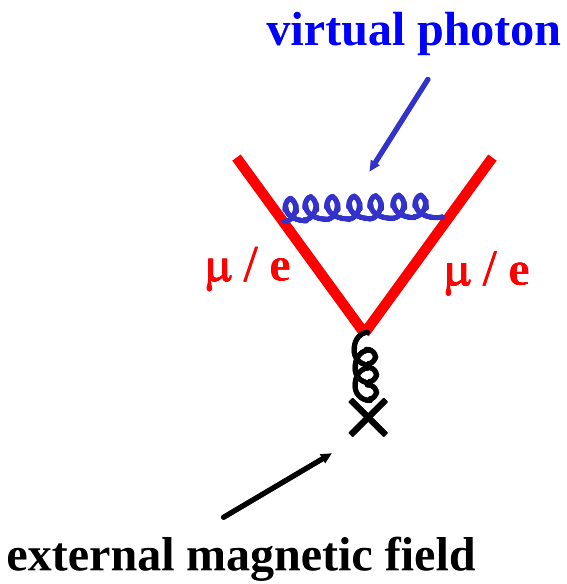
$$\Delta E = 2\mu B = 2\frac{g}{2}\frac{e}{2M}B$$

add perturbation of virtual fields and particles

with perturbation energy levels repel so g must increase

g-factor anomaly

(this figure is a Feynman diagram)



lowest order contribution
Schwinger (1947)

$$a = \frac{1}{2\pi}\frac{e^2}{\hbar c} \approx \frac{1}{800} \approx 0.001$$

fine structure constant $\alpha \equiv \frac{e^2}{\hbar c} \approx \frac{1}{137}$

Standard Model contributions to g-factor anomaly
QED + hadronic + weak

Interaction	Field	Particles
QED	photons	$e^+ \ e^- \ \mu^+ \ \mu^- \ etc$
strong	gluons	$\pi^+ \ \pi^- \ \pi^o \ \ quarks \ etc$
weak	$W^+ \ W^- \ Z$	$e^+ \ e^- \ \mu^+ \ \mu^- \ etc$ $\nu_e \ \overline{\nu}_e \ \nu_\mu \ \overline{\nu}_\mu$

measure the g-factor anomaly to search for new physics

$$\boxed{a^{\text{"NEW"}}} = a^{\text{experiment}} - a^{\text{theory}}$$

QED + WEAK + HADRONIC

What is the sensitivity to new physics?
How accurate is the experiment?
How accurate is the theory?

muon anomaly versus electron anomaly

- Electron is stable and common

- Electron anomaly ha been measured to 4 parts in 1,000,000,000

- Hadronic contribution only 2 ppb

- Weak contribution only 0.03 ppb

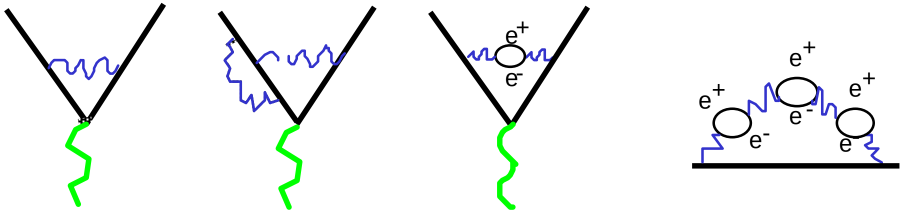
- Coupling to virtual particles is $\propto \left(\frac{M_\ell}{M_X} \right)^2 \quad \ell = e / \mu$

- So, muon anomaly is 40,000 more sensitive

Standard Model calculation of muon anomaly - QED

QED contribution is well known and understood
dominant contribution 99.9930% of anomaly

a(QED)=11 658 470.57(0.29)$\times 10^{-10}$ (0.025 ppm)

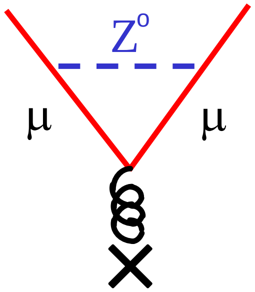


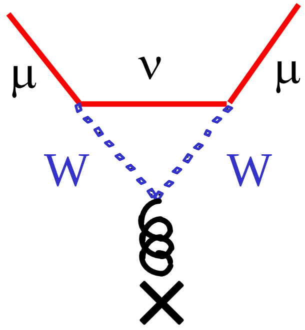
representative diagrams--hundreds more

Standard Model calculation of muon anomaly - Weak

Weak contribution is also well known and understood
1.3 parts per million contribution

$a(\text{Weak})=15.1(0.4)\times 10^{-10}$ (0.03 ppm)



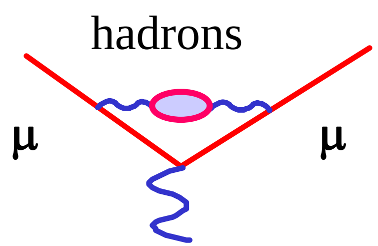


representative diagrams--many more

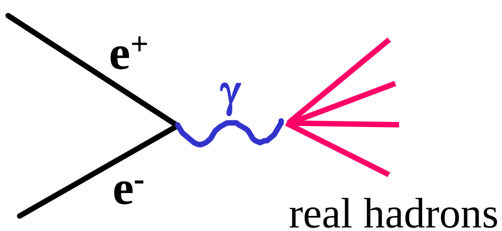
Standard Model calculation of muon anomaly - Hadronic

Hadronic contribution cannot be calculated from QCD (yet)

dominant contribution

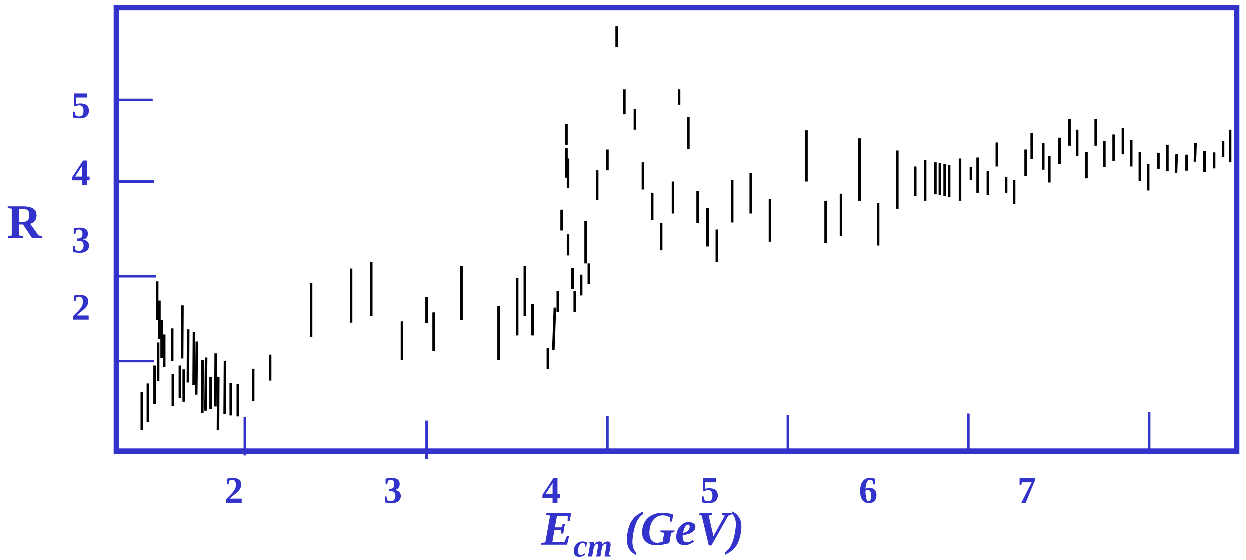


theory needs



available from experiment

Standard Model calculation of muon anomaly - Hadronic

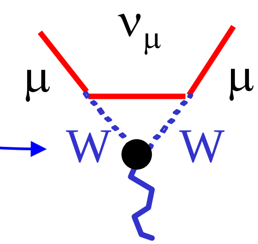


$$\sigma_H(s) \text{ obtained by: } R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

$$\Delta a_\mu^{had} = \left(\frac{1}{4\pi^3}\right) \int_{4m_\pi^2}^{\infty} \sigma_H(s) K(s) ds$$

Speculations of physics beyond the Standard Model

•W, Z structure

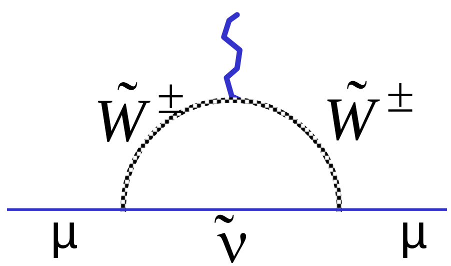


$$\Delta\left(\frac{g_W - 2}{2}\right) \approx 0.02$$

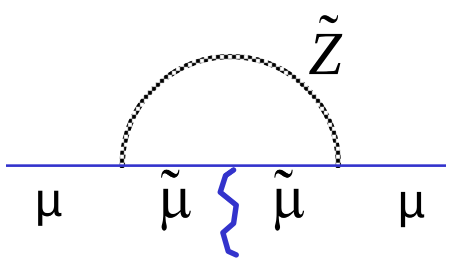
•Muon substructure

$$a(\Lambda) \approx \frac{M^2}{\Lambda^2} \quad \Lambda \approx 4\,\text{Tev}$$

•Supersymmetry



chargino-sneutrino one-loop



neutralino-smuons one-loop

Elements of g-2 experiment

- **Polarized muons**

  pion decay produces polarized muons
- **Precession gives (g-2)**

  in a storage ring the spin precesses relative to the momentum at a rate $\propto$ (g-2)
- **Parity violation**

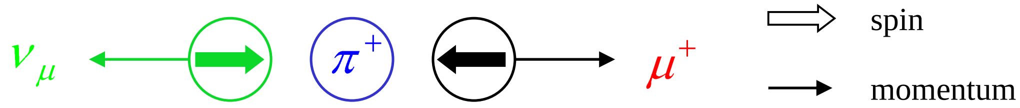
  positron energy indicates the direction of the muon spin
- **P_μ The magic momentum**

  at one special momentum, the precession is independent of the electric focussing fields

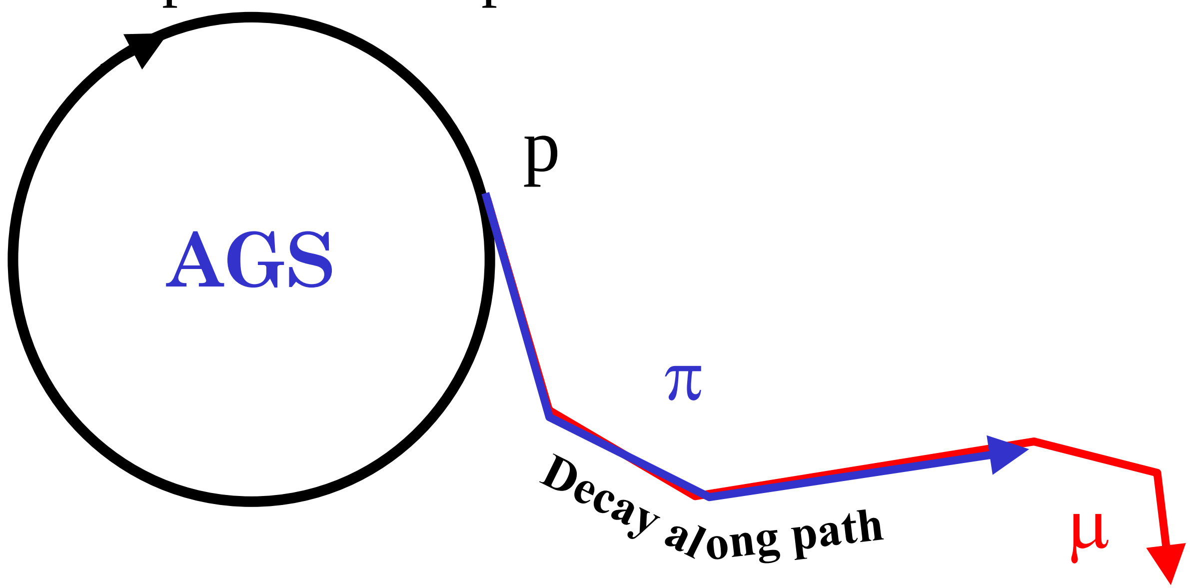
Polarized Muons

- Pion decay is the source of polarized muons

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

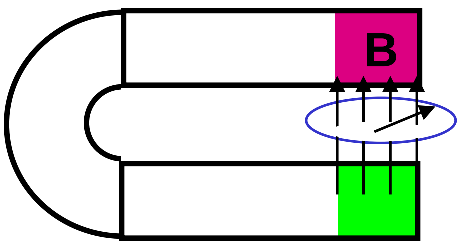


- Pions are produced in proton nucleus collisions

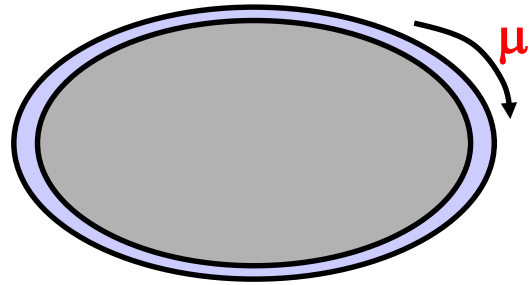


Precession in Uniform Field of Storage Ring

at rest



in flight



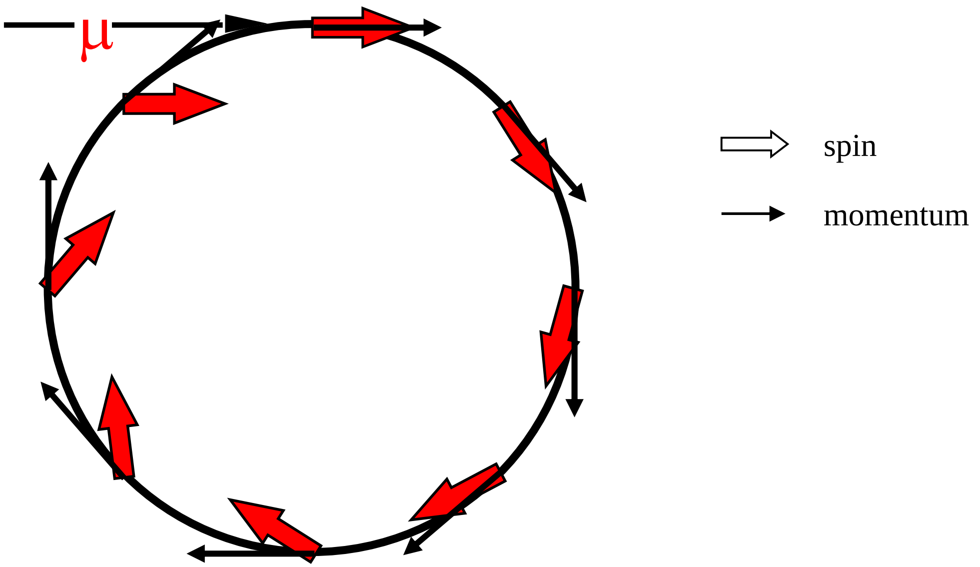
$$\omega_s = \frac{g}{2}\frac{e}{M}B$$

$$\omega_s = \left[1 + \gamma\frac{g-2}{2}\right]\frac{e}{m}\frac{1}{\gamma}B$$

cyclotron frequency

$$\omega_c = \frac{e}{M}\frac{1}{\gamma}B$$

Difference Frequency ω_a



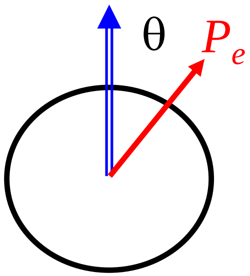
simulation

$$\omega_a = \omega_s - \omega_c = \frac{g-2}{2} \frac{e}{M} B$$

difference frequency ∝ g-2, not g, independent of γ !

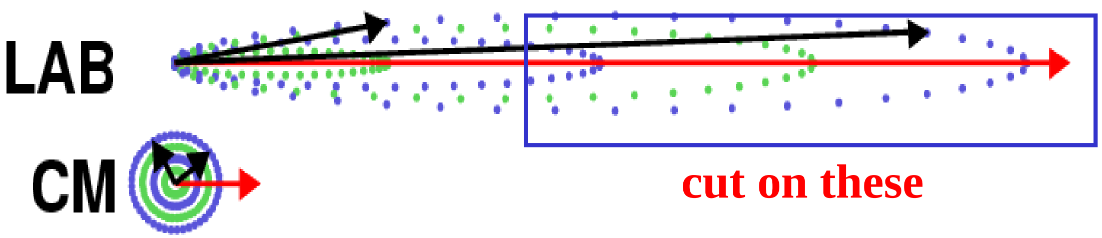
Measuring the Spin Direction

$$\mu^+ \rightarrow \nu_e + \bar{\nu}_\mu + e^+$$



In the COM: $$\frac{dN}{dE\,d\Omega} = n(E)\left(1 + A(E)\cos\theta\right)$$

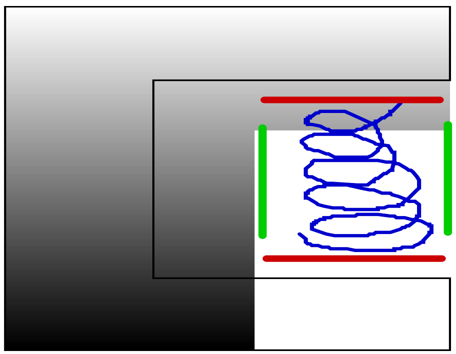
High energy positrons in the LAB frame are emitted at forward angles in the CM frame



We measure

$$N(t) = N e^{-t/\gamma\tau}\left(1 + A\cos\omega_a t\right)$$

Storing the Muons
The Magic Momentum



Problem: Orbiting particles hit the top or bottom

Solution: Build a trap with electric quadrupoles

Moving muons “see” electric field like another magnetic field…and their spins react...

$$\vec{\omega}_a = \frac{e}{M}\left[a\,\vec{B} - \left(a - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{v}}{c^2} \times \vec{E} \right]$$

$\gamma = 29.3$

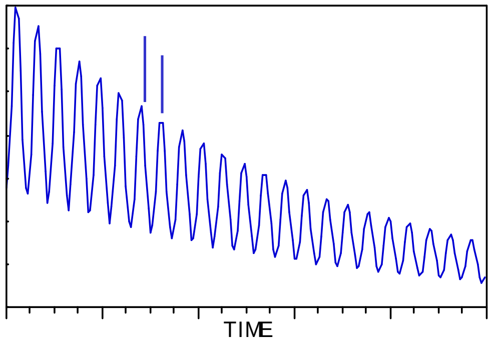
$\tau = 64\ \mu s$

vanishes at 3.094 GeV/*c*

Getting the Answer

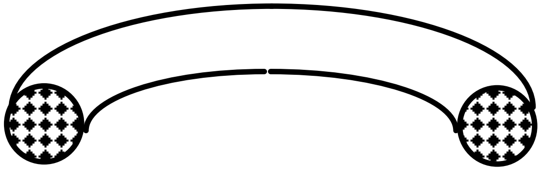
Precession frequency

ω_a



Magnetic field map

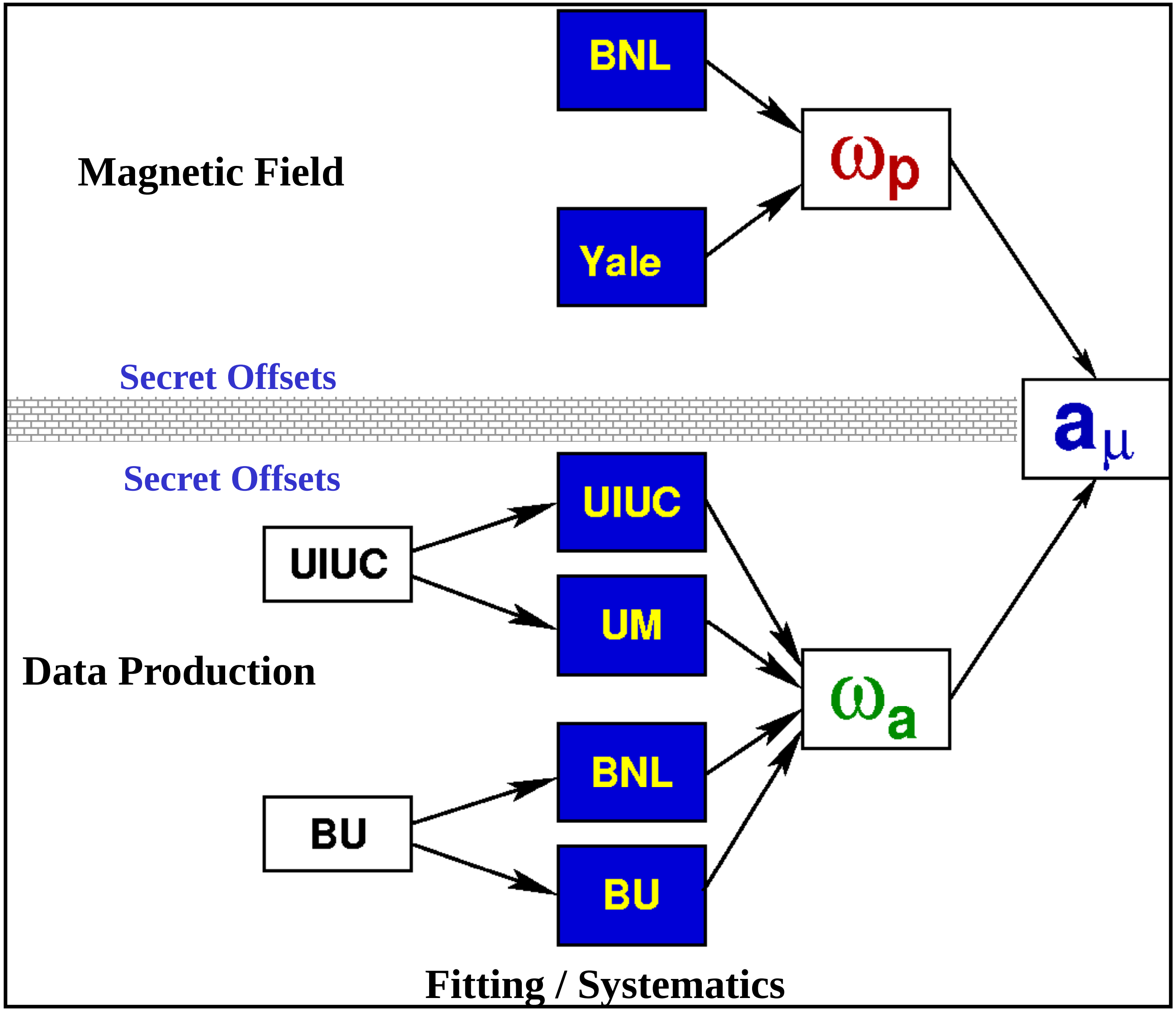
ω_p



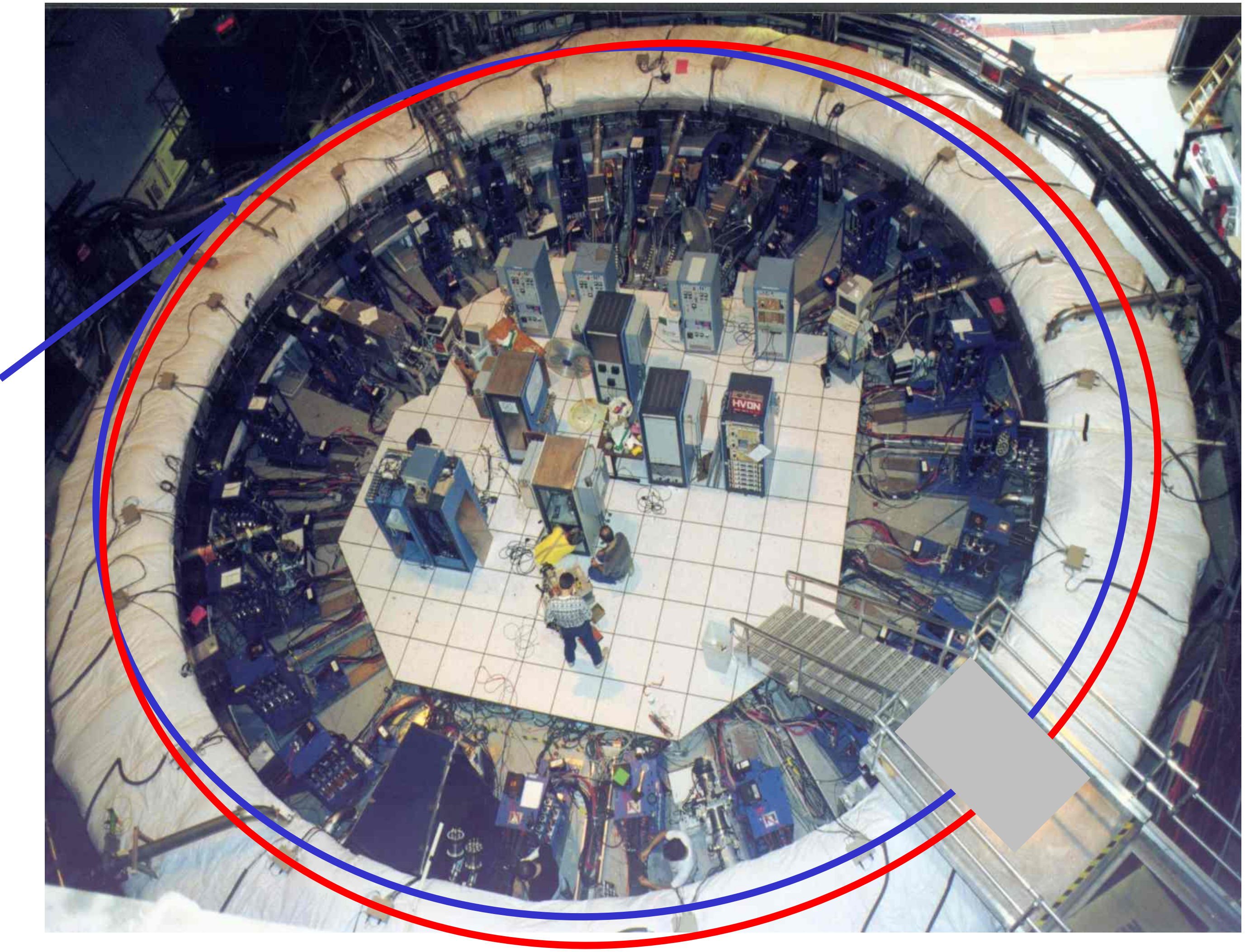
$$B = \frac{\hbar \omega_p}{2\mu_p}$$

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu} B}$$

BNL E821 Analysis Strategy

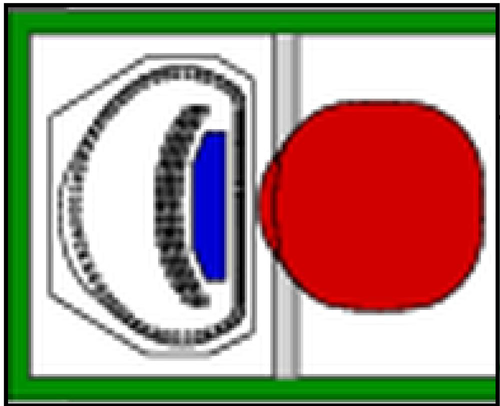


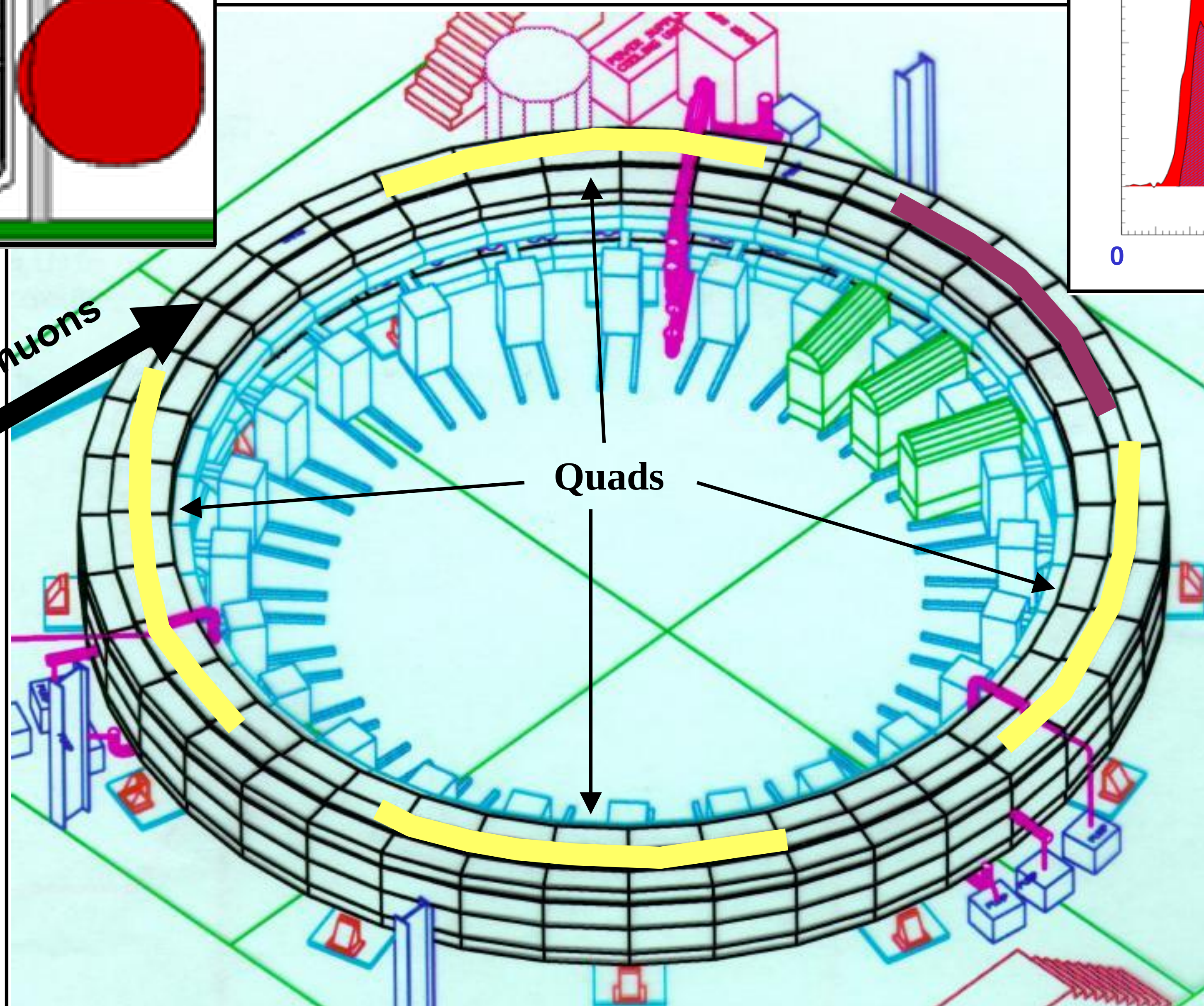
Storage Ring / Kicker

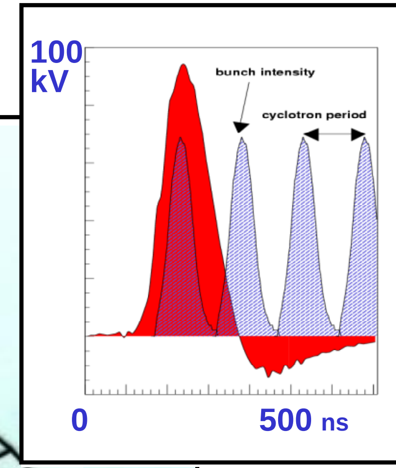


Radius	7112 mm
Aperture	90 mm
Field	1.45 T
P_m	3.094 GeV/*c*

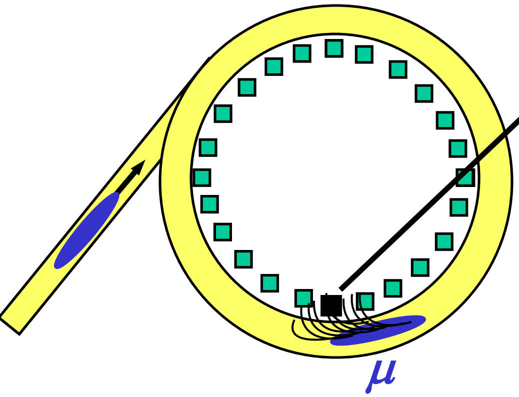
BNL Storage Ring

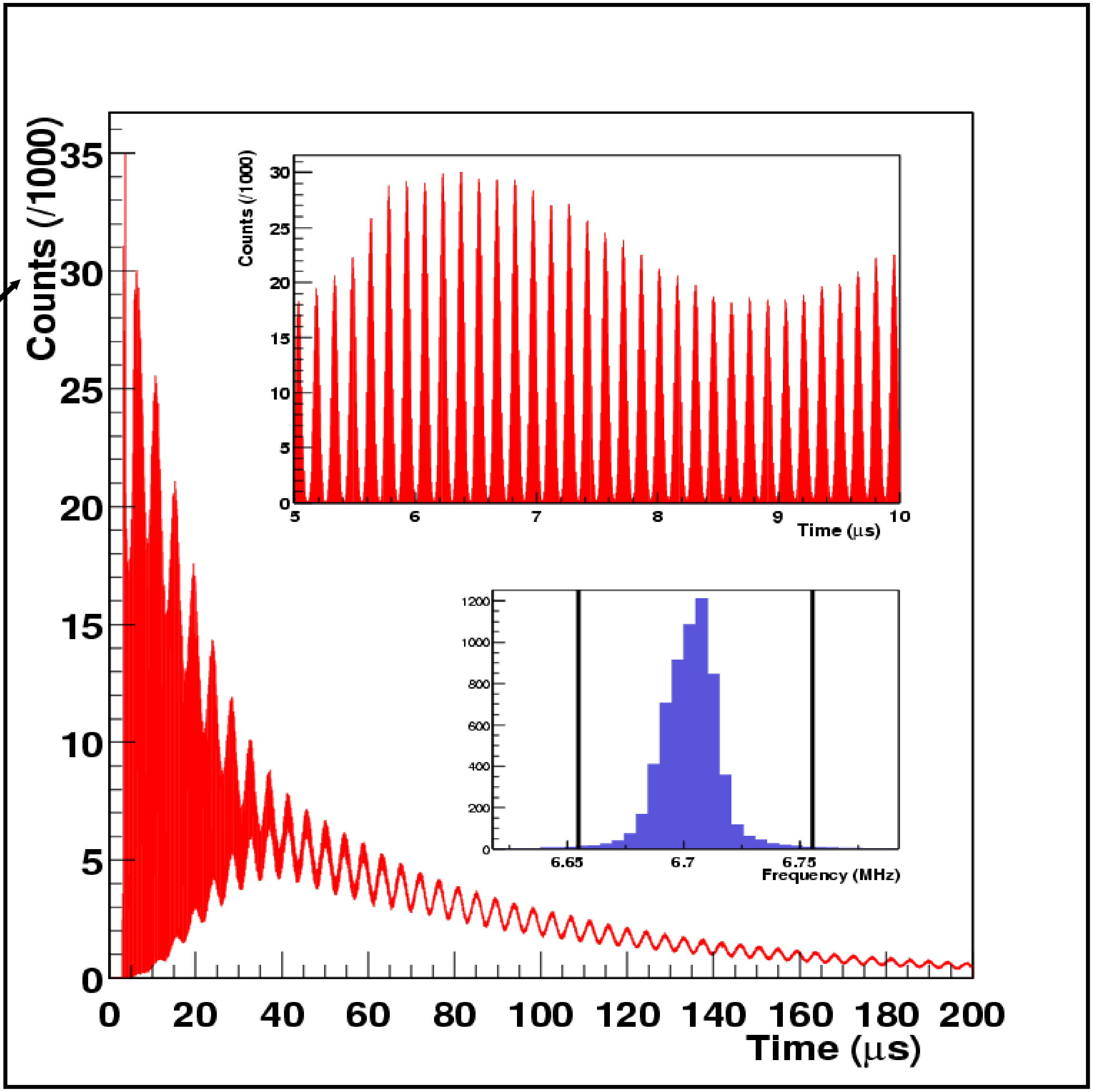




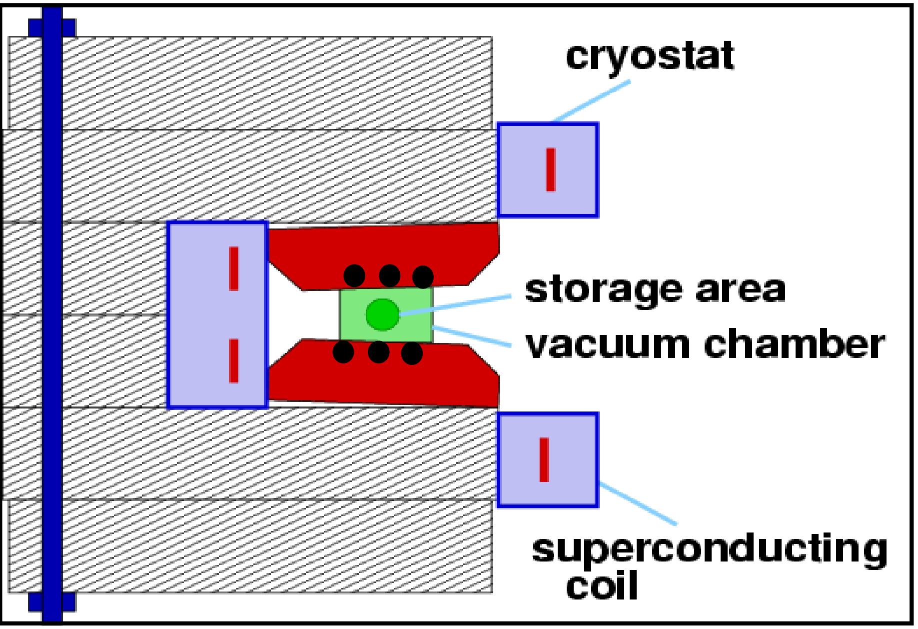


Fast Rotation

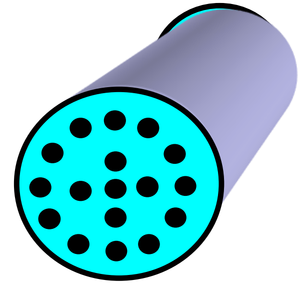




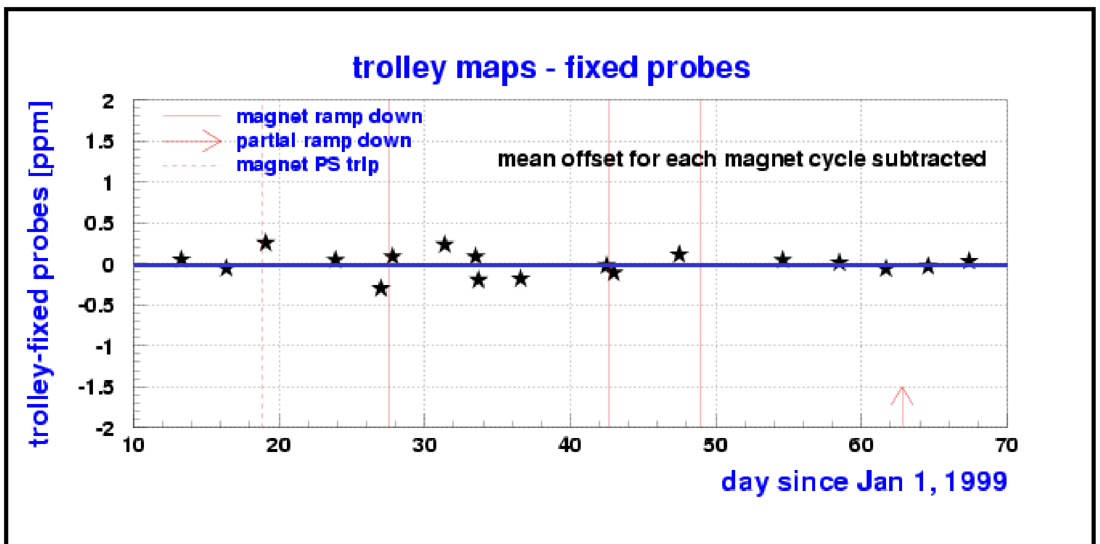
Magnetic Field



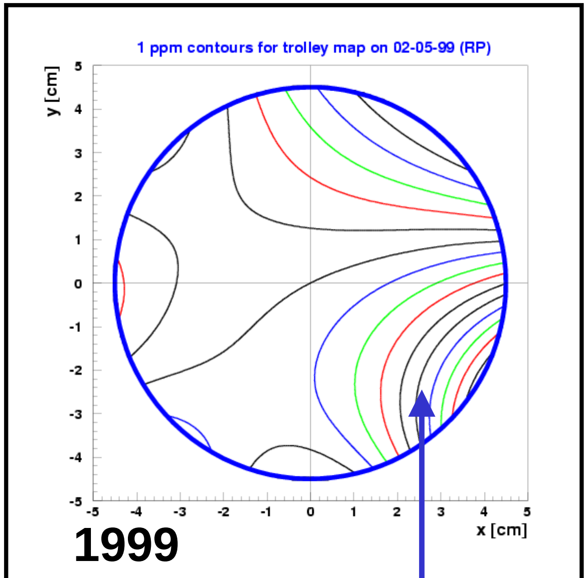
Measured *in situ* using an NMR trolley

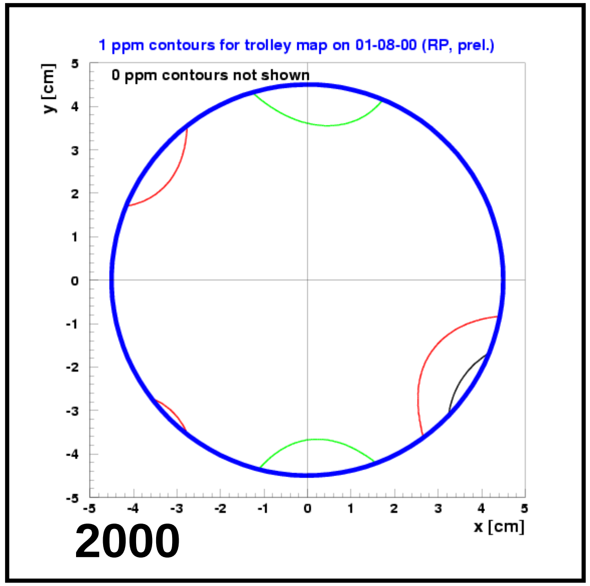


Continuously monitored with > 360 fixed probes mounted above and below the storage region

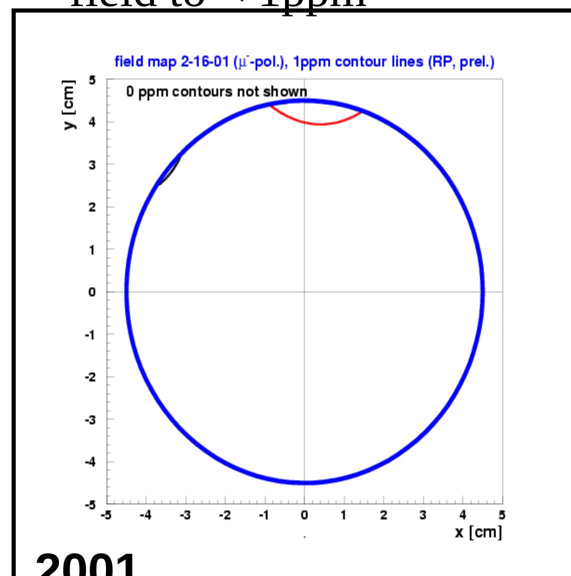


Field Contours Averaged around Ring





Regardless of muon orbit, all muons see the same field to < 1ppm



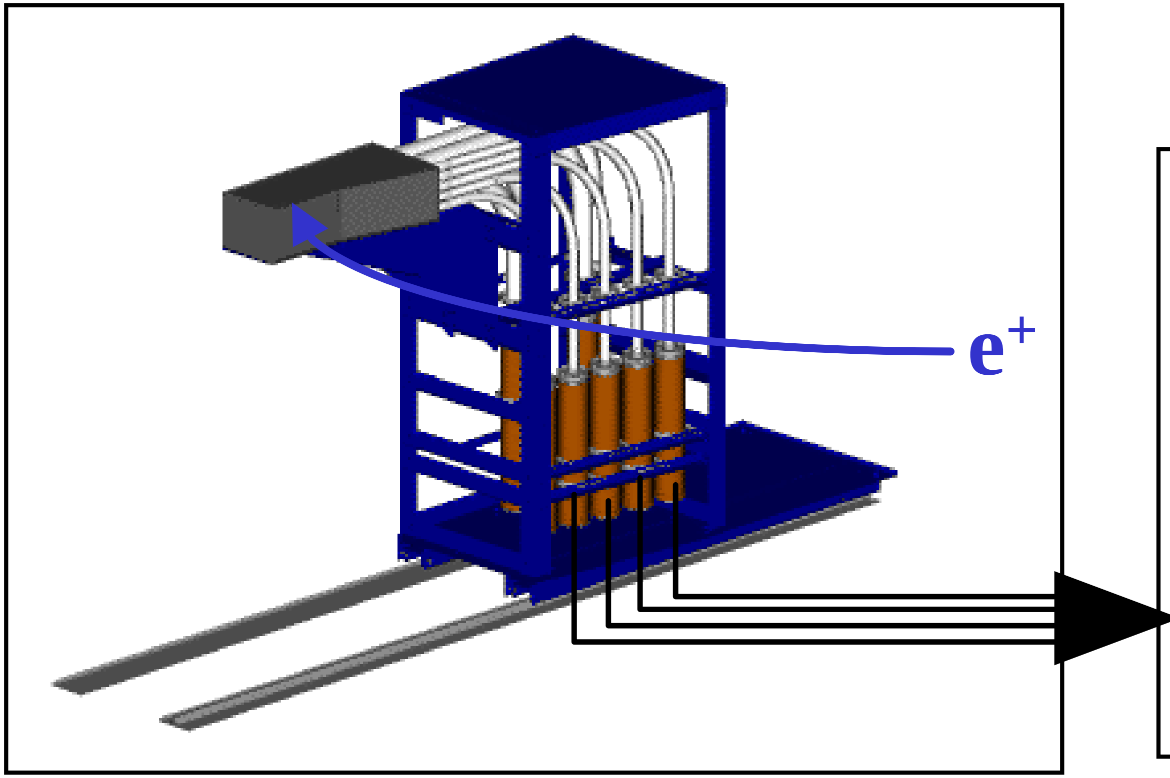
inflector shield problem

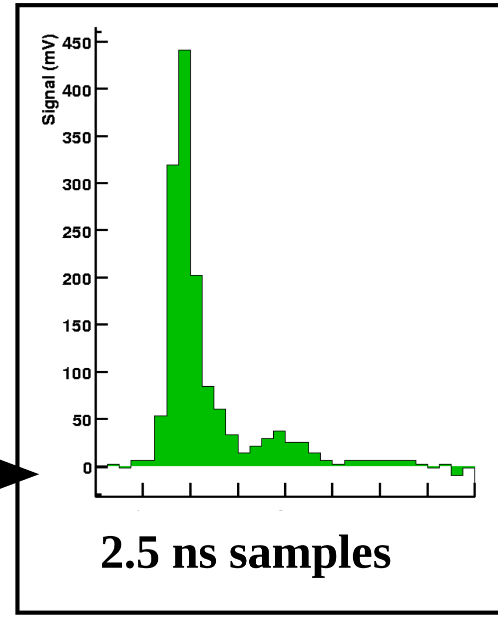
Systematic Errors for “ω_p”

		Size [ppm]
1)	absolute calibration	0.05
2)	trolley probe calibration	0.15
3)	trolley measurement of B_0	0.10
4)	interpolation with fixed probe	0.10
5)	muon distribution	0.03
6)	others	0.15

Total Systematic Error $\delta\omega_p = 0.24$ ppm

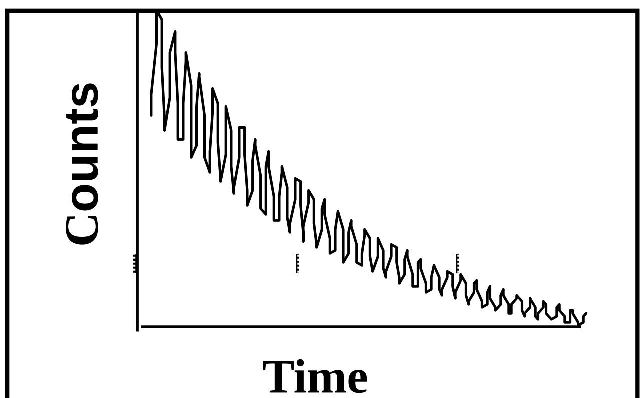
Measuring the difference frequency "ω_a"



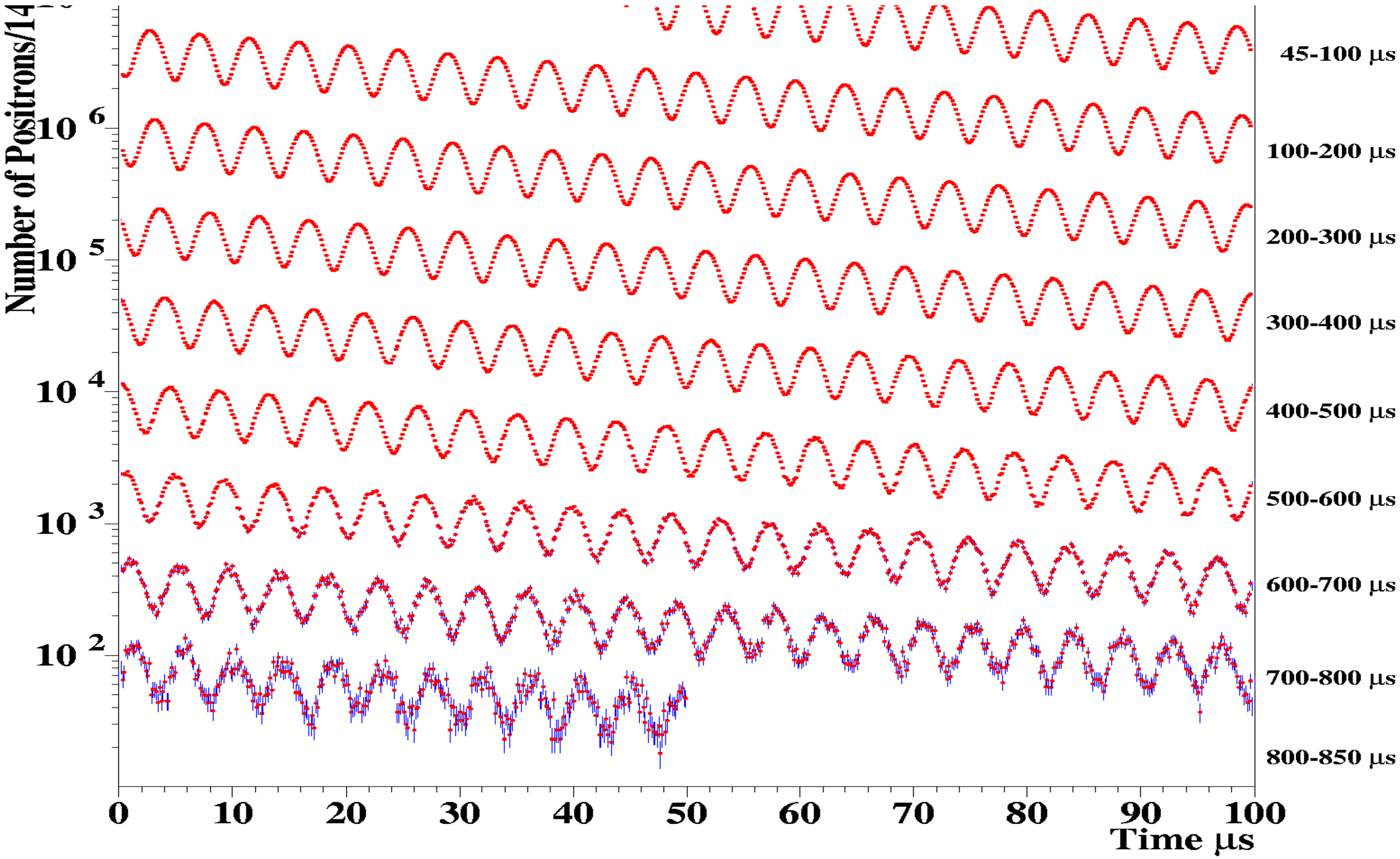


< 20 ps shifts

< 0.1% gain change

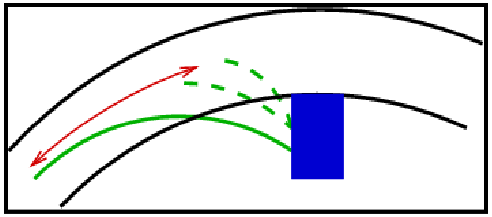


4,000,000,000 e^+ with E > 2 GeV



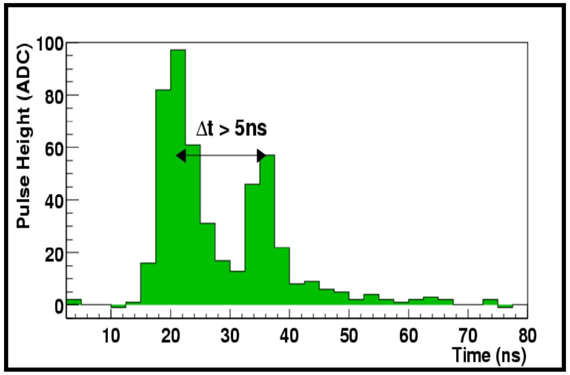
$$N(t) = Ne^{-t/\gamma\tau}\left(1 + A\cos\omega_a t\right)$$

One Analysis Challenge: Pileup Subtraction

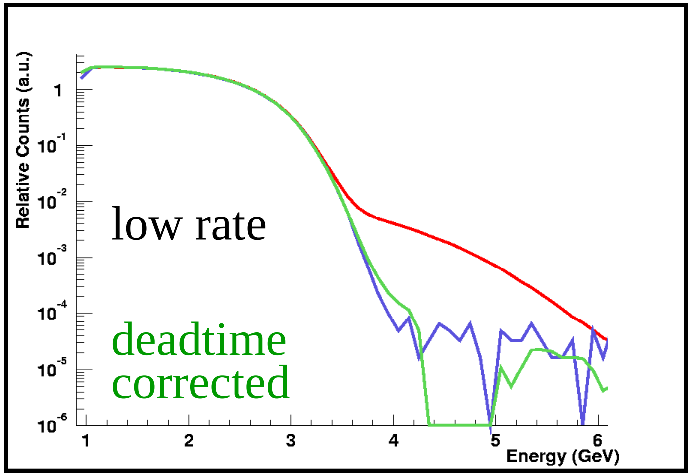


Phase shift can be seen here

Two are close; we can still separate these



With two software deadtimes, we extrapolate to zero deadtime and make a pileup-free histogram



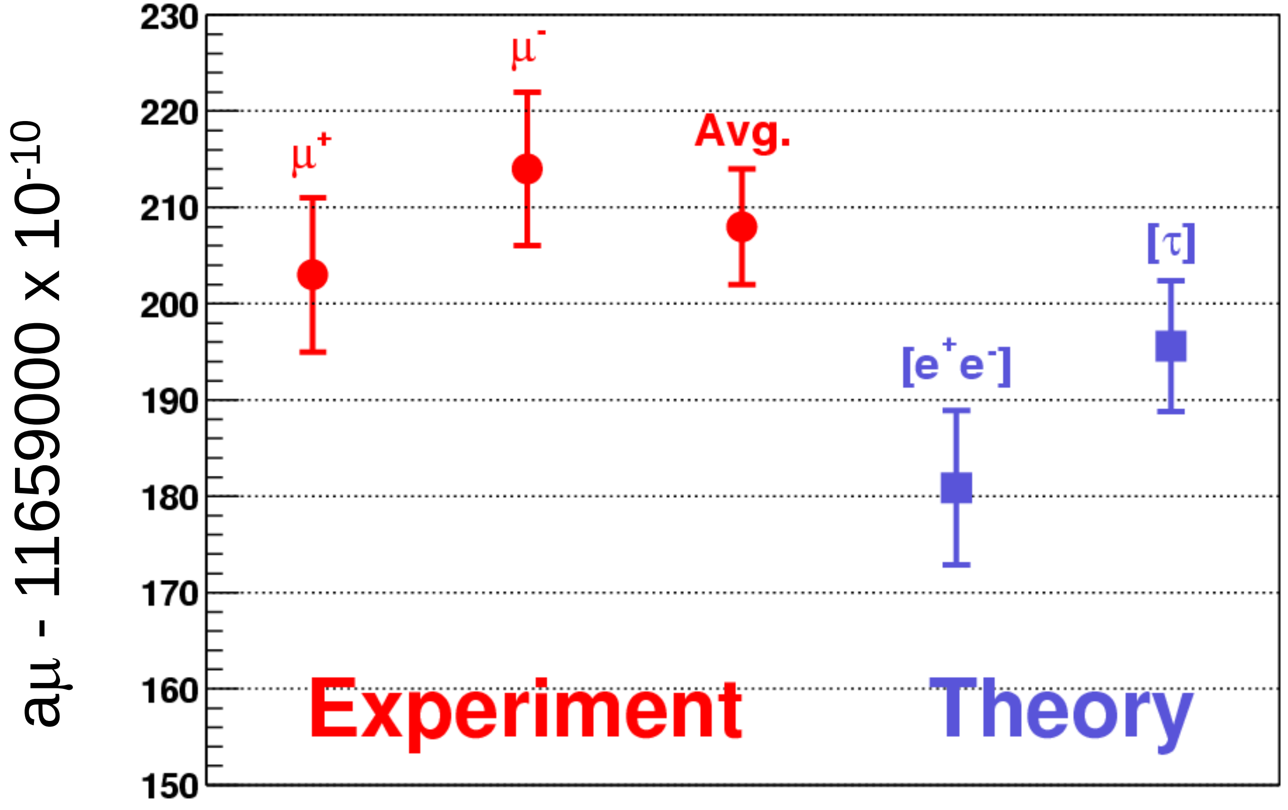
Energy Spectrum

Systematic Errors for "ω_a"

	Size [ppm]
1) coherent betatron oscillations	0.21
2) pileup	0.13
3) gain changes	0.13
4) lost muons	0.10
5) binning and fitting procedure	0.06
6) others	0.06

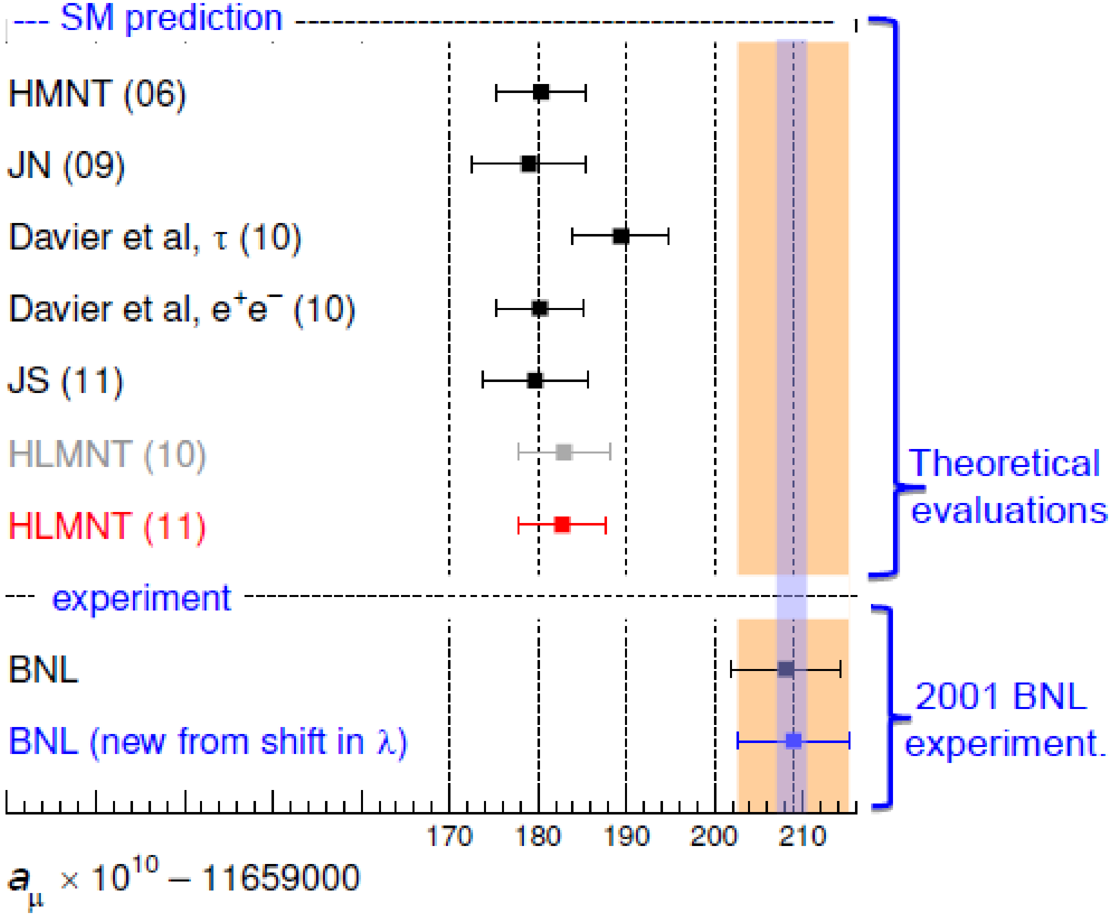
Total Systematic Error $\delta\omega_a = 0.31$ ppm

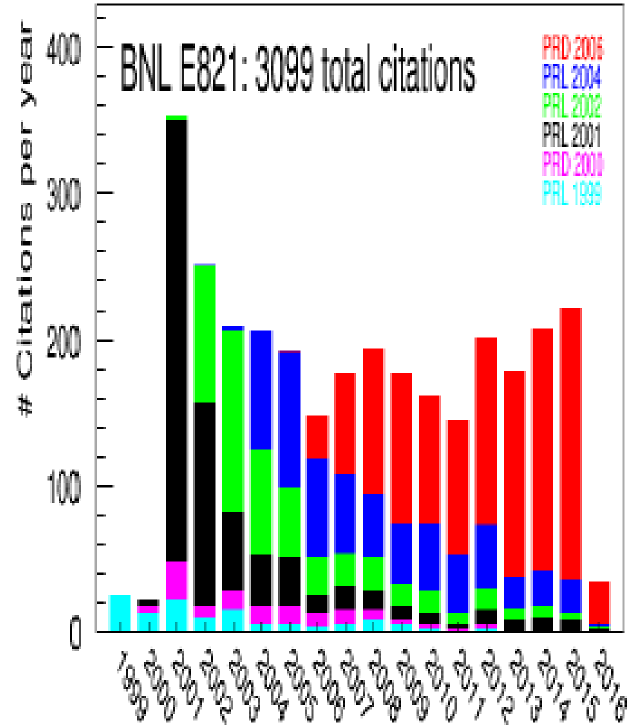
Results from the 2000/2001 runs & World Average



World average a_μ = 11659208(6) x 10^{-10}

Muon anomalous magnetic moment, a_μ.





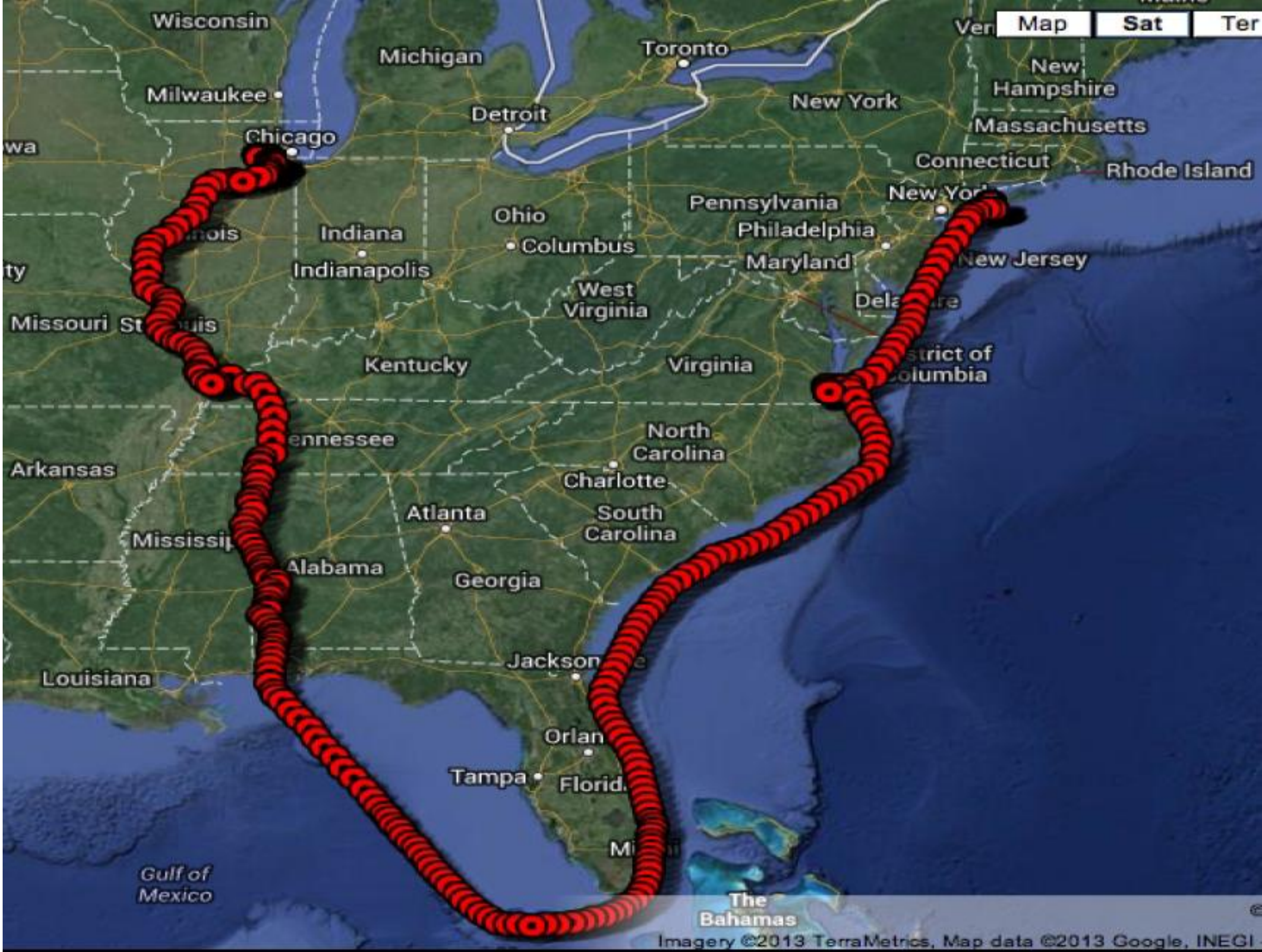
a_μ^{SM} = 116 591 802 (49) x 10^{-11} **(0.42 ppm)**

a_μ^{EX} = 116 592 089 (63) x 10^{-11} **(0.54 ppm)**

- longstanding 3.5σ discrepancy with standard model prediction.
- goal of FNAL g-2 expt to reduce the experimental uncertainty by fourfold.

The Big Move from Brookhaven to Fermilab

http://muon-g-2.fnal.gov/bigmove/gallery.shtml



Map
Sat
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Wisconsin
Michigan
Toronto
Milwaukee
Detroit
New York
New Hampshire
Massachusetts
Chicago
Connecticut
Rhode Island
Pennsylvania
Ohio
Indiana
Philadelphia
Columbus
Maryland
New Jersey
Indianapolis
West Virginia
Missouri
Kentucky
Virginia
North Carolina
Arkansas
Charlotte
Atlanta
South Carolina
Alabama
Georgia
Louisiana
Tampa
Gulf of Mexico
The Bahamas
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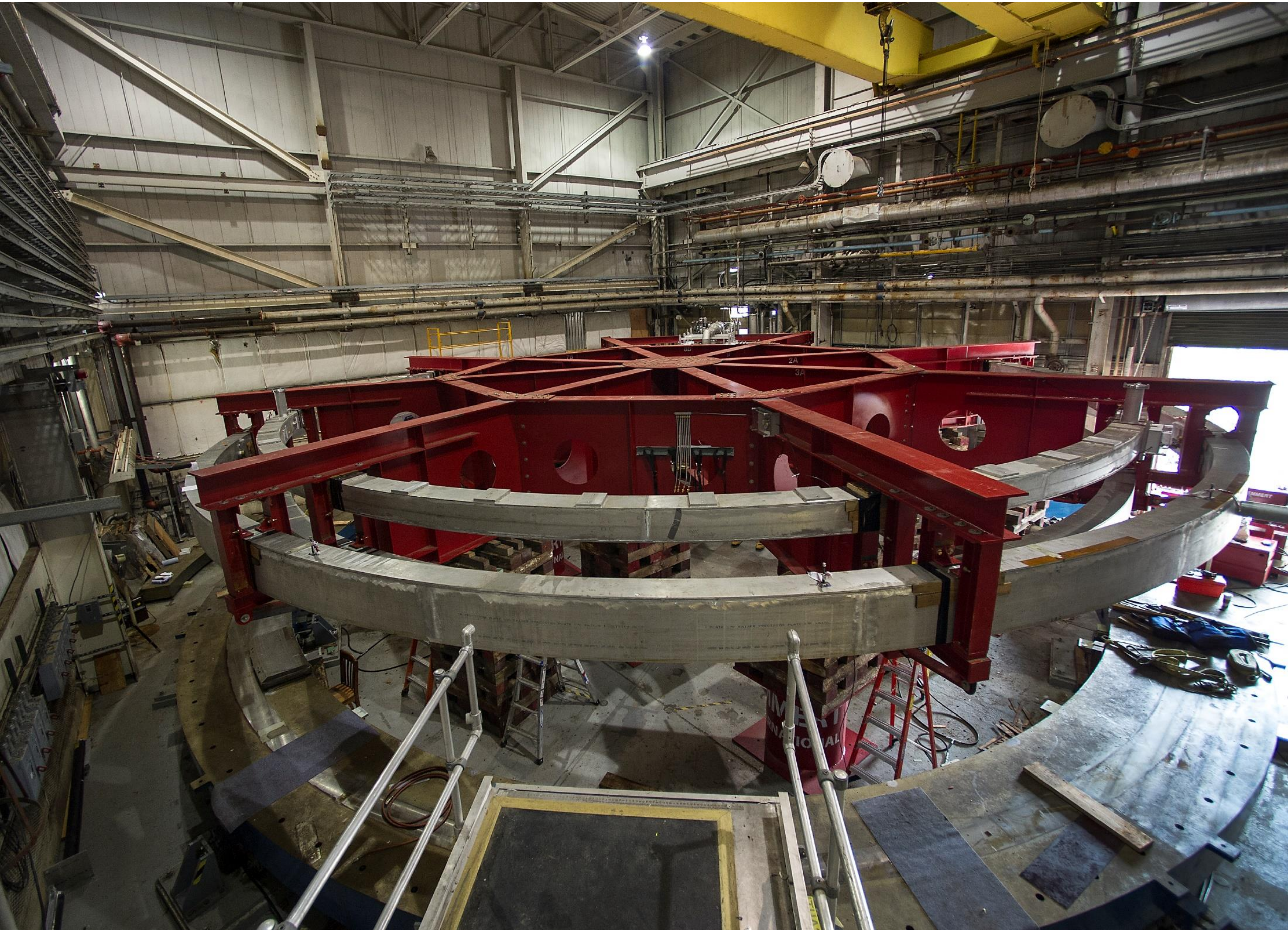


Muon g-2
g-2
http://muon-g-2.fnal.gov
BROOKHAVEN


OVERSIZE LOAD


MARQUE
EMMERT


OVERSIZE LOAD
EMMERT
Driving Discovery in Particle Physics


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INTERNATIONAL


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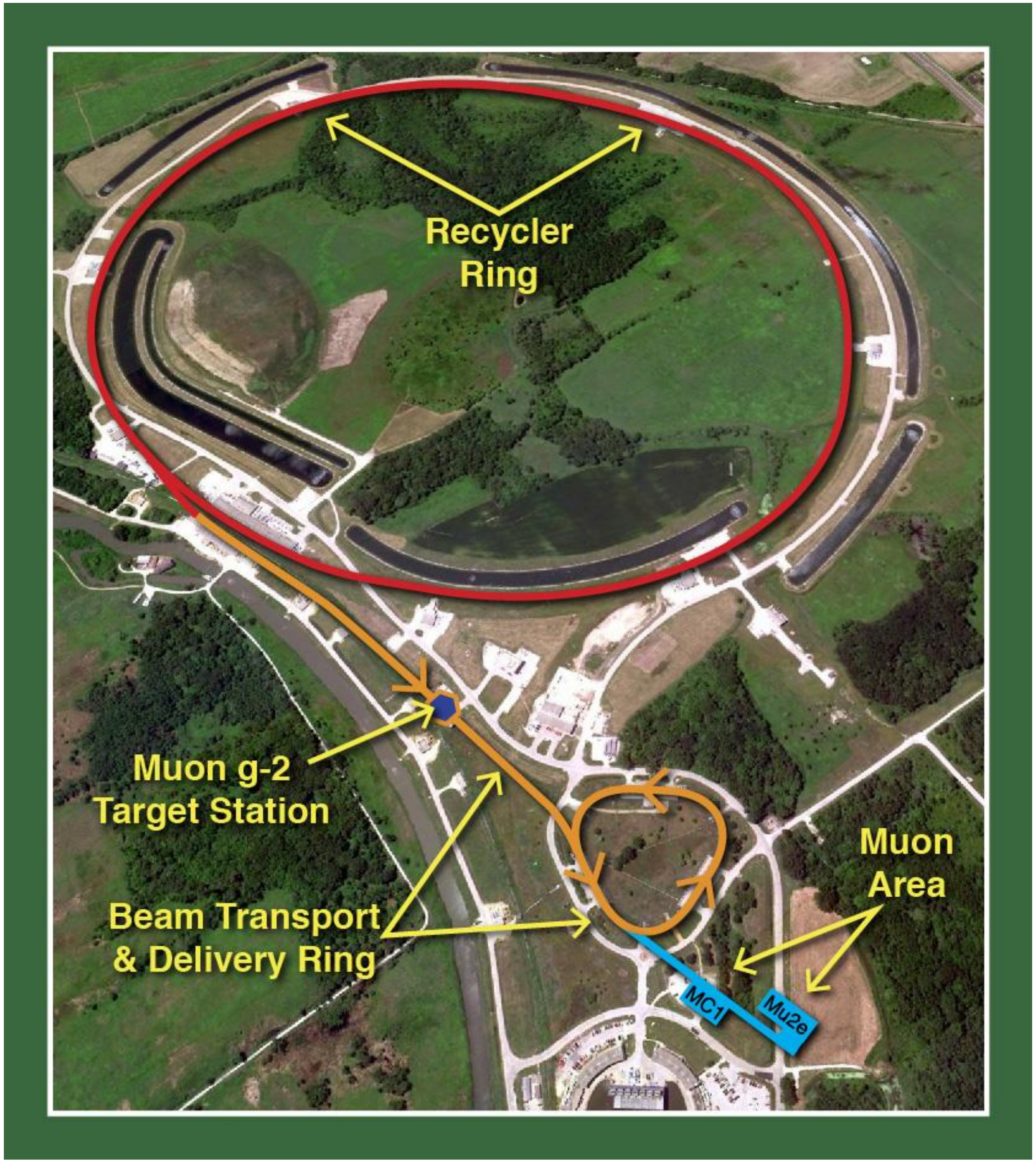
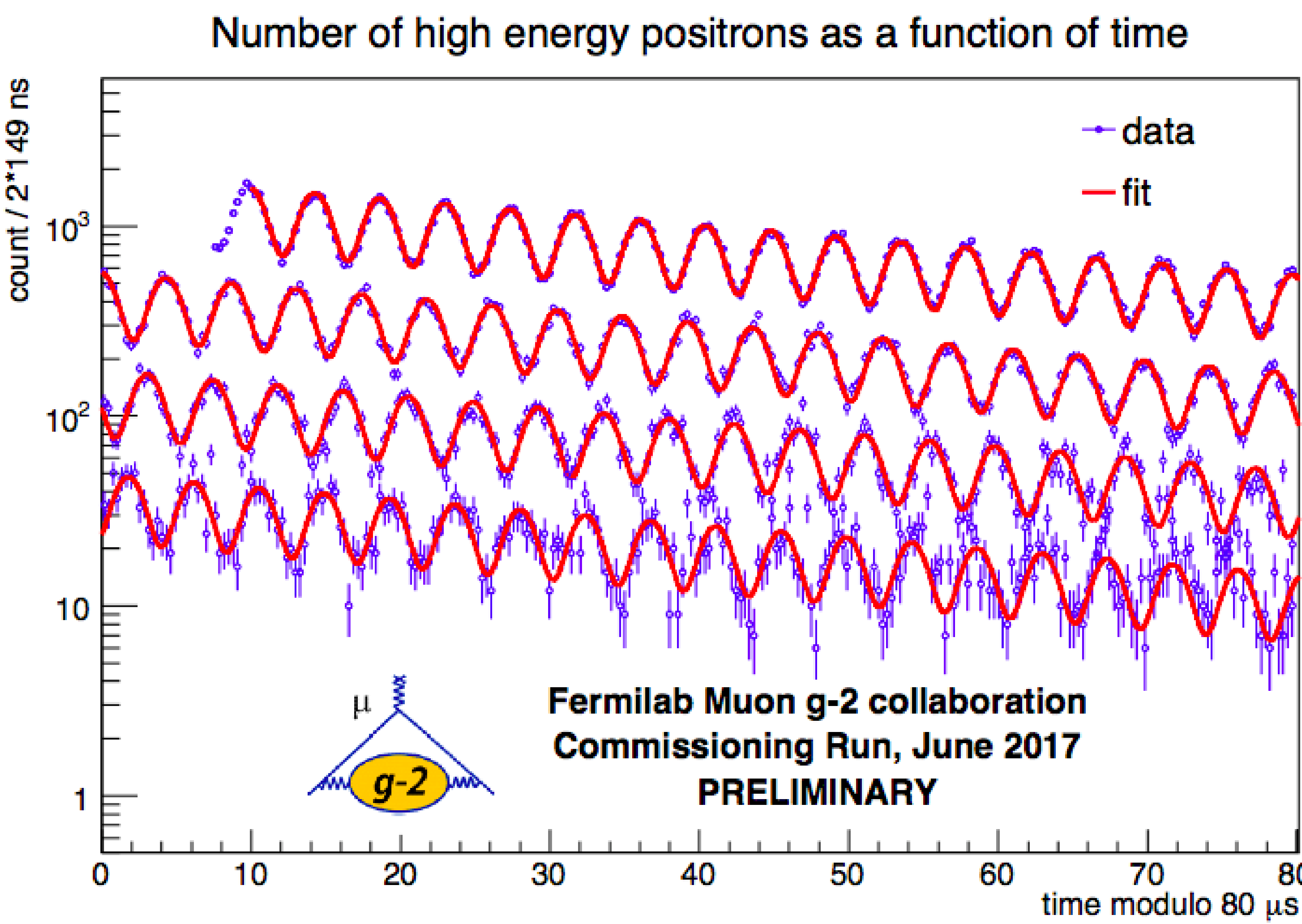

Recycler
Ring
Muon g-2
Target Station
Beam Transport
& Delivery Ring
Muon
Area
MC1
Mu2e


Number of high energy positrons as a function of time
count / 2*149 ns
data
fit
time modulo 80 μs
0
10
20
30
40
50
60
70
80
1
10
10^2
10^3
μ
g-2
Fermilab Muon g-2 collaboration
Commissioning Run, June 2017
PRELIMINARY