Noise

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Noise : Nuisance and Tool

Outline:

**Broad categories**
- fundamental equilibrium noise
- noise reflecting system physics
- bad contacts, etc.

**Case studies in applications:**
- Noise and extraneous dirt: defects in SiO$_2$ etc.
- Noise and dirty thermodynamics: e.g. manganites
- Noise out of equilibrium: ferroelectric Barkhausen
Where does noise come from?

- **White noise (often not a mystery):**
  - Look at a resistor in an amplifier circuit

The voltage (or air pressure, etc) changes quickly from one random value to a new, independent random value. *Why?*
Noise and the laws of thermodynamics

- ANY two resistors with the same resistance at the same temperature MUST have the same sort of noise before a current is applied to them, even if one is made of gold and one of salt water! Why?
- Let’s say one was noisy and the other quiet. Then when hooked together, the noisy one would drive more currents through the quiet one than vice versa. Currents heat up resistors. So the quiet one would heat up and the noisy one would cool down. But a basic law of thermodynamics says that two objects at the same temperature don’t spontaneously go to different temperatures. Therefore they must have the same amount of noise.
- But no law like that applies when current is forced through them. (Refrigerators work.)
Equilibrium basics

The magnitude of the noise is given by equipartition.
\[ \langle (\delta V)^2 \rangle = kT/C \]

The time course is just exponential decay, with RC time constant.

So the autocorrelation function is
\[ \langle (\delta V(t) \delta V(t+\tau)) \rangle = (kT/C)e^{-\tau/RC} \]

The spectrum \( S(f) \) is just the Fourier transform of the autocorrelation function:
\[ S(f) = 4kTR \quad (\text{up to } f \sim 1/RC) \]

Similar Fluctuation-dissipation relations hold for magnetism, dielectrics, mechanical systems etc.

Limited new info from noise
Frequency spectra: \( S(f) \)

\[
V(t) = a_1 \cos(2\pi \cdot 1\text{Hz}) + a_2 \cos(2\pi \cdot 2\text{Hz}) + \text{etc}
\]

\[
S(1\text{Hz}) = a_1^2 \quad S(2\text{Hz}) = a_2^2 \quad \text{etc}
\]

Write the signal as a sum of waves at a set of equal spaced frequencies. \( S(f) \) gives how the square of the size of the components depends on \( f \).

White noise: same amount of power in each equal frequency range 20Hz-30 Hz, 30 Hz-40 Hz, etc (like white light, except different range)

1/f noise: same amount in each OCTAVE:
20 Hz-40 Hz, 40 Hz-80 Hz, etc

Playing the tape back at double speed doesn’t change the sound!
Another fact to intrigue to theorists.
Noise pictures

- White noise
- 1/f noise
- 1/f^2 noise

VOLTAGE vs TIME

NOISE POWER per OCTAVE vs frequency
Non-equilibrium basics

- Some noise is intrinsically non-equilibrium, driven. e.g.
  - Shot noise (photons, electrons,..)
    - $S_l(f) = 2lq$ for current, in simple case
  - Barkhausen domain flips in magnets
  - Sliding charge density waves

- Some is just sampled by non-equilibrium means. e.g.
  - most 1/f noise in resistors
  - Particle density fluctuations in fluids (via light scattering)
1/f noise basics

- $\delta R$ almost always measured out of equilibrium
  - but that rarely matters, as confirmed by
    - Linearity of $\delta V$ in $I$
    - Independence of ac or dc measurements
    - Occasional equilibrium measurements via $\delta(kTR)$

- Other variables (magnetic $\mu$, capacitor $V$) are measured in equilibrium.
- Spectra are often remarkably close to $1/f$, but not usually exactly so
- The deviations from $1/f$ often shift around like simple thermally activated kinetics (Dutta-Horn)

FIG. 1. (a) The basic experimental configuration and typical observations of $1/f$ noise. Schematic diagram of the simplest measuring apparatus for $1/f$ noise. $R_s$ is a large, constant resistor. The unlabeled resistor is the sample. Various modifications, such as the use of ac currents with phasesensitive detection, bridge circuits, and multiprobe samples, are common. (b) An actual fluctuating voltage from a silicon resistor with about 100 $\mu$A of current (1 V average bias), measured in a setup like that shown in part (a). (c) Noise spectra from two thick-film resistors, shown over a very broad range of frequencies. The upper plot is taken from an IrO$_2$-based film at $T = 556$ K, the lower from a ruthenate-based film at $T = 300$ K. Each point in each spectrum represents the average square of the Fourier transforms of 1200 1024 point traces, such as that in part (b). Several such spectra, taken at different sampling rates, are stitched together for each broad-band spectrum shown (from Pellegrini, Saletti, Terrini, and Prudenziati, 1983).
So what’s rattling?

In silicon with an oxide layer electrons jump in and out of traps in the oxide.

In copper, defects in the crystal structure move around.

In chromium, domains of a type of magnetism change their alignment back and forth.

And all give the same shape of spectrum: $1/f$. 
1/f noise: the simplest ingredients

- electron traps in amorphous SiO$_2$
- collection of simple parallel noise sources
- equilibrium thermodynamics and kinetics
- random trap depths
- random trap positions
- random barrier heights
- No important correlations among those random variables
  - Measurable from E and T dependences

Why 1/f?
Could 1/f noise just come from summing the switchers?

- It sure looks that way
  - E.g in silicon-on-sapphire resistors (1983):

![Graph](image.png)

**FIG. 4.** Power spectra at room temperature for seven different samples having approximately the same size and geometry.
Quantum noise

- At low temperatures, you still get 1/f noise, but the rattles don’t occur by getting enough thermal energy to go over the barrier. Things tunnel through, quantum mechanically. (electrons in and out of traps in Nb$_2$O$_5$, Rogers and Buhrman, 1985)

![Graph showing the abrupt change from thermally activated behavior above to nonactivated behavior below $T \sim 15$ K.](image)
The secret of 1/f noise

- Ingredient (e.g. two-state)

\[ S(f) = \int \frac{s\left(\frac{f}{f_c}\right)}{f_c} \rho(f_c) df_c \quad \text{e.g. } s\left(\frac{f}{f_c}\right) = \frac{4}{1 + \left(\frac{f}{f_c}\right)^2} \]

\[ f_c = f_A e^{-E_A/kT} \quad f_A \approx 10^{12} \text{ Hz} \]

\[ \rho(f_c) = \frac{kT \rho(E_A)}{f_c} \quad \text{i.e. } \frac{d \ln(f)}{df} = \frac{1}{f} \]

\[ S(f) \approx \frac{kT \rho\left(kT \ln\left(\frac{f_A}{f}\right)\right)}{f} \]

- \( f_c \) depends exponentially on a distributed energy, tunneling distance, etc.

- Change variables
  - Bernamont, 1939; McWhorter, 1951

- 1/f with log corrections
Where does the 1/f ‘secret’ that apply?
Quasi-equilibrium systems

• (Almost?) all 1/f noise in metals
  – $^2$Defect motions (~all metals)
  – $^{1,2}$Domain motions (SDW, FM,…)
  – $^2$Glassy TLS
  – $^{1,2}$Spinglassy collective modes….

• $^2$1/f noise in semiconductors
  – (especially traps in SiO$_2$ )

• $^2$disordered phase transitions
  – Manganites…..

• $^1$Dielectric 1/f noise
  – Relaxor ferroelectrics

Strongly driven systems
e.g. depinned CDWs or vortex lattices, usually show big deviations from 1/f$^{1.0}$

$^1$ direct equilibrium fluctuation-dissipation: $S_V(f) \sim kT \varepsilon''/C_f$, $S_{\mu}(f) \sim kTV\chi''/f$

$^2$ indirect $\delta V=I\delta R$, I is non-equilibrium probe of equilibrium noise
Manganites: inhomogeneity and thermodynamics

- Thermodynamics not clear from macro-measurements of $R(H,T)$ and $M(H,T)$
  - Disorder messes things up
  - Noise shows what’s up:
    little pieces of 1\textsuperscript{st}-order transition

Well defined $\Delta E$, $\Delta S$, $\Delta \mu$
between states
Barkhausen Noise in Ferroelectrics

Noise shows size of units involved in different stage of conversion of glassy state to ferroelectric state.
Big domains form before most of sample goes FE. Rates *not* limited by nucleation. Some domains melt *after* main melting → important heterogeneity.
Xinyang's data: notice anything?
Summary

• Noise provides a good probe of
  – Conduction mechanisms (shot noise)
  – Domain dynamics (Barkhausen)
  – Defect dynamics (1/f noise in metals)
  – Subtle phase transitions (CR films,…)
  – Hidden order (spinglasses)
  – Charge density wave dynamics (TaS$_3$)
  – ……. 