Uncertainties in Measurements

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uncertainties

- any measurement must include uncertainties
  - any report must include a discussion of the uncertainties
- two types:
  - statistical:
    - uncertainties based on the number of observations
    - uncertainty usually goes like the $\sqrt{N}$, these describe $1\sigma$ uncertainties
  - systematic:
    - uncertainties inherent in the methods, equipment, stability, external conditions …
    - these are typically more challenging to identify and to quantify
    - focus on the most important sources
- measurements are often limited by one or the other
  - if your measurement is statistics limited, try to take more data, if you can significantly improve uncertainties (doubling data, improves uncertainties by 40%)
  - if your measurement is systematics limited, taking more data won’t help
Search for the Standard Model Higgs Boson in the Diphoton Decay Channel with 4.9 fb$^{-1}$ of $pp$ Collision Data at $\sqrt{s} = 7$ TeV with ATLAS

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A search for the standard model Higgs boson is performed in the diphoton decay channel. The data used correspond to an integrated luminosity of 4.9 fb$^{-1}$ collected with the ATLAS detector at the Large Hadron Collider in proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 7$ TeV. In the diphoton mass range 110–150 GeV, the largest excess with respect to the background-only hypothesis is observed at 126.5 GeV, with a local significance of 2.8 standard deviations. Taking the look-elsewhere effect into account in the range 110–150 GeV, this significance becomes 1.5 standard deviations. The standard model Higgs boson is excluded at 95% confidence level in the mass ranges of 113–115 GeV and 134.5–136 GeV.

Abstract

A search for the Standard Model Higgs boson in proton-proton collisions with the ATLAS detector at the LHC is presented. The datasets used correspond to integrated luminosities of approximately 4.8 fb$^{-1}$ collected at $\sqrt{s} = 7$ TeV in 2011 and 5.8 fb$^{-1}$ at $\sqrt{s} = 8$ TeV in 2012. Individual searches in the channels $H \rightarrow ZZ^{(*)}\rightarrow 4\ell$, $H \rightarrow gg$ and $H \rightarrow WW^{(*)}\rightarrow e\nu\nu$ in the 8 TeV data are combined with previously published results of searches for $H \rightarrow ZZ^{(*)}$, $WW^{(*)}$, $b\bar{b}$ and $\tau^+\tau^-$ in the 7 TeV data and results from improved analyses of the $H \rightarrow ZZ^{(*)}\rightarrow 4\ell$ and $H \rightarrow gg$ channels in the 7 TeV data. Clear evidence for the production of a neutral boson with a measured mass of 126.0 ± 0.4 (stat) ± 0.4 (sys) GeV is presented. This observation, which has a significance of 5.9 standard deviations, corresponding to a background fluctuation probability of $1.7 \times 10^{-5}$, is compatible with the production and decay of the Standard Model Higgs boson.
counting experiments

\[ P_n(rt) = \frac{(rt)^n}{n!} e^{-rt}, \quad n = 0, 1, 2, \ldots \]

\[ rt = (\text{decay rate})(\text{time}) = \text{number of counts} \]

- random processes follow Poisson distribution
- nuclear decay is one such process, but this applies to many counting experiments
- asymmetric distribution at small number of counts
  - you can’t observe negative counts
- becomes Gaussian as rt increases
- distribution is a probability distribution, not the number of counts
  - \( \sigma/\mu = 1/\sqrt{rt} \) → larger rt, smaller uncertainty on \( \mu \)

\[ \sum_{n=0}^{\infty} P_n(rt) = 1, \text{ probabilities sum to 1} \]

\[ <n> = \sum_{n=0}^{\infty} n \cdot P_n(rt) = rt, \text{ the mean} \]

\[ \sigma = \sqrt{\sum_{n=0}^{\infty} (n - <n>)^2 P_n(rt)} = \sqrt{rt}, \text{ standard deviation} \]
accuracy and precision

• precision:
  • measurements close together
• accuracy:
  • measurements that contain the true value inside the uncertainty
• want to be both accurate and precise!
• in this class you will try to be accurate, but other measurements will typically be more precise than we can do with this equipment
error propagation

\[ \Delta z = \sqrt{(\Delta x)^2 + (\Delta y)^2} + \ldots \]

\[ \frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} + \ldots \]

addition/subtraction \hspace{2cm} multiplication/division

• these formula are true if x and y are independent of each other

• if you have correlated measurements then you must deal with the covariance

• many automated programs will do this for you, but you must figure out if you have correlated measurements

• think about a measurement with a lot of background:

• if \( \Delta x \) & \( \Delta y \) are large then \( \Delta z \) will be large when \( z = x - y \)

• clear why minimizing background is very important for many measurements!
• fitting:
  • you provide the functional form—the fit should be meaningful
  • many implementations of chi2 minimization fitting around
  • need to understand how well the fit describes your data
  • this will only take into account statistical uncertainties, not systematics

\[ \chi^2 = \sum_{i=1}^{n} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \]
$A e^{-x/\tau}$  $\tau = -3$

$N = 100$

$\tau = -2.24 \pm 0.27$
$\chi^2$/dof = $5.5/9$
fit probability = 0.79

$N = 1000$

$\tau = -2.87 \pm 0.10$
$\chi^2$/dof = $25.3/13$
fit probability = 0.02

$N = 100000$

$\tau = -2.994 \pm 0.010$
$\chi^2$/dof = $15.1/13$
fit probability = 0.30
thinking about systematics

• stability:
  • if you repeat a measurement, will you get the same answer?
  • how uncertain is your calibration?
  • if you repeated it, would you get the same calibration
  • what happens if you take the same measurement on different days? do you get the same answer?
omitting data because it doesn’t conform to your expectations isn’t scientific

if something looks off, try to understand why

what other things can you check?

are you getting results consistent with yesterday? is the data overall consistent?

can you go back to some control measurement where you know the answer?

write everything down!

if you need to omit data document why

be aware of confirmation biases!
uncertainties

- uncertainties are inherent in all measurements
- it is typical in experimental physics that the majority of the time is spent on uncertainty analysis
- always question and think about your data
  - think of the questions you would ask if it was someone else’s result
- use appropriate significant figures!
  - don’t tell me you have measured $x = 3.948532 \pm 0.3$
  - $L = (1.979 \pm 0.012)m$ or $L = (1.98 \pm 0.8)m$
  - the difference being if the first significant digit of the uncertainty is small or large
• many books written about uncertainty analysis
• Bevington and Taylor are some of the most popular
• systematic uncertainties depend on the kind of measurement you are doing
• include in your report a discussion of how you evaluated your systematic uncertainties
• think critically about your data, but do not let your biases dictate which data you use
• write everything down so you know can know if there is something going on in your measurement