Introduction to entanglement
Making entanglement in the lab
Quantum teleportation

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Quantum states describing more than one system can be entangled

System A (a photon)
System B (another photon)

$|\Psi\rangle_{AB} = \text{Two-photon state of A and B}$

- States that can be written $|\Psi\rangle_{AB} = |\varphi^1\rangle_A |\varphi^2\rangle_B$ are **separable**
  - **Example**: $|\Psi\rangle_{AB} = |H\rangle_A |V\rangle_B$

- States that cannot be written this way are **entangled**
  - **Example**: $|\Psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B)$
Measurement outcomes are *random* and *correlated*

\[ |\Psi\rangle_{AB} = \frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B + |V\rangle_A |H\rangle_B) \]

- 50% chance to measure H or V for either photon (random)
- But the photons always have orthogonal polarization (correlated)

But classical things can be random and correlated too... what’s special about entanglement?
Quantum cakes

Lucy

Does the cake taste **good** or **bad**?

Ricardo

Has the cake **risen** or not **risen** early?

Only one measurement can be made on any particular cake!
Lucy and Ricardo randomly decide which measurement to make on each cake, and record their results.

1. They both check their ovens midway
   - 9% of the time, both cakes rise early (the rest of the time, only one or neither does)

2. One checks midway and the other waits
   - Lucy’s cake rises early \(\rightarrow\) Ricardo’s tastes good
   - Ricardo’s cake rises early \(\rightarrow\) Lucy’s tastes good

3. They both wait
   - Do the cakes taste good or bad?
When they both check midway...
9% of the time, both cakes *rise early*

Whenever Lucy’s cake *rises early*, Ricardo’s cake tastes *good*
Whenever Ricardo’s cake *rises early*, Lucy’s cake tastes *good*

When they both wait...
How often do both cakes taste good?

Both cakes *never* taste good!
This experiment isn’t really possible with cakes, but it is possible with photons

Tasted good = 0° (horizontal)
Tasted bad = 90° (vertical)

Rose early = -50.8°
Didn’t rise early = 39.2°
Summary: quantum mechanics violates **local realism**, and we can prove it (with a Bell test or something like it)
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Downconversion produces pairs of photons

Momentum conservation:

\[ k_1 \quad k_2 \]
\[ k_{\text{laser}} \]

Energy conservation:

\[ \omega_{\text{laser}} \quad \omega_1 \quad \omega_2 \]

Source: David Guzman, Universidad de las Andes
Downconversion is polarization-dependent
Two crystals can create polarization entanglement

\[
\begin{align*}
\text{Horizontal} & \quad \text{Vertical} \\
|V\rangle & \rightarrow |H\rangle|H\rangle \\
|H\rangle & \rightarrow |V\rangle|V\rangle
\end{align*}
\]

Superposition \rightarrow Polarization entanglement

\[
|H\rangle + e^{i\phi}|V\rangle \rightarrow |V\rangle|V\rangle + e^{i\phi}|H\rangle\langle H|
\]
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Lucy wants to communicate an unknown quantum state to Ricardo.

Bell-state measurement

\[ |\phi^\pm\rangle = \frac{|HH\rangle \pm |VV\rangle}{\sqrt{2}} \]

\[ |\psi^\pm\rangle = \frac{|HV\rangle \pm |VH\rangle}{\sqrt{2}} \]

Classical Communication

Unitary Transformation

\[ |\psi\rangle_3' = \alpha |H\rangle_3 + \beta |V\rangle_3 \]

\[ |\psi\rangle_{23} = \frac{1}{\sqrt{2}} (|H_2 H_3\rangle + |V_2 V_3\rangle) \]
Summary

• Entangled systems can’t be completely described independently (not separable)
• Entanglement is a type of correlation between quantum systems that is stronger than any classical correlation, and violates local realism
• Entanglement is fairly easy to create in the lab
• Entanglement plays a central role in quantum information applications