# Qualitative Studies with Microwaves

**Physics 401, Spring 2019**

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AT URBANA-CHAMPAIGN

# Qualitative Studies with Microwaves

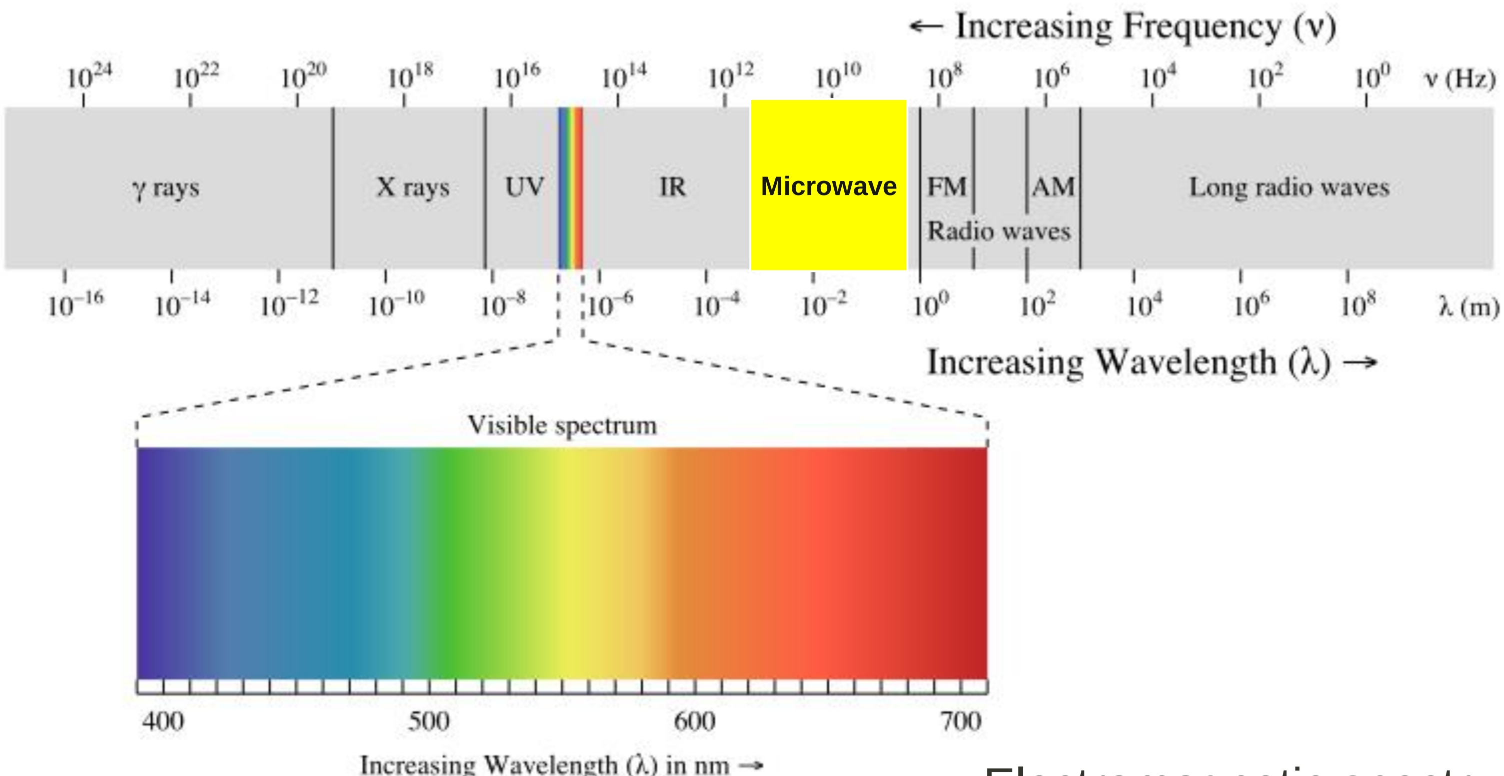
**The main goals of the Lab:**

- ✓ **Refreshing the memory about the electromagnetic waves propagation**
- ✓ **Microwaves. Generating and detecting of the microwaves**
- ✓ **Microwaves optics experiments**

**This is two weeks Lab**

# Microwaves place in the electromagnetic spectrum

**The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.**




Electromagnetic spectrum*

# Application of the microwaves

Microwave oven (2.45GHz)

Communication
(0.8-2.69GHz)

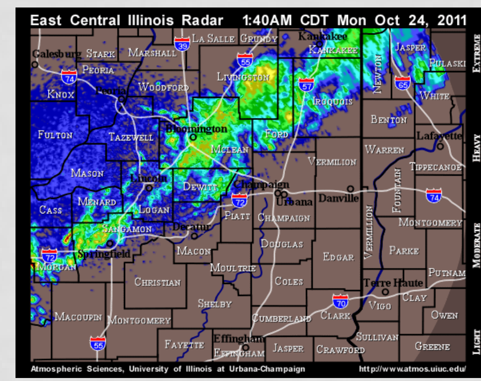
Satellite TV (4-18GHz)

Radar
(up to 110GHz)



Motion detector (10.4GHz)


GPS 1.17-1.575 GHz




Weather radar (8-12Ghz)

*by courtesy Wikipedia

# Maxwell equations

$$\nabla \vec{D} = \rho \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3)$$

$$\nabla \vec{B} = 0 \quad (2)$$

$$\nabla \times \vec{H} = J + \frac{\partial \vec{D}}{\partial t} \quad (4)$$

**James Clerk Maxwell (1831–1879)**

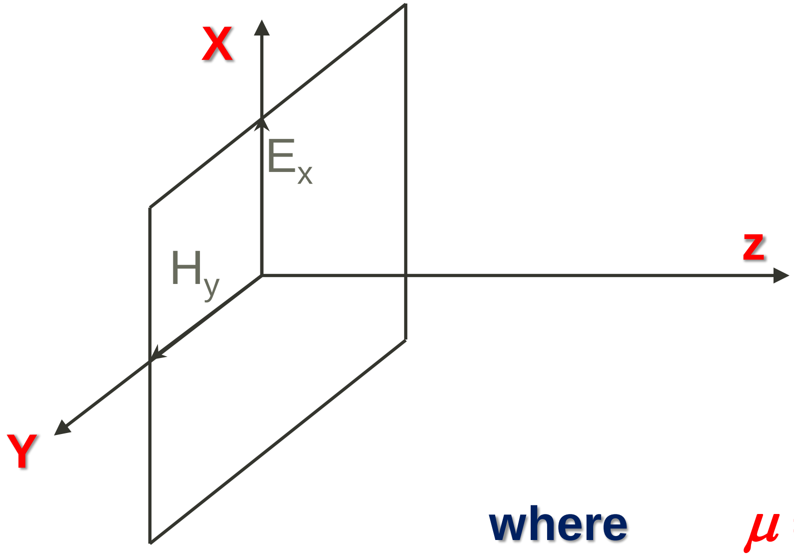
**If $\rho = 0$ and $J = 0$ and taking in account that $\vec{D} = \varepsilon \vec{E}$, $\vec{B} = \mu \vec{H}$ (1) and (4) can be rewritten as**

$$\nabla \vec{D} = \varepsilon \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = 0$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

# Plane wave

Now assuming that plane wave propagate in z direction and what leads to $E_y$=$E_z$=0 and $H_x$=$H_z$=0




Now (3) and (4) could be simplified as

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \qquad (5)$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_y}{\partial t} \qquad (6)$$

where $\mu = \mu_o \mu_r$ $\varepsilon = \varepsilon_o \varepsilon_r$

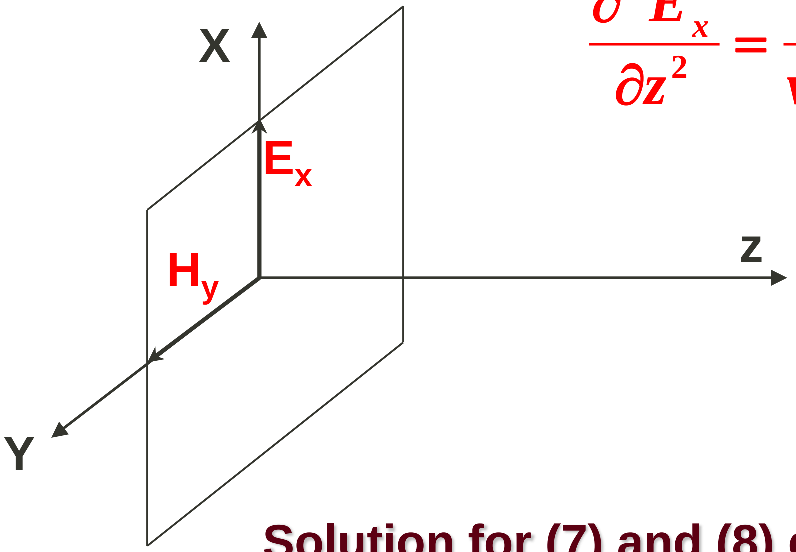
$\mu_0$ is the free space permeability, $\varepsilon_0$ is the free space permittivity
$\mu_r$ is permeability of a specific medium , $\varepsilon_r$ is permittivity of a specific medium

# Plane wave

Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 E_x}{\partial t^2} \quad (7) \qquad \frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 H_y}{\partial t^2} \quad (8)$$

where $v = \frac{1}{\sqrt{\varepsilon\mu}}$




$$E_x = E_{x0}\cos(\omega t - kx)$$
$$H_y = H_{y0}\cos(\omega t - kx)$$

Solution for (7) and (8) can found as $H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x$ or $E_x = ZH_y$

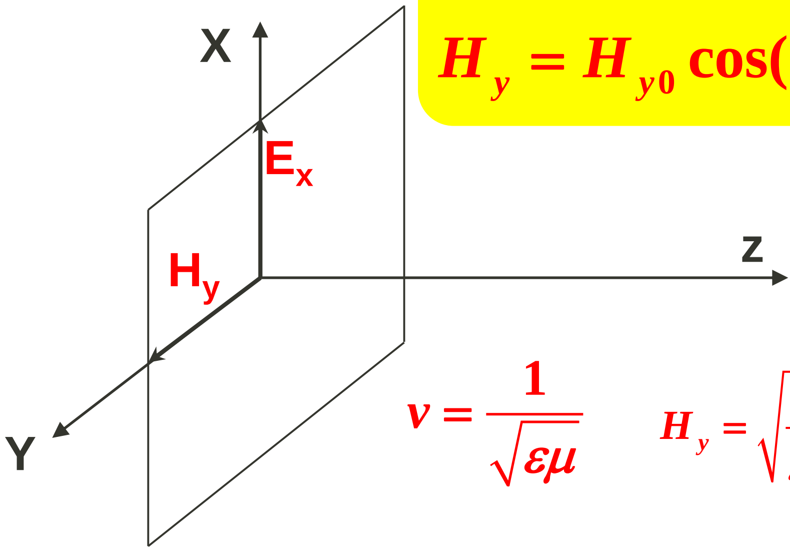
where $Z = \sqrt{\frac{\mu}{\varepsilon}}$ known as characteristic impedance of medium

k is wave vector and is defined as $k = \frac{2\pi}{\lambda}$ or $k = \frac{\omega}{v}$

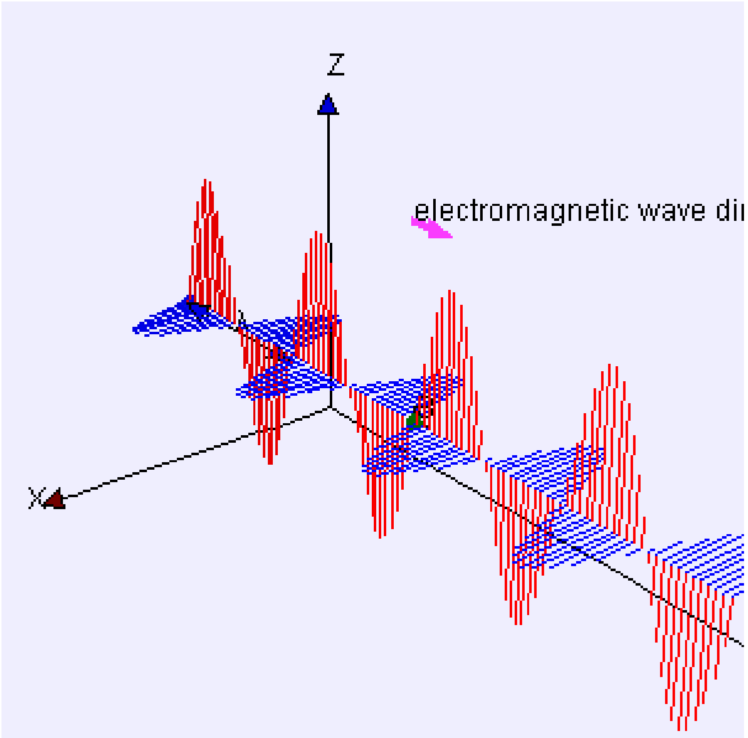
For free space ($\varepsilon_r=1$ and $\mu_r=1$) $Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 ohms$

# Plane wave

$$E_x = E_{x0}\cos(\omega t - kx)$$
$$H_y = H_{y0}\cos(\omega t - kx)$$




$$v = \frac{1}{\sqrt{\varepsilon\mu}} \qquad H_y = \sqrt{\frac{\varepsilon}{\mu}}E_x$$




$$Z = \sqrt{\frac{\mu}{\varepsilon}} \qquad E_x = ZH_y \qquad k = \frac{2\pi}{\lambda} \text{ or } k = \frac{\omega}{v}$$

$$Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 ohms$$

*by courtesy Wikipedia

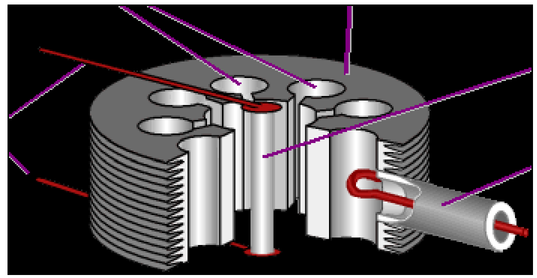
**For free space ($\varepsilon_r$=1 and $\mu_r$=1)**

# Generating of the microwaves

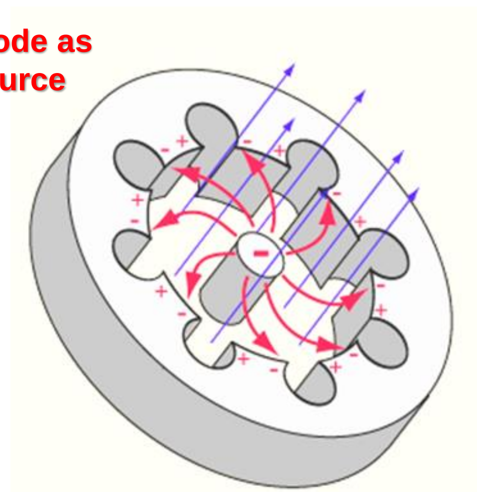
**Vacuum tubes: klystron, magnetron, traveling wave tube**

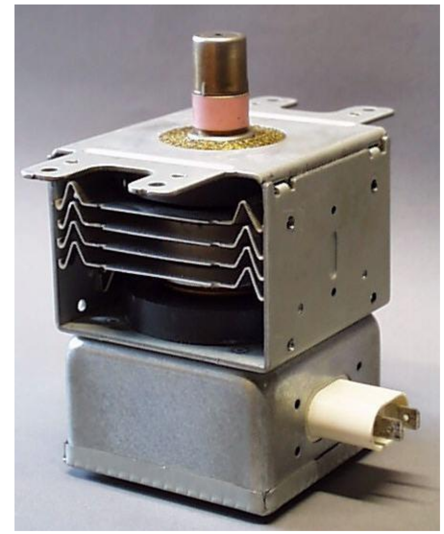
**Solid state devices: FET, tunneling diodes, Gunn diodes**


**Tunable frequency from 9 to 10GHz; maximum output power 20mW**



**Heated cathode as electron source**

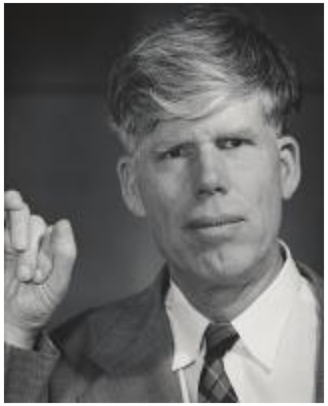




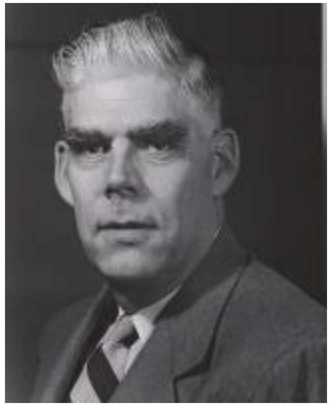
**Microwave oven magnetron; typical power 0.7-1.5kW**

# Klystron. A piece of history.

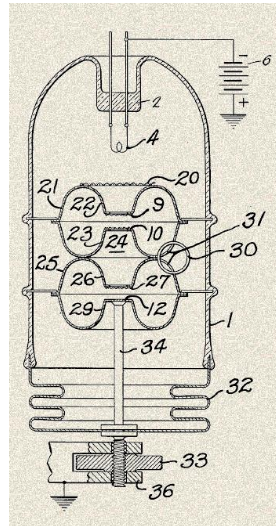
varian



**Russell Harrison Varian** (April 24, 1898 – July 28, 1959)



**Sigurd Fergus Varian** (May 4, 1901 – October 18, 1961)




Patented May 20, 1941

2,242,275

UNITED STATES PATENT OFFICE

2,242,275

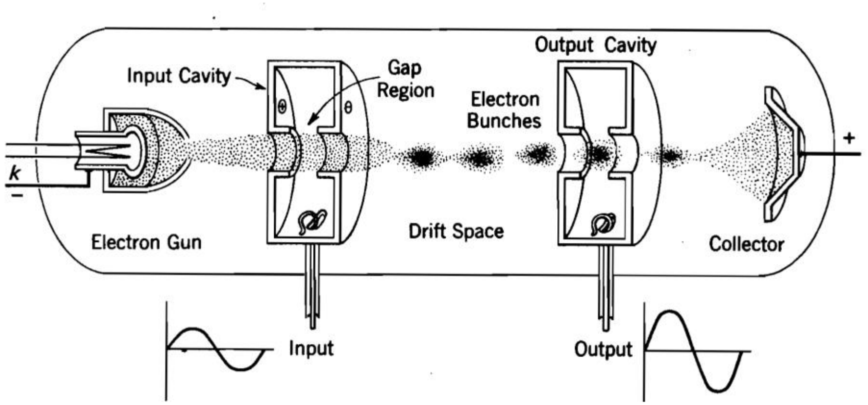
ELECTRICAL TRANSLATING SYSTEM AND METHOD

Russell H. Varian, Stanford University, Calif., assignor to The Board of Trustees of The Leland Stanford Junior University, Stanford University, Calif., a corporation of California

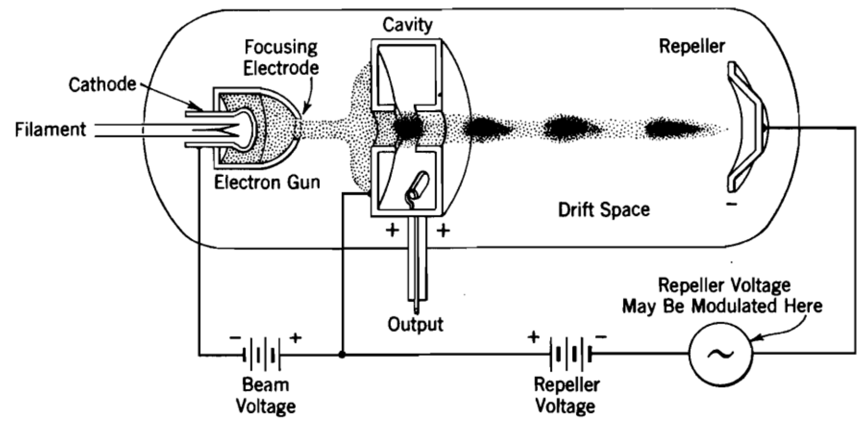
Application October 11, 1937, Serial No. 168,355

**Varian Brothers...Klystron Tube (1940)**

# Generating of the microwaves. Klystron.




**Single transit klystron**



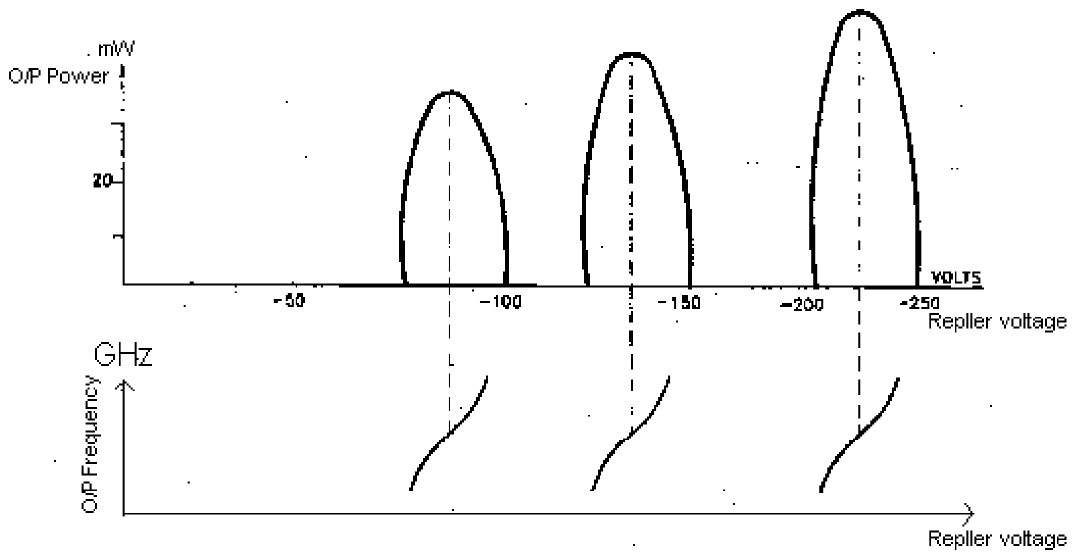

**Reflection klystron**

**Advantages:** well defined frequencies, high power output

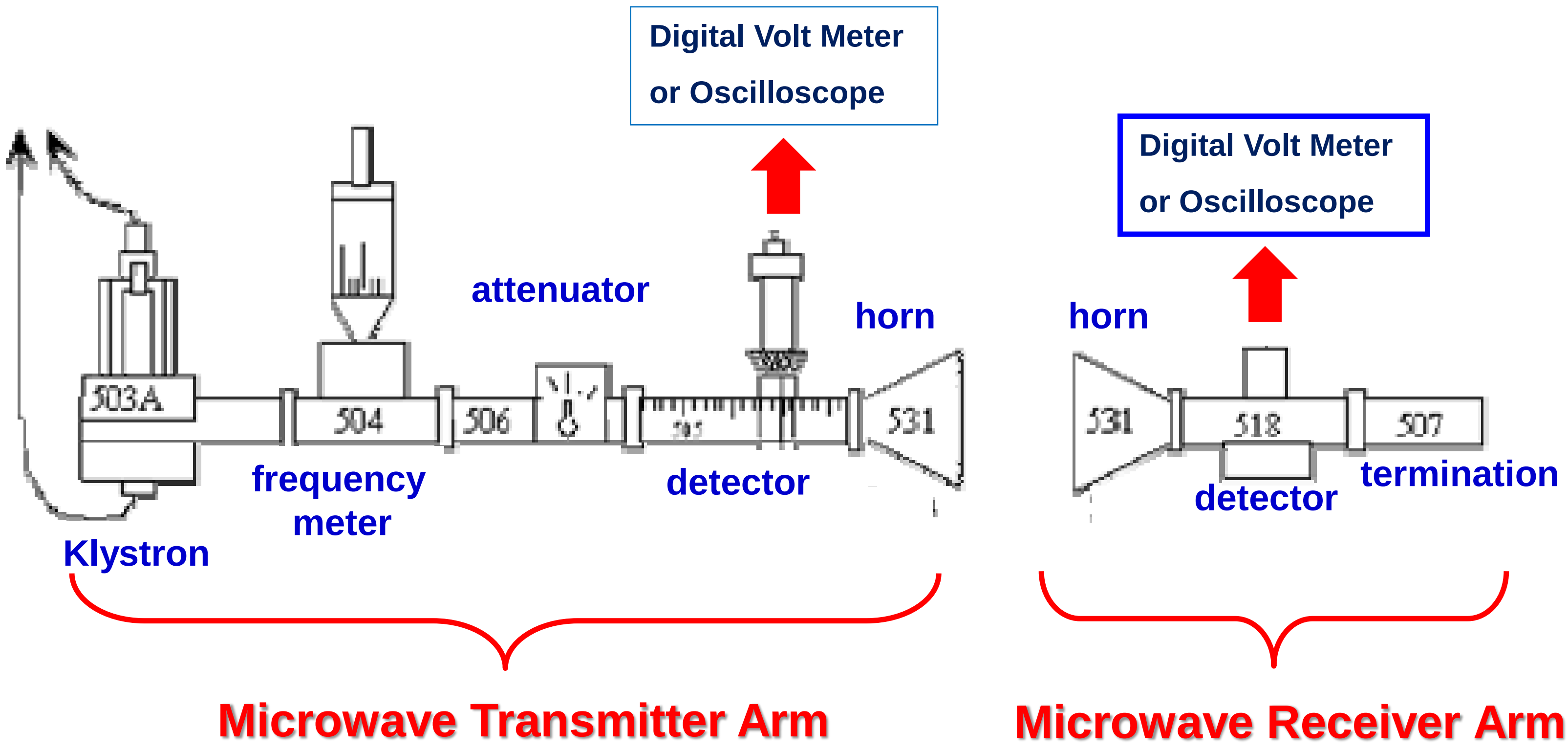
High power klystron used in Canberra Deep Space Communications Complex (courtesy of Wikipedia)

# 2K25 Klystron


**GENERAL CHARACTERISTICS**
**Frequency Range ······················8,500 to 9,660 Mc**
**Cathode Oxide-coated, indirectly heated**
**Heater Voltage·································6.3Volts**
**Heater Current························0.44 Amperes**

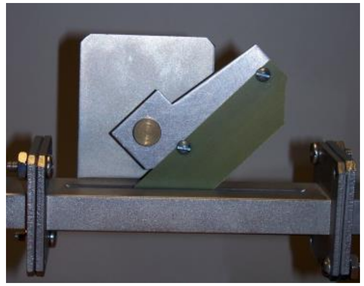


# Experimental setup. Main components.



# Experimental setup. Main components.

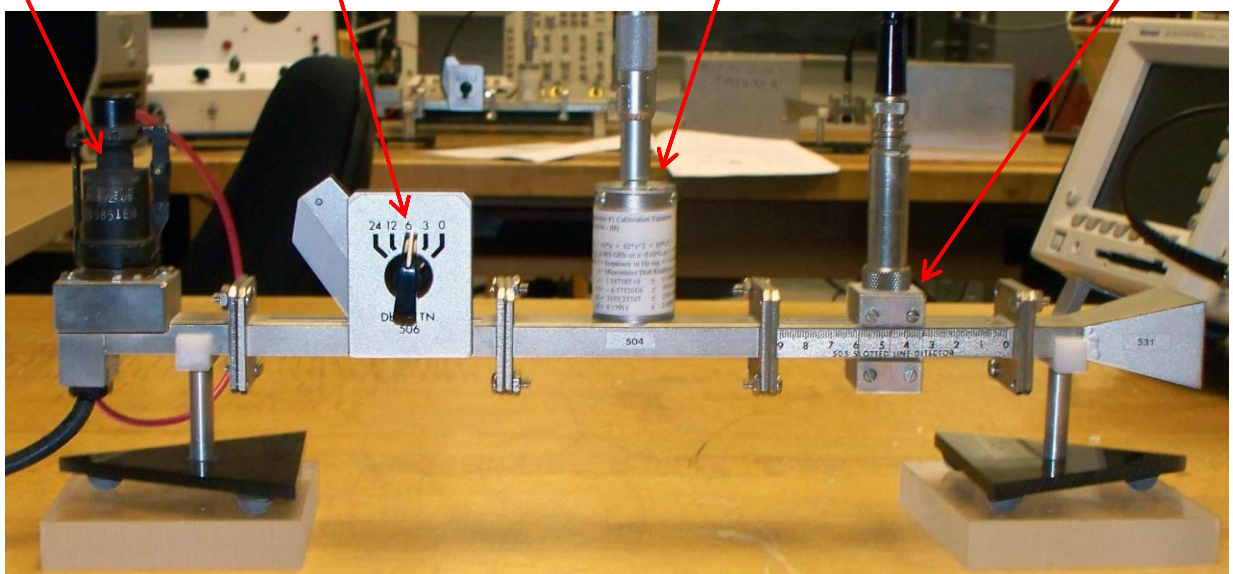
Klystron



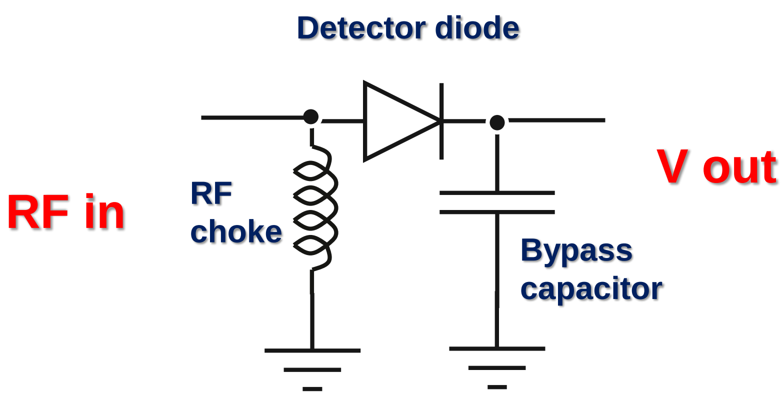
Attenuator

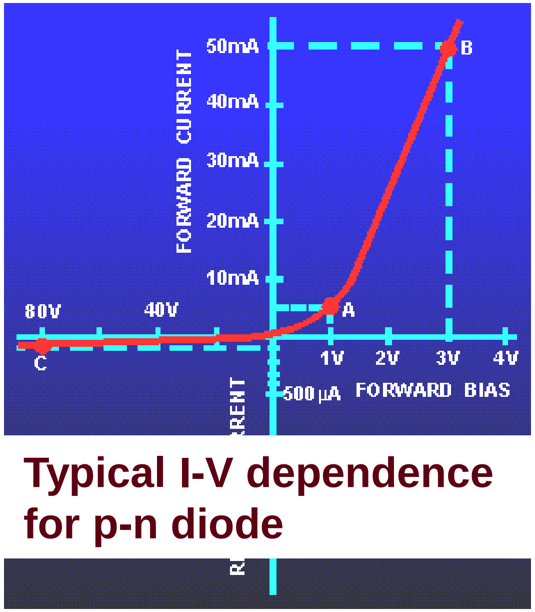
Frequency meter

detector



# Detecting of the microwaves




Typical I-V dependence for p-n diode

$$I = I_0\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Taylor expansion for exp function will give

$$\exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

$$I \propto aV + bV^2 + \ldots$$

0(DC)

If V=$V_0$sinωt

$$b * \frac{V^2{}_0}{2}(1 - \cos 2\omega t)$$

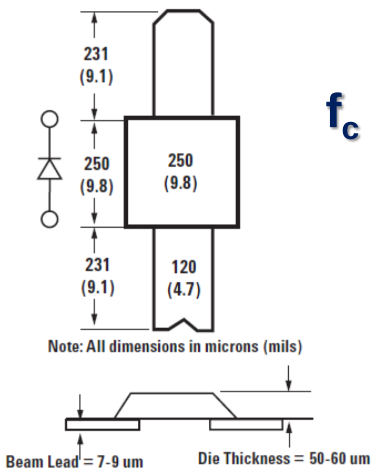
And finally

$$I_{DC} \propto b\frac{V_0{}^2}{2} + \ldots$$

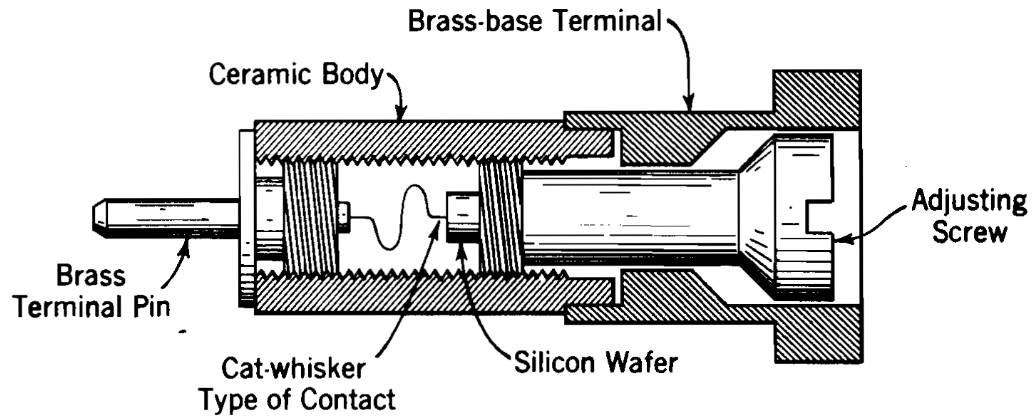
illinois.edu

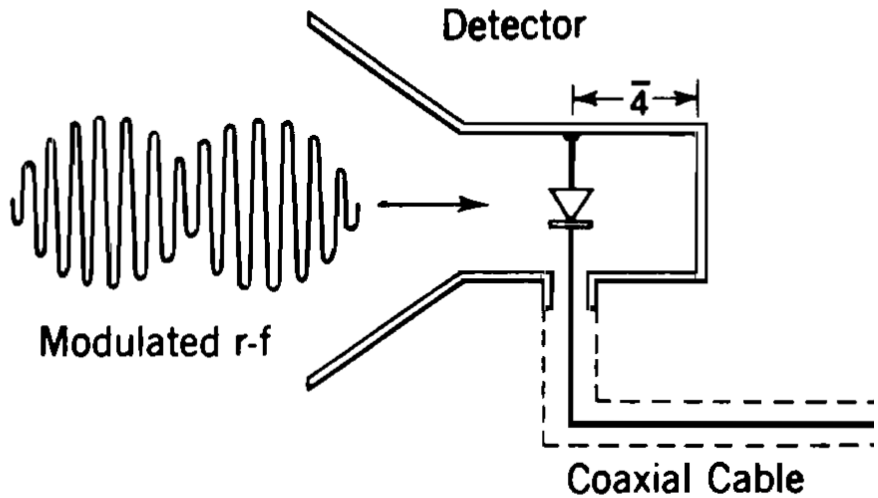
# Detecting of the microwaves

**HSCH-9161**
**HSCH-9162**
**GaAs Detector Diode**

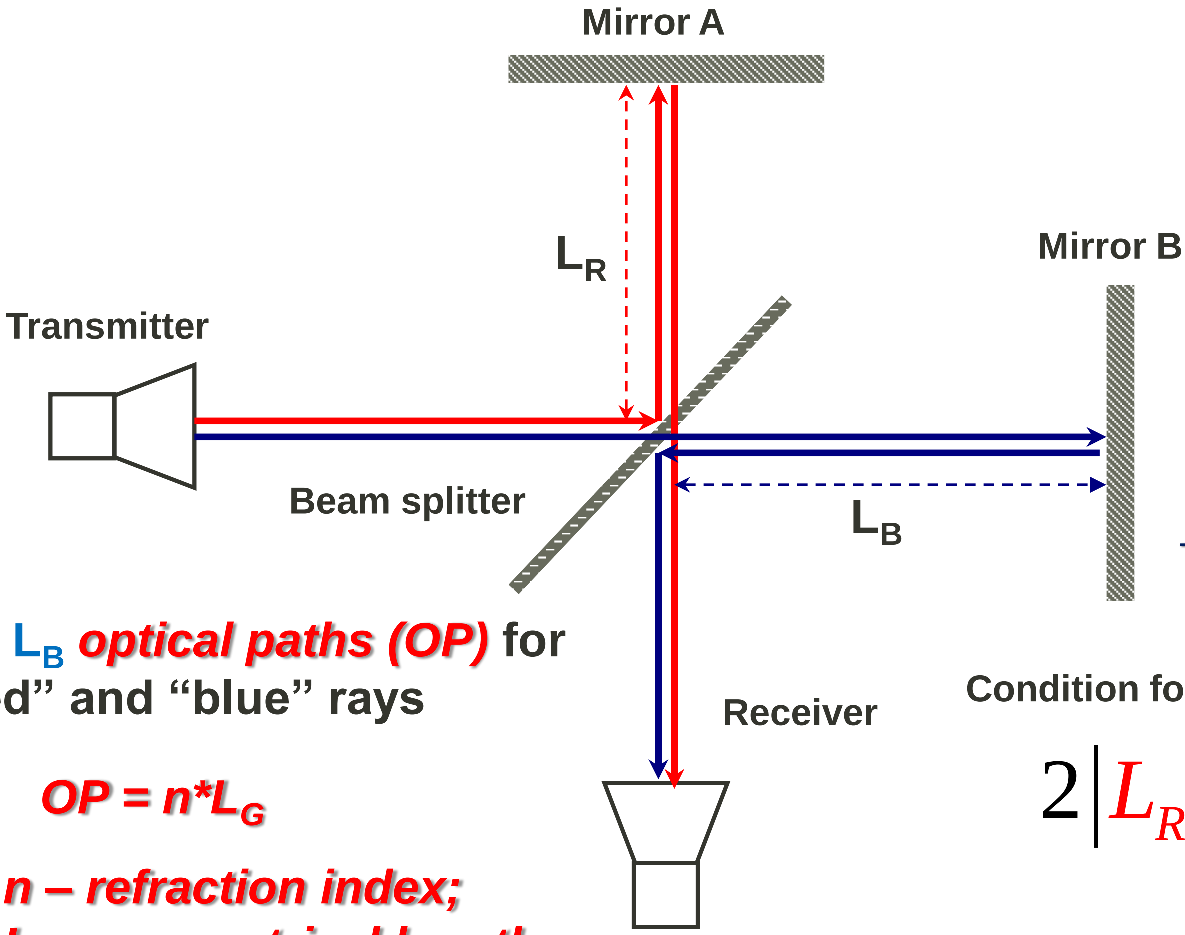


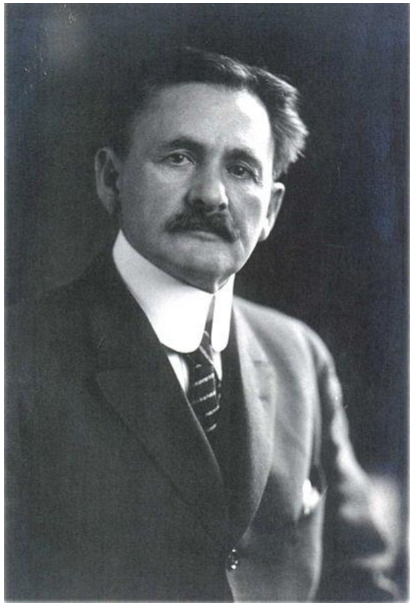

**$f_c \sim 200GHz$**



# Experiments: Michelson interferometer



Albert Abraham Michelson

(1852 - 1931)

The Nobel Prize in Physics 1907

$L_R$, $L_B$ ***optical paths (OP)*** **for "red" and "blue" rays**

***OP = n*$L_G$***

***n – refraction index;***
***$L_G$ – geometrical length***

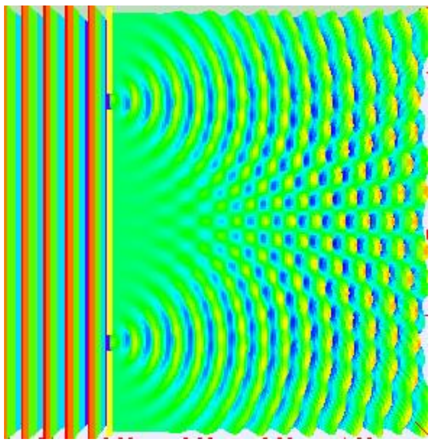
**Condition for constructive interference**

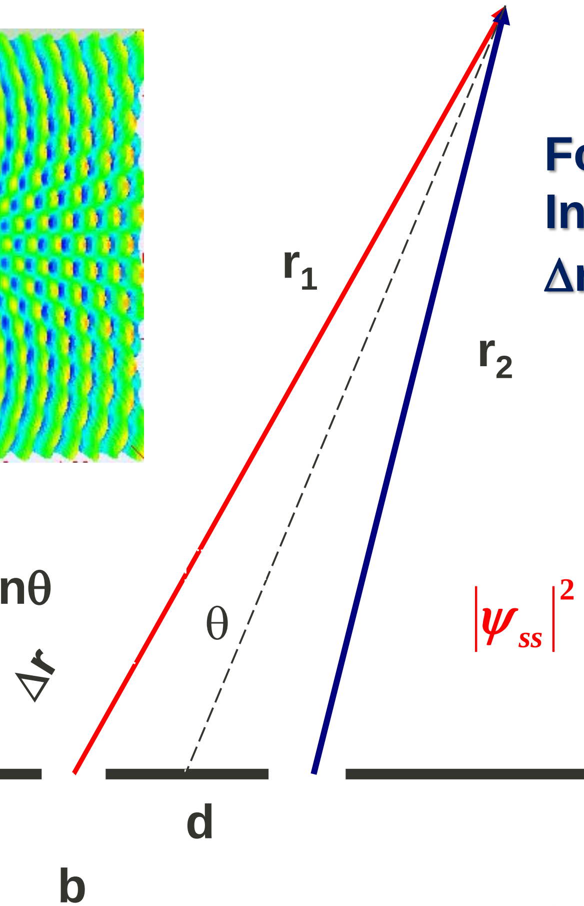
$$2\left|L_R - L_B\right| = k\lambda$$

# Experiments: Michelson interferometer


**Physics 403 Lab Michelson interferometer setup**

# Experiments: Double slit Interference. T. Young 1801






$\Delta r = r_1 - r_2 = d\sin\theta$

**For constructive Interference**
**$\Delta r = n\lambda$ or $d\sin\theta = n\lambda$**

Thomas Young
(1773 – 1829)

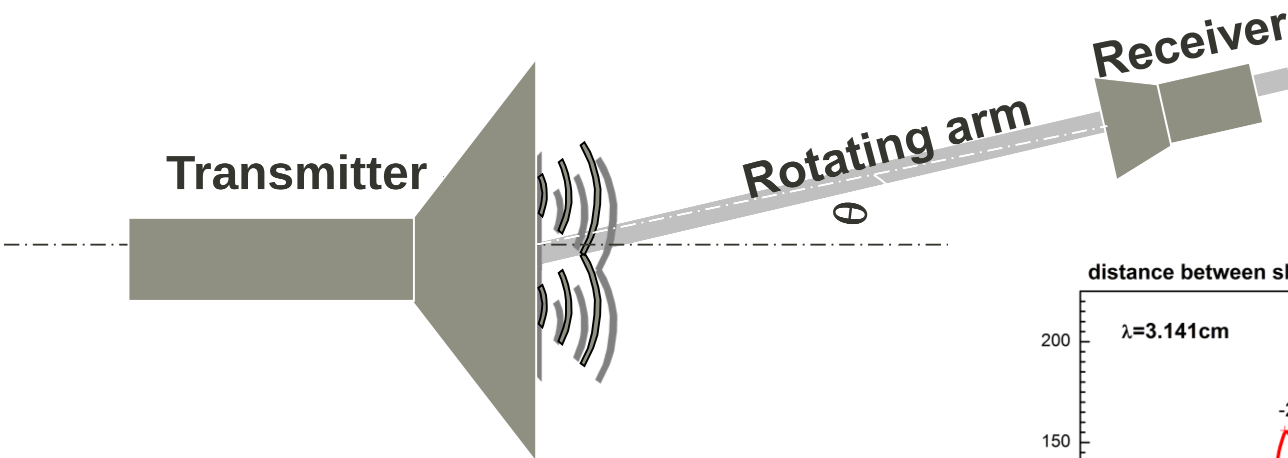
**The measured envelope of the diffraction pattern can be defined as:**

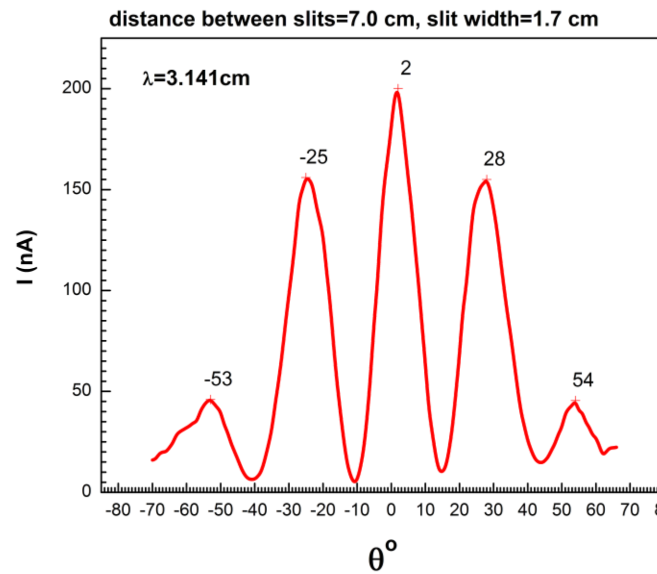
$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \times \cos^2[(kd\sin(\theta/2)]$$

where $x = kb\sin(\theta/2)$ and

$k = \dfrac{2\pi}{\lambda}$ is wave vector of the plane wave

# Experiments: Double slit interference



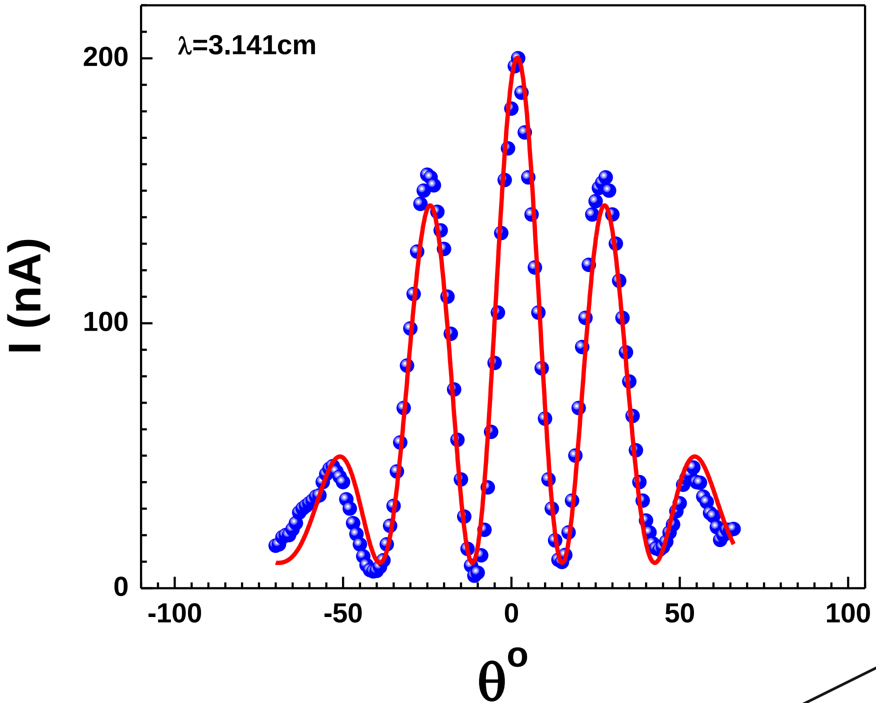


**Physics 401 Lab setup and example of the data**

# Experiments: Double slit interference. Fitting

$$|\psi_{ss}|^2 = |\psi_0|^2 \left(\frac{\sin x}{x}\right)^2 \times \cos^2[(kd\sin(\theta/2)]$$

$$x = kb\sin(\theta/2)$$




| Model | Two_slit (User) | | |
|---|---|---|---|
| Equation | y=I0*(sin(K1*sin(pi*x/360+f))/(K1*sin(pi*x/360+f)))^2 *(cos(K2*sin(pi*x/360+f)))^2+I00 | | |
| Reduced Chi-Sqr | 94.62111 | | |
| Adj. R-Square | 0.96659 | Value | Standard Error |
| | I0 | 190.6014 | 3.042882 |
| | K1 | 4.384042 | 0.074754 |
| | K2 | 13.51332 | 0.052244 |
| | f | -0.01525 | 7.19E-04 |
| | I00 | 9.572049 | 1.440409 |

**Fitting equation**

$$y = I0 \cdot \left(\frac{\sin\left(K1\sin\left(\frac{\pi x}{360}+f\right)\right)}{K1\, sin\left(\frac{\pi x}{360}+f\right)}\right)^2 \cos^2\left(K2\sin\left(\frac{\pi x}{360}+f\right)\right) + I00$$

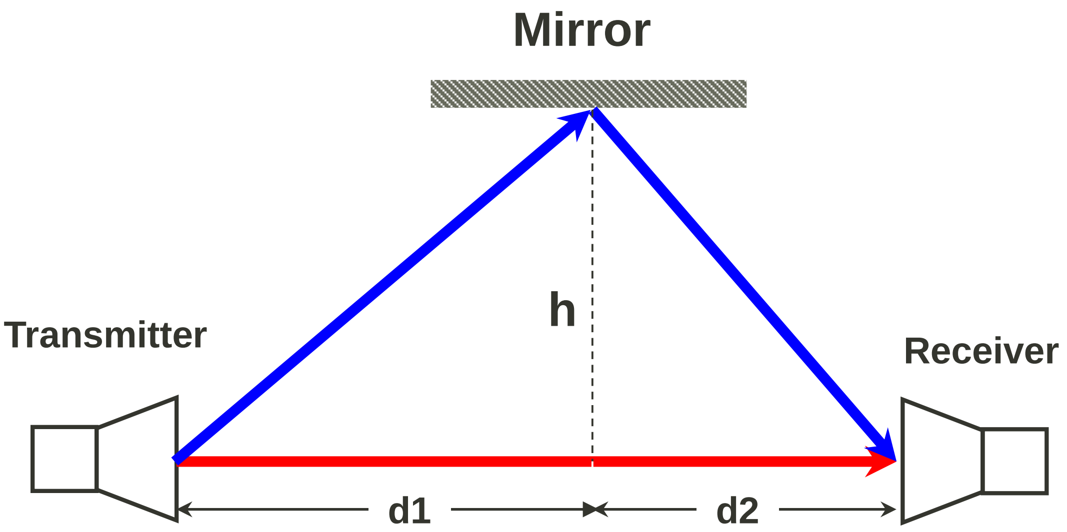
**Here in fitting expression:**

$I_0 = |\psi_0|^2$;

$K1 = kb$;

$K2 = kd$

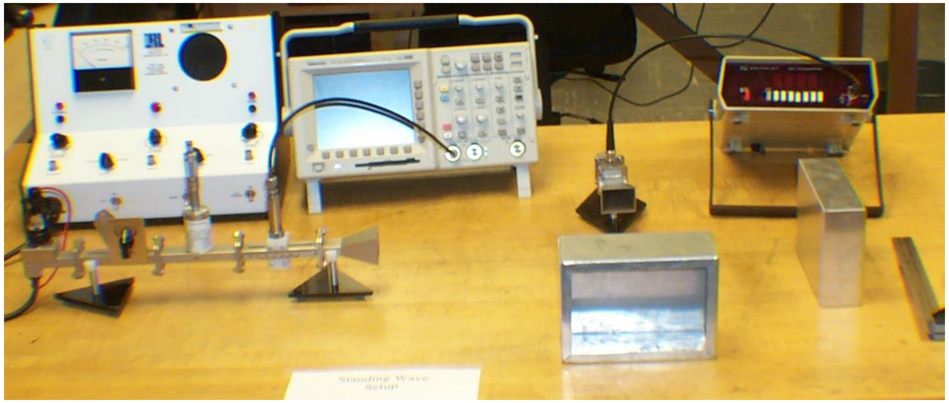
# Lloyd's Mirror experiment




**Humphry Lloyd**
**1802-1881**

**Difference of the wave paths of "red" and "blue" rays is:**

$$\Delta S = \sqrt{h^2 + d1^2} + \sqrt{h^2 + d2^2} - (d1 + d2)$$



**Lab setup picture**

**For constructive interference**

$$\Delta S = n\lambda$$

# Total internal reflection experiment. Snell's law

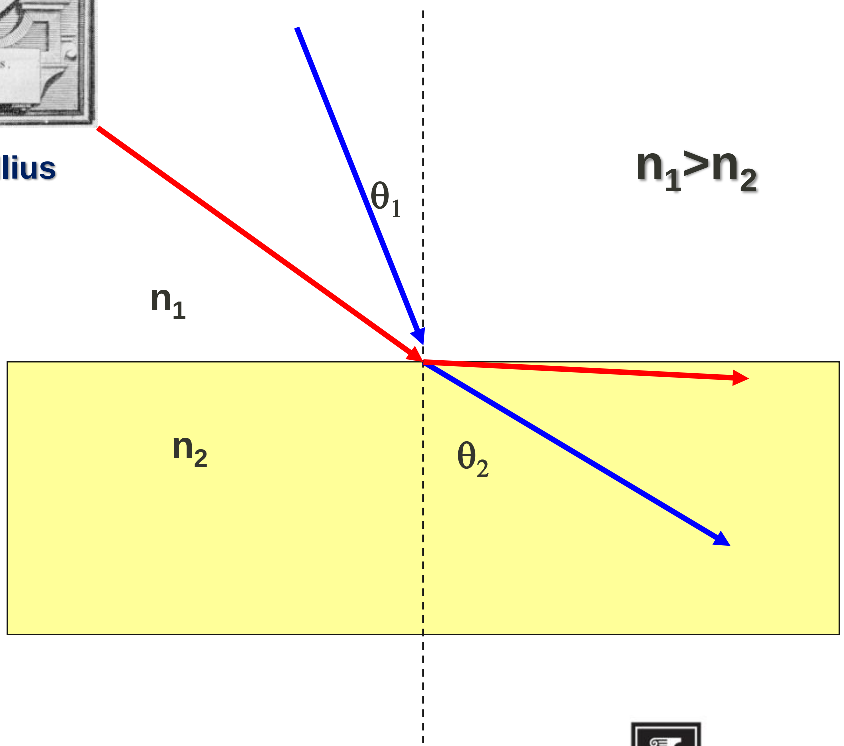

**Willebrord Snellius**
**1580-1626**

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

**Snell's law**

**Claudius Ptolemaeus**
**after AD 83–*c*.168)**

$n_1 > n_2$

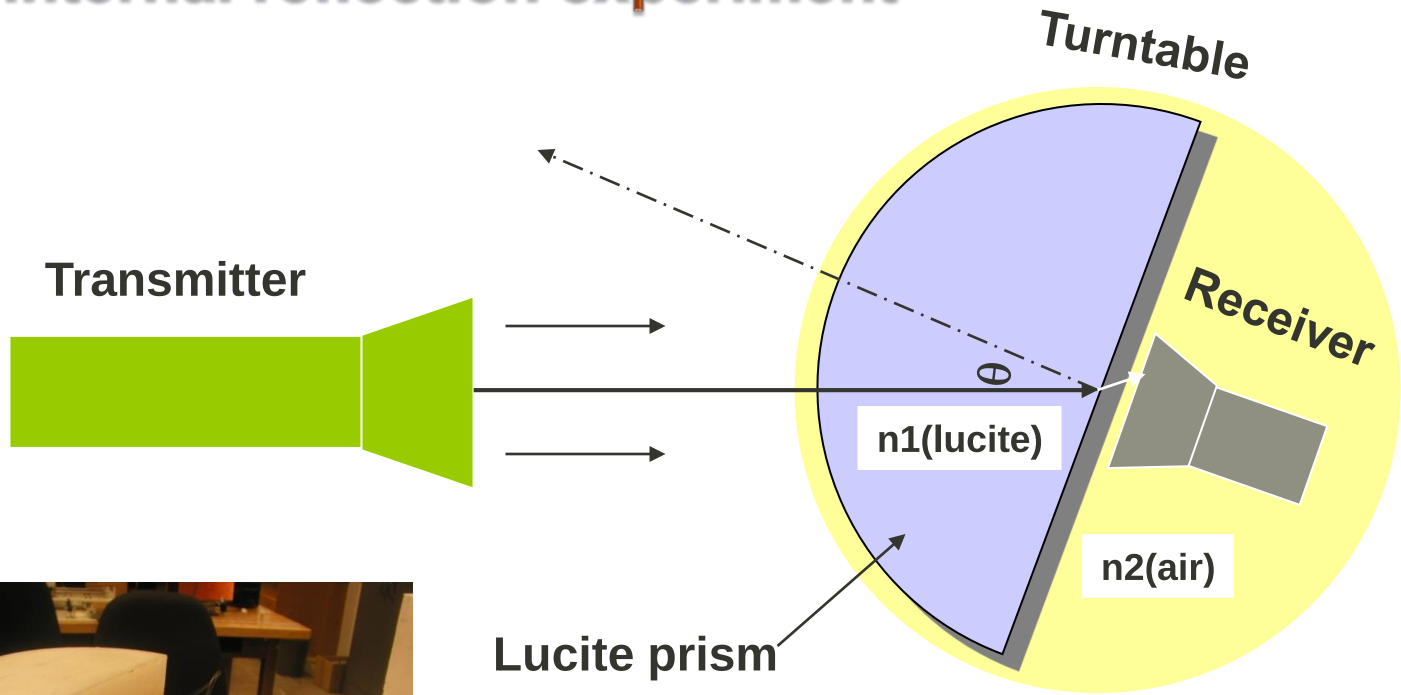


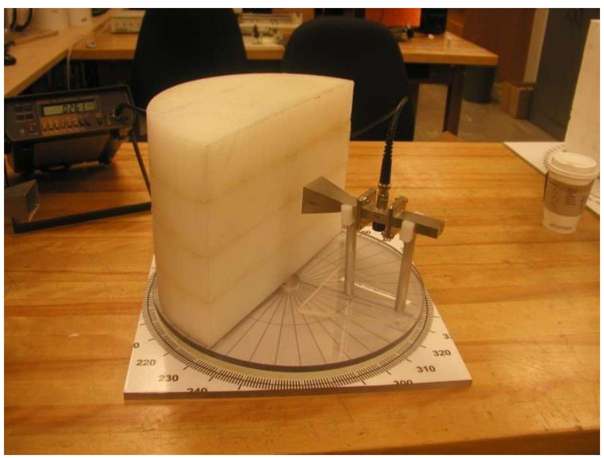

**Equation for critical angle:**

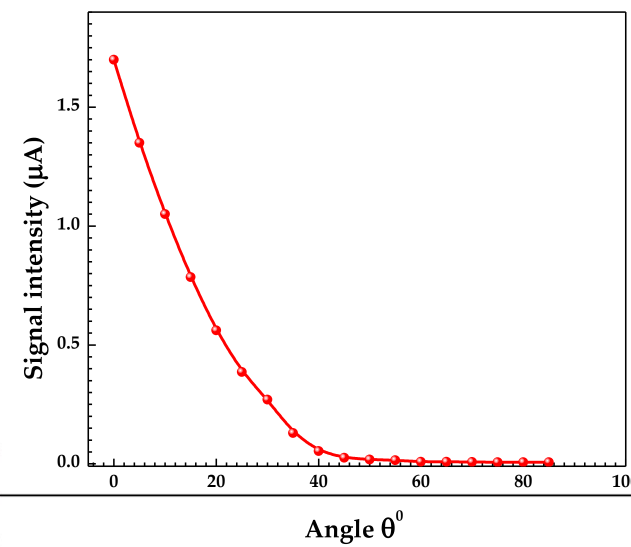
$$n_1 \sin\theta_c = n_2 \sin 90^o$$

$$\theta_c = \sin^{-1}(n_2/n_1)$$

# Total internal reflection experiment

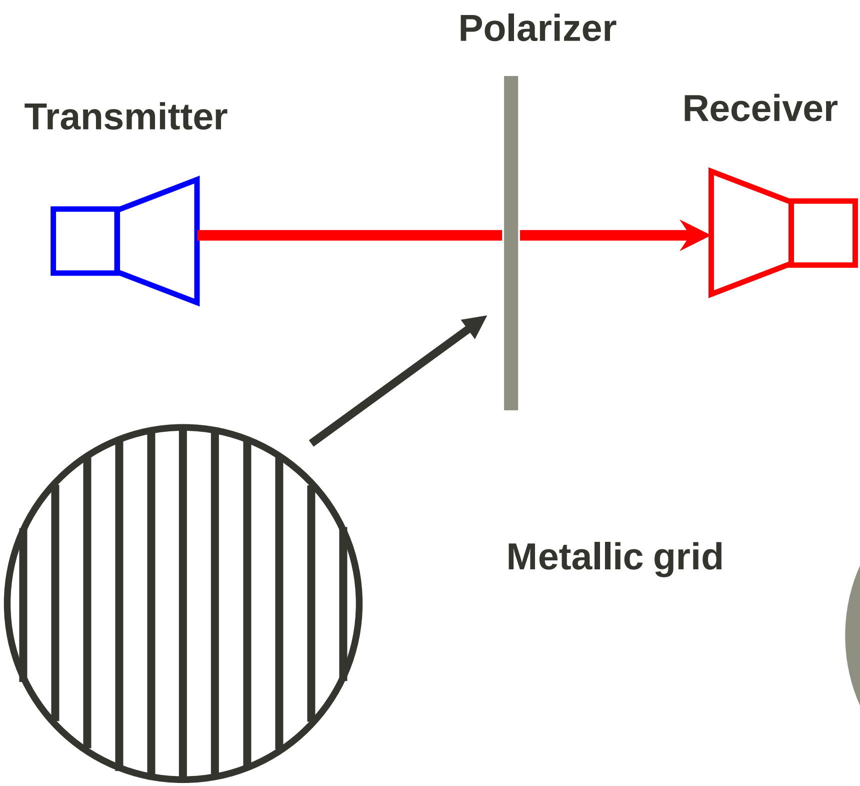


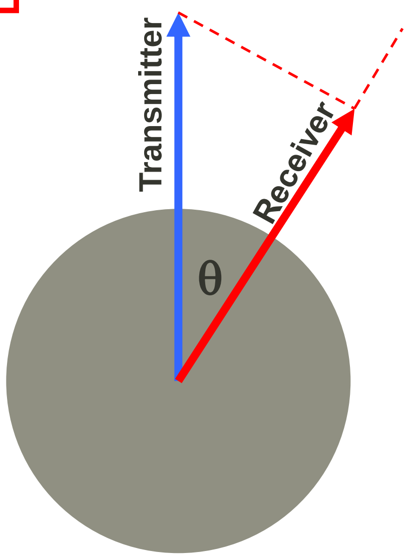






Experimental setup and the example of the data

# Microwave polarization




Etienne-Louis Malus

1775 – 1812

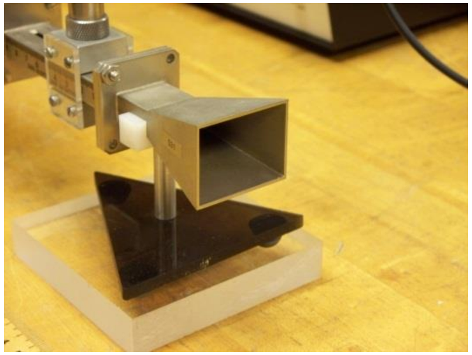
## Malus law

$$E=E_0\cos\theta$$

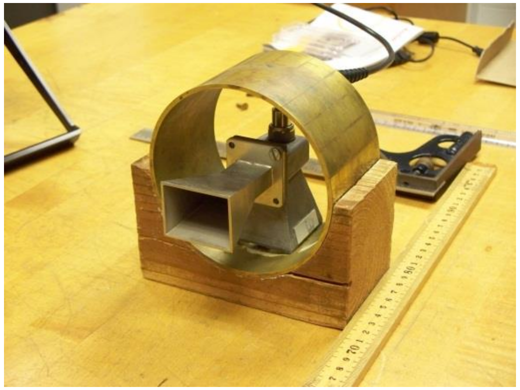
$$I\propto E^2$$

$$I=I_0\cos^2\theta$$

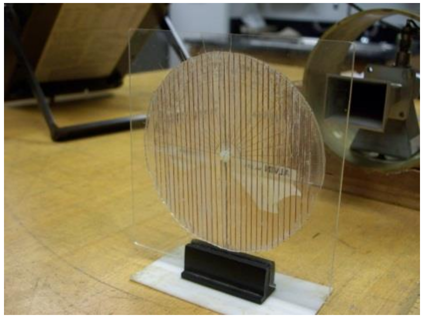
# Microwave polarization



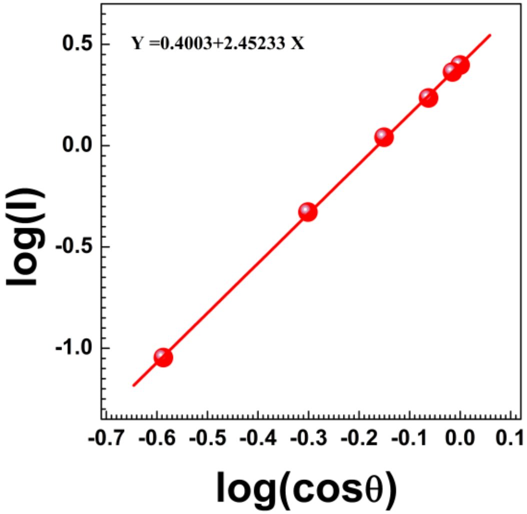
Transmitter



Rotatable receiver



Polarizer

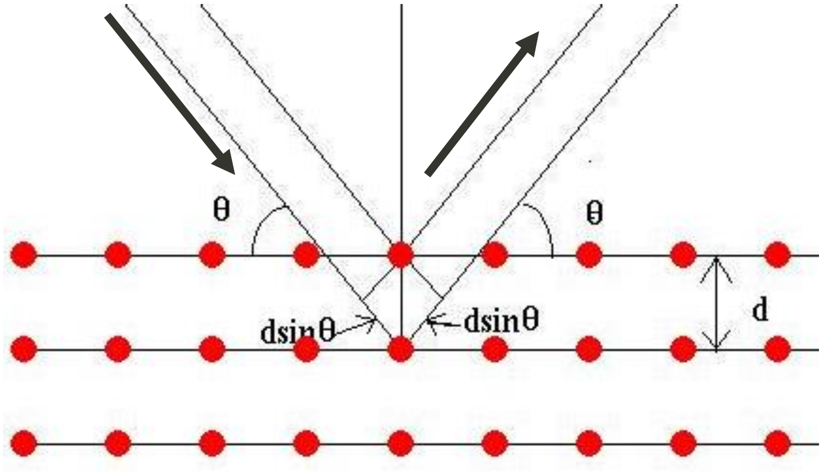


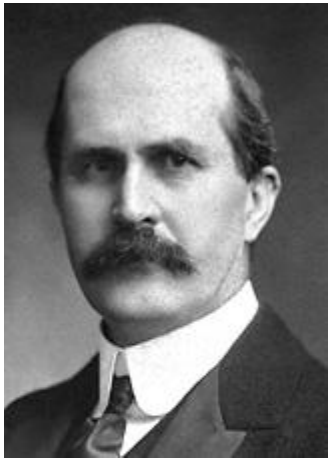

Experimental data

$$I=I_0\cos^2\theta$$

# Bragg diffraction

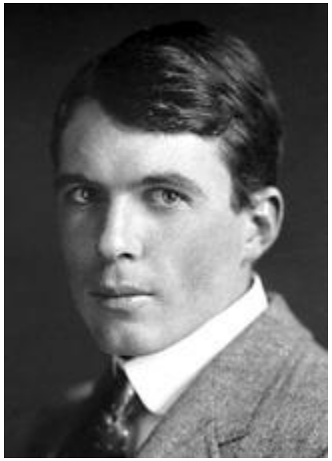
Interference of the EM waves reflected from the crystalline layers



Sir William Henry Bragg

1862-1942



William Lawrence Bragg

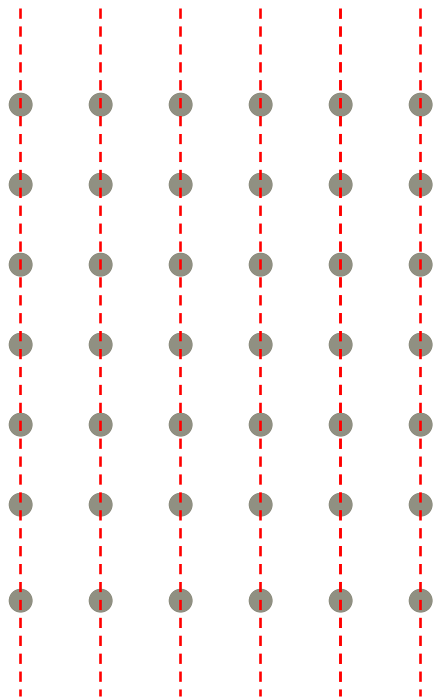
1890-1971

**The Nobel Prize in Physics 1915**

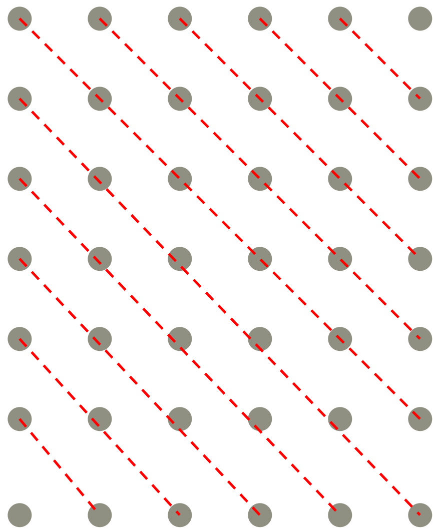
**"for their services in the analysis of crystal structure by means of X-rays"**

$$n\lambda = 2d\sin\theta$$ *Bragg's Law*

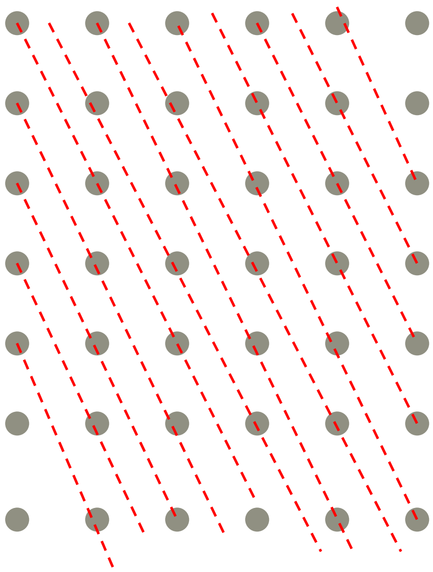
# Bragg diffraction



(100)



(110)



(210)

**Different orientations of the crystal**

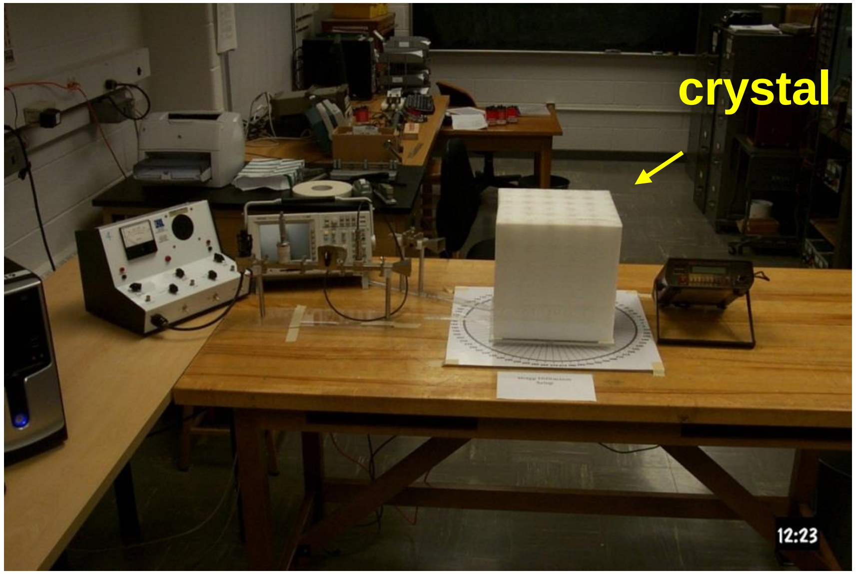
# Bragg diffraction

In our experiment λ~3cm;
For cubic symmetry the angles of Bragg peaks can be calculated from:

$n\lambda=2d\sin\theta$

$\lambda<2d$

$$\left(\frac{\lambda}{2d}\right)^2=\frac{\sin^2\theta}{h^2+k^2+l^2}$$

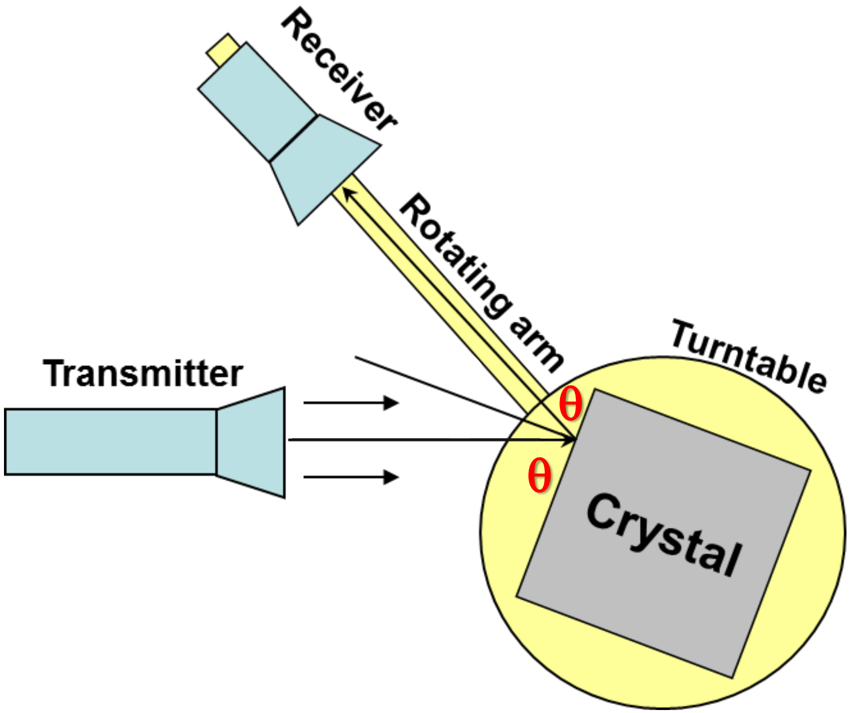


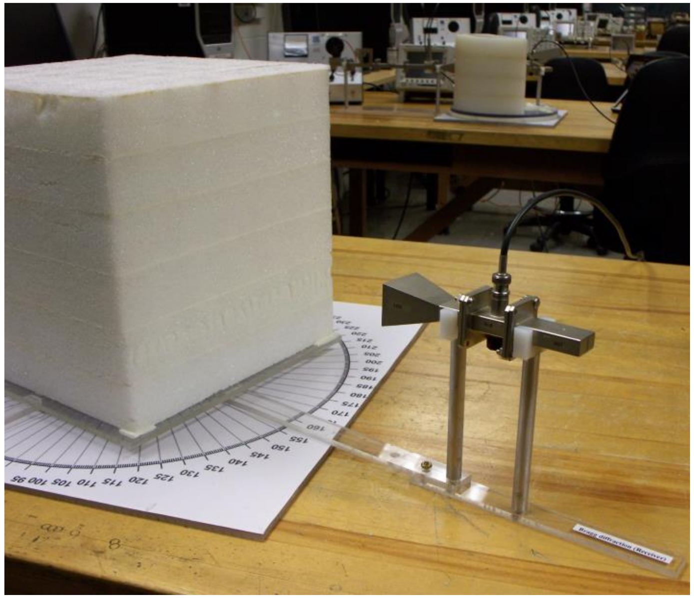

Experimental setup

where h,k,l are the Miller Indices.

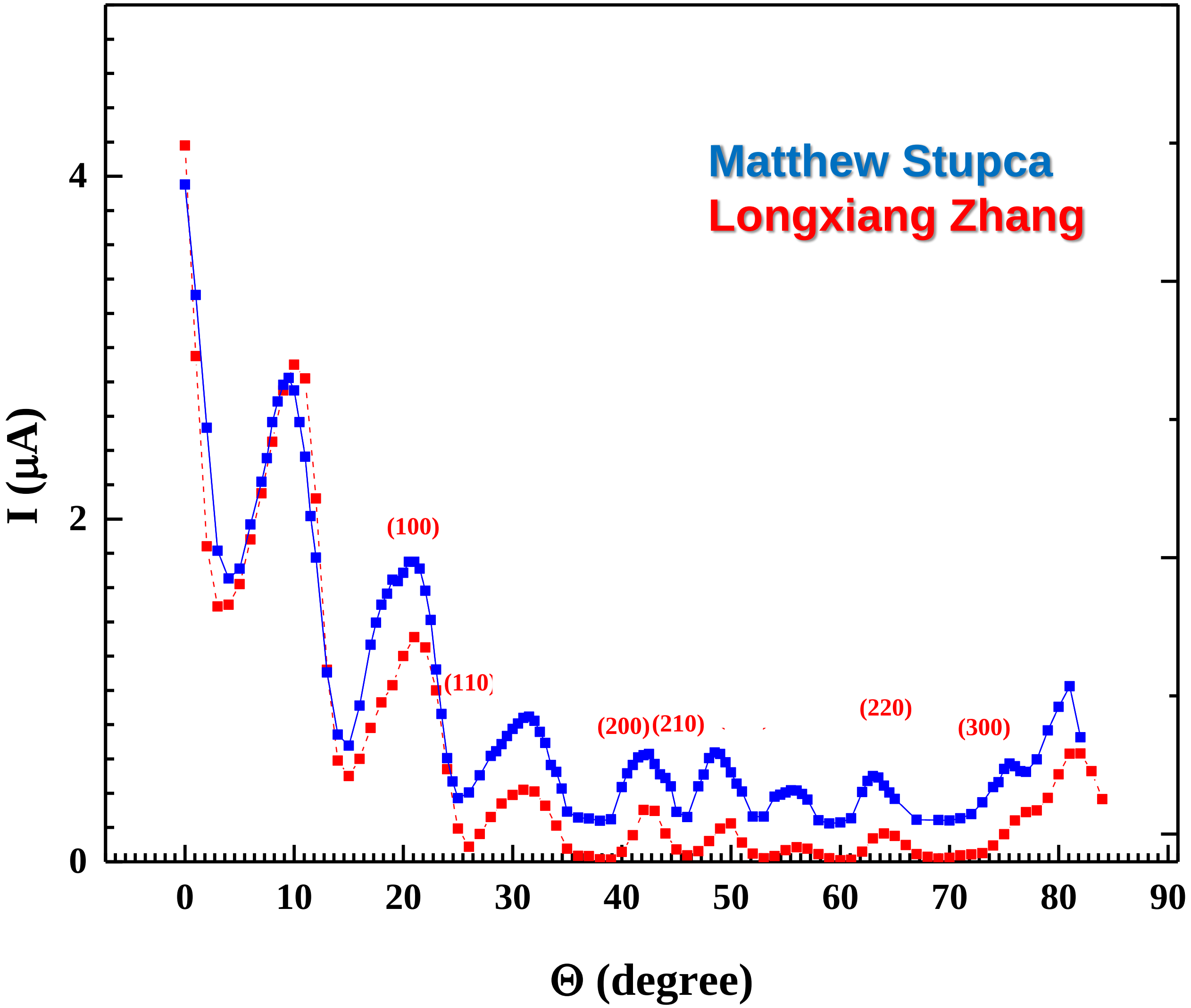
For crystal with d=5cm and λ=3cm the 3 first Bragg peaks for (100) orientation can be found at angles: ~17.5°; 36.9° and 64.2°

# Bragg diffraction



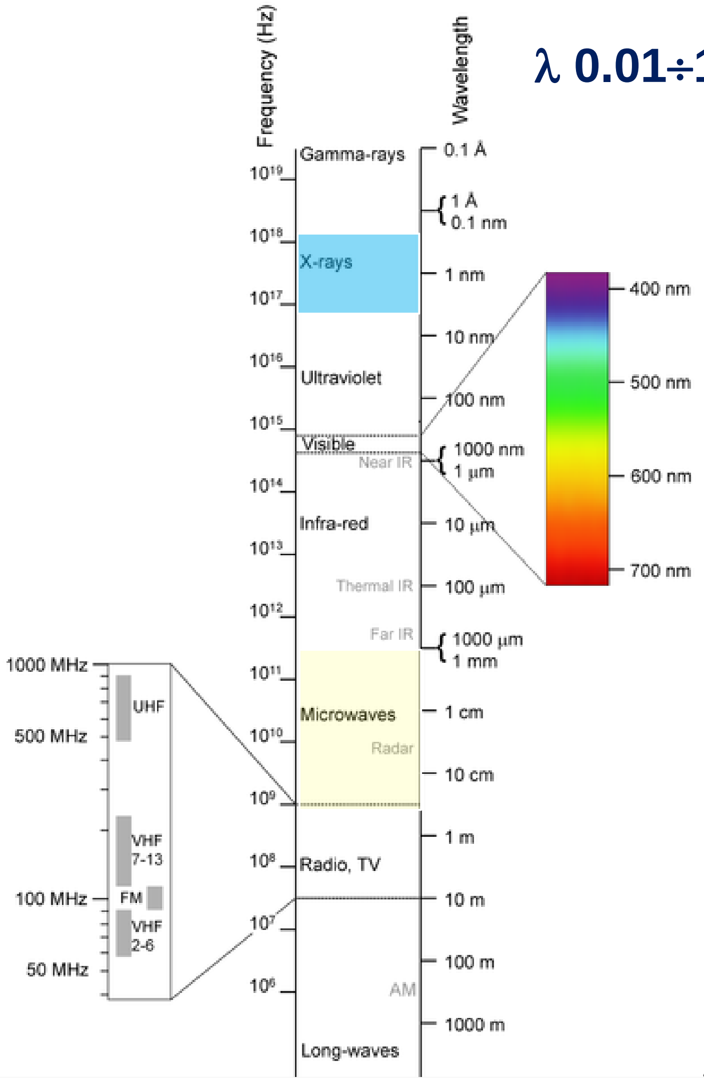
# Bragg diffraction. Results.*



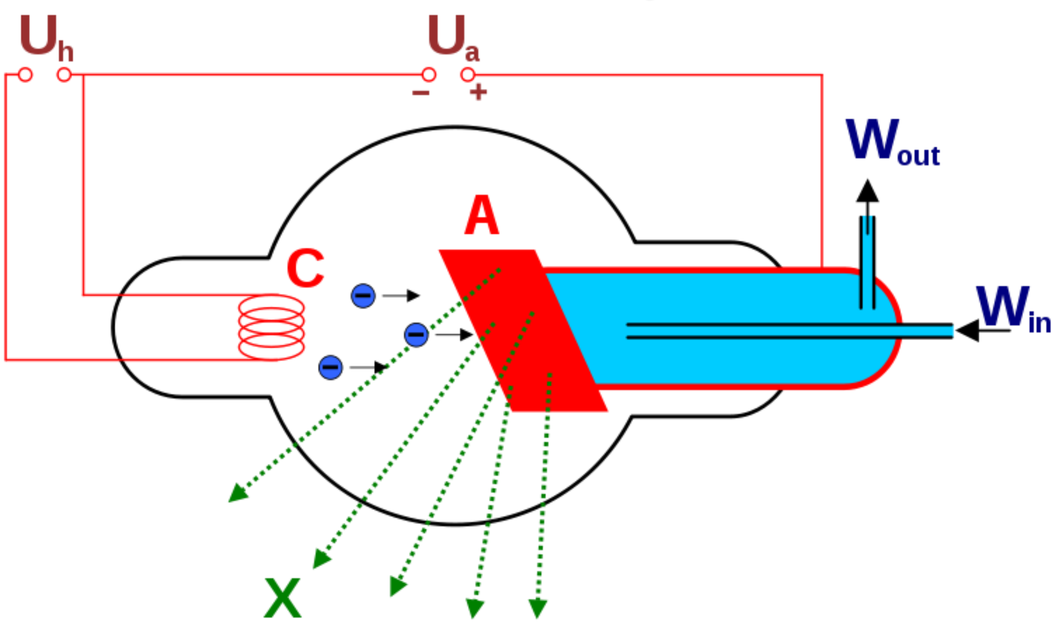

*courtesy of Matthew Stupca

# Bragg diffraction. X-rays.

$\lambda$ 0.01÷10nm




X-ray tube




*courtesy of Wikipedia

# Bragg diffraction. X-rays.

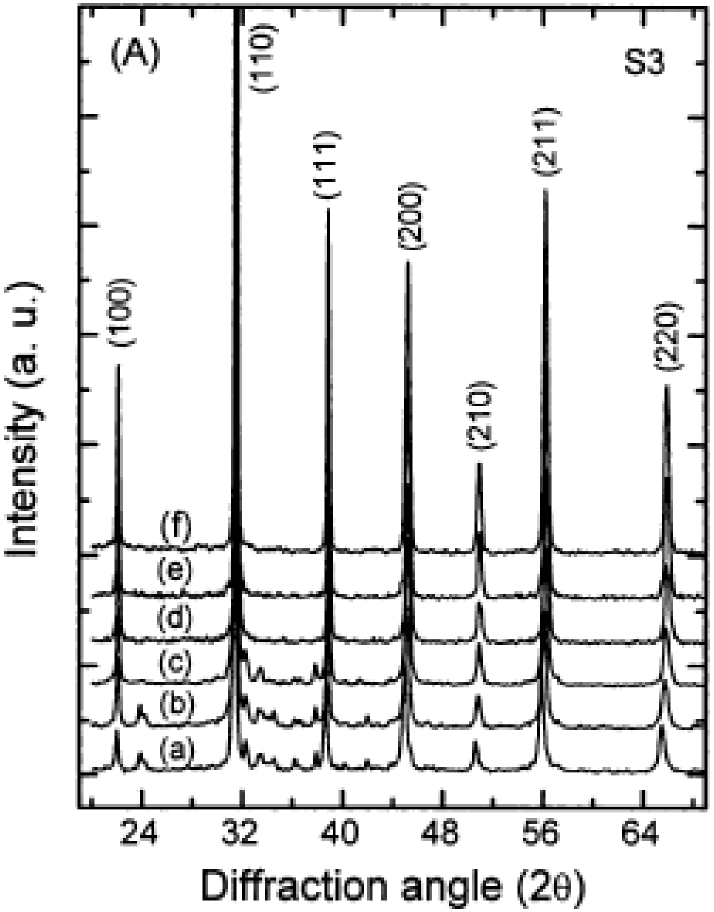
**X-ray K-series spectral line wavelengths (nm) for some common target materials**

| Target | $K\beta_1$ | $K\beta_2$ | $K\alpha_1$ | $K\alpha_2$ |
|---|---|---|---|---|
| Fe | 0.17566 | 0.17442 | 0.193604 | 0.193998 |
| Co | 0.162079 | 0.160891 | 0.178897 | 0.179285 |
| Ni | 0.15001 | 0.14886 | 0.165791 | 0.166175 |
| Cu | 0.139222 | 0.138109 | 0.154056 | 0.154439 |
| Zr | 0.70173 | 0.68993 | 0.78593 | 0.79015 |
| Mo | 0.63229 | 0.62099 | 0.70930 | 0.71359 |

David R. Lide, ed. (1994). *CRC Handbook of Chemistry and Physics 75th edition*. CRC Press. pp. 10–227

***courtesy of Matthew Stupca**

# Bragg diffraction. X-rays.



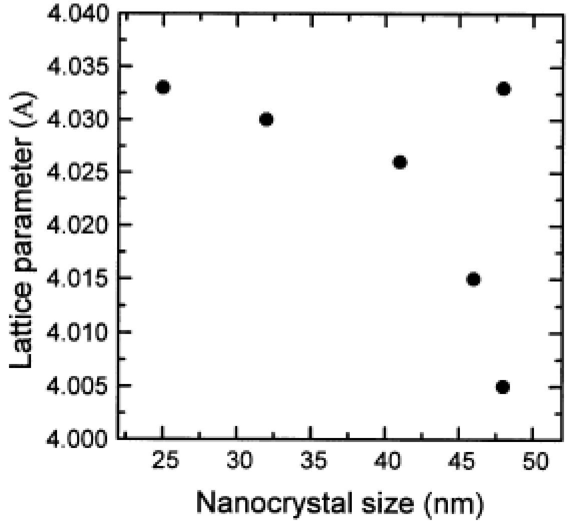




Fig. 4. Lattice parameter $c$ versus the grain size in the $BaTiO_3$ nanocrystal.

F
a
fi

Solid State Communications 119 (2001) 659–663

Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang[a,*], Zhen Yin[a], Qiang Chen[a], Weifeng Zhang[b], Wanchun Chen[c]

***courtesy of Matthew Stupca**

# Comments and suggestions

1. Klystron is very hot and the high voltage (~300V) is applied to repeller.
2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.
3. Do not put on the tables any extra stuff – this will cause extra reflections of microwaves and could result in smearing of the data.
4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week

Good luck !