

## Subharmonics in Torsional Oscillator Experiment

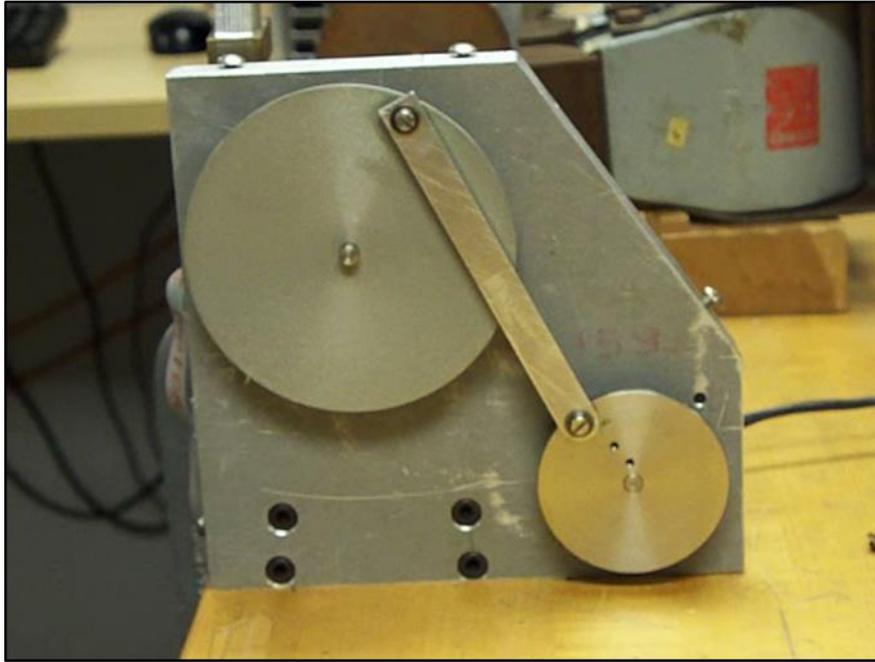


Fig.1. Kinematics of the piano wire drive

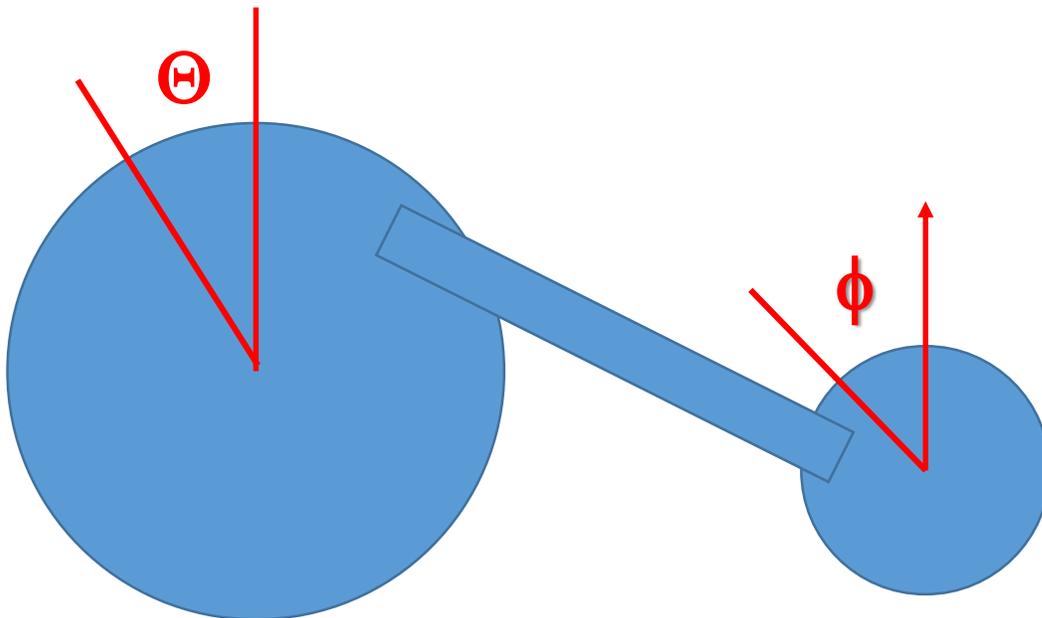


Fig.2.  $\phi$  is driven by the step motor sinusoidal with angular velocity  $w$  but on piano wire are applied  $\omega$  and also two harmonics  $2\omega$  and  $3\omega$  because of specific of the geometry of design.

The drive motion of the wire can be written as:

$$\Theta = \Theta_1 \sin(\omega t) + \Theta_2 \sin(2\omega t + \phi_1) + \Theta_3 \sin(3\omega t + \phi_2)$$

$\Theta_2$  is about 6.7% of  $\Theta_1$  and  $\Theta_3$  is about 2.8%. The presence of the high harmonics could be observed by applying the FFT on time domain trace of the piano wire drive motion.

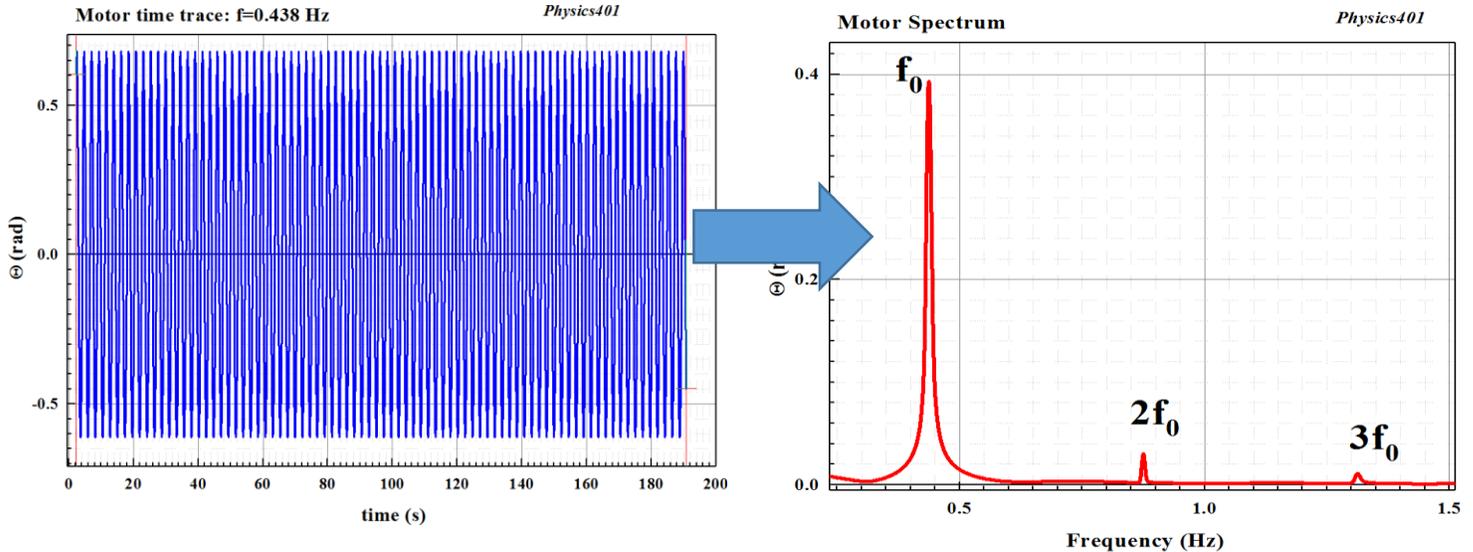
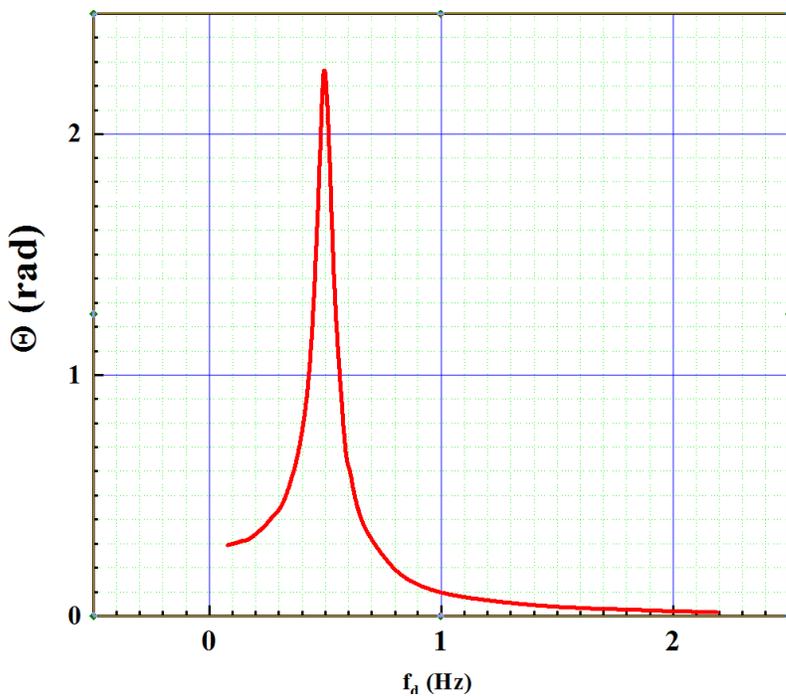


Fig.3. Time domain and frequency domain images of the piano wire drive motion



Now we have to recall the shape of the resonance curve of the motion of the pendulum and let us analyze how the pendulum will respond on the driving signal (with harmonics) of different frequency

Fig. 4. Resonance curve of the pendulum motion

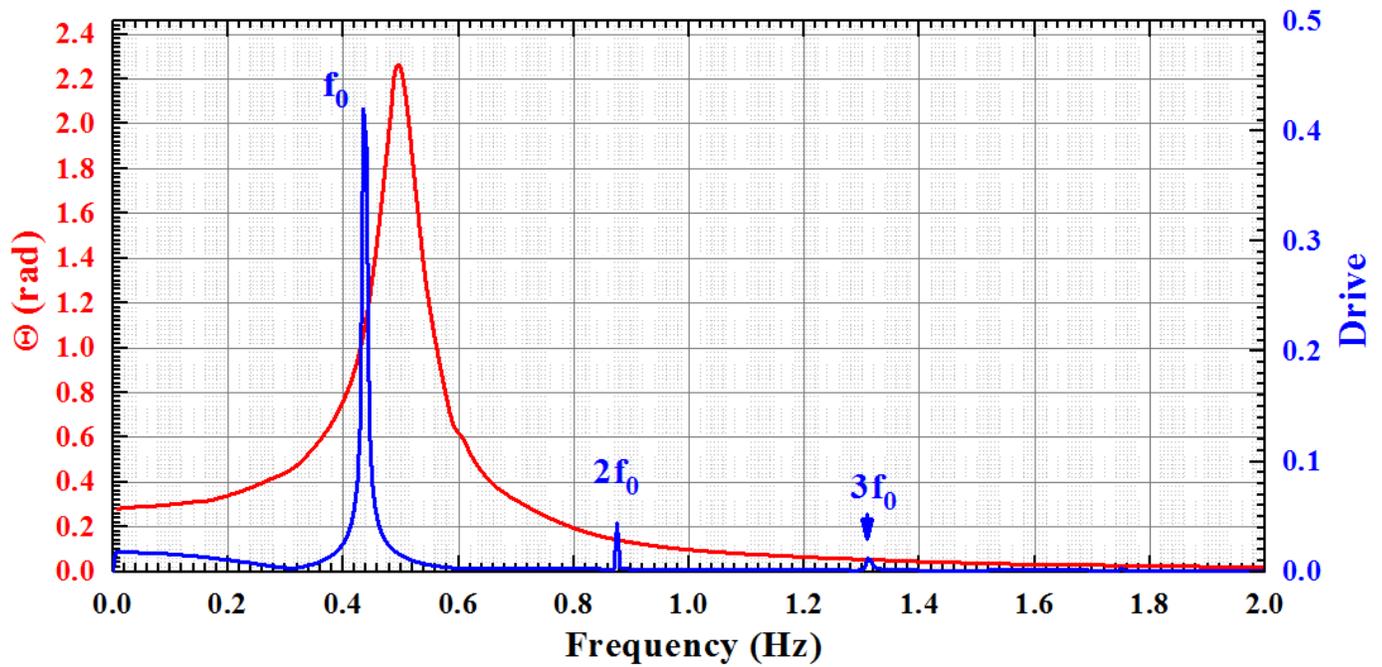


Fig.5. Driving the pendulum at a frequency close to resonance. The two highest harmonics of the drive system will not effect too much the motion of the pendulum because pendulum response on these frequencies is rather small.

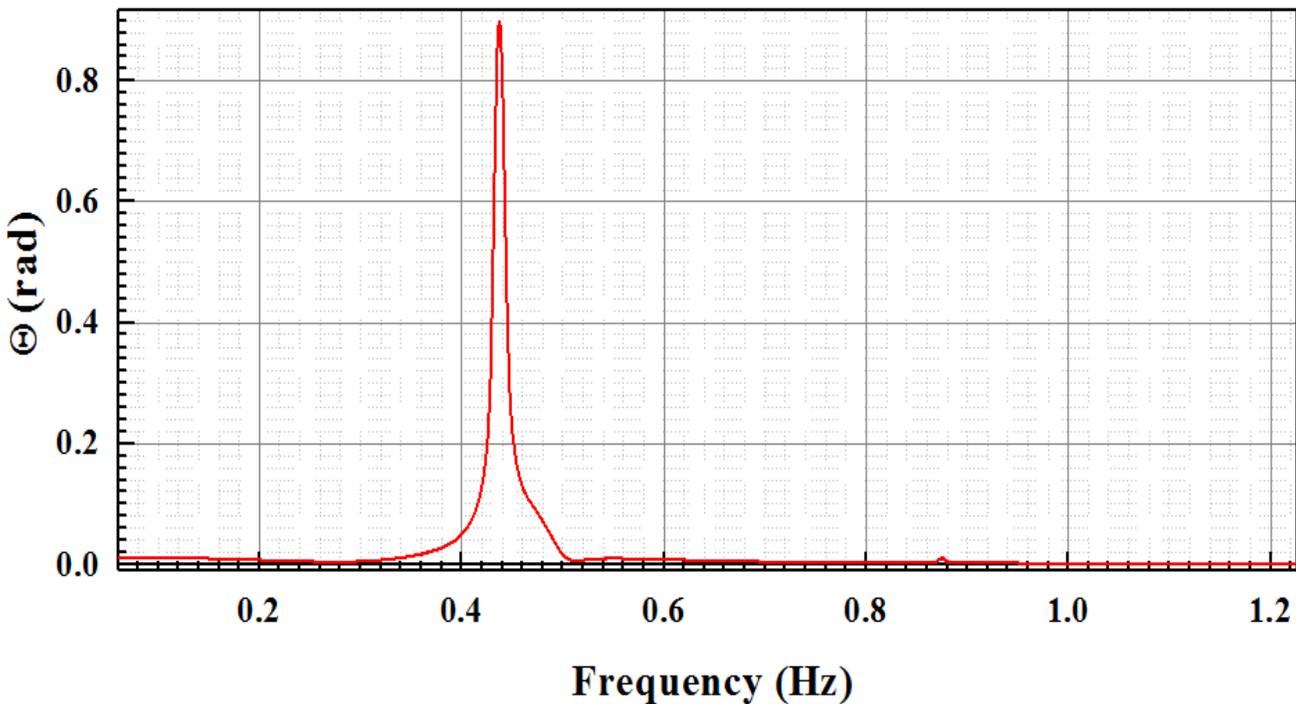
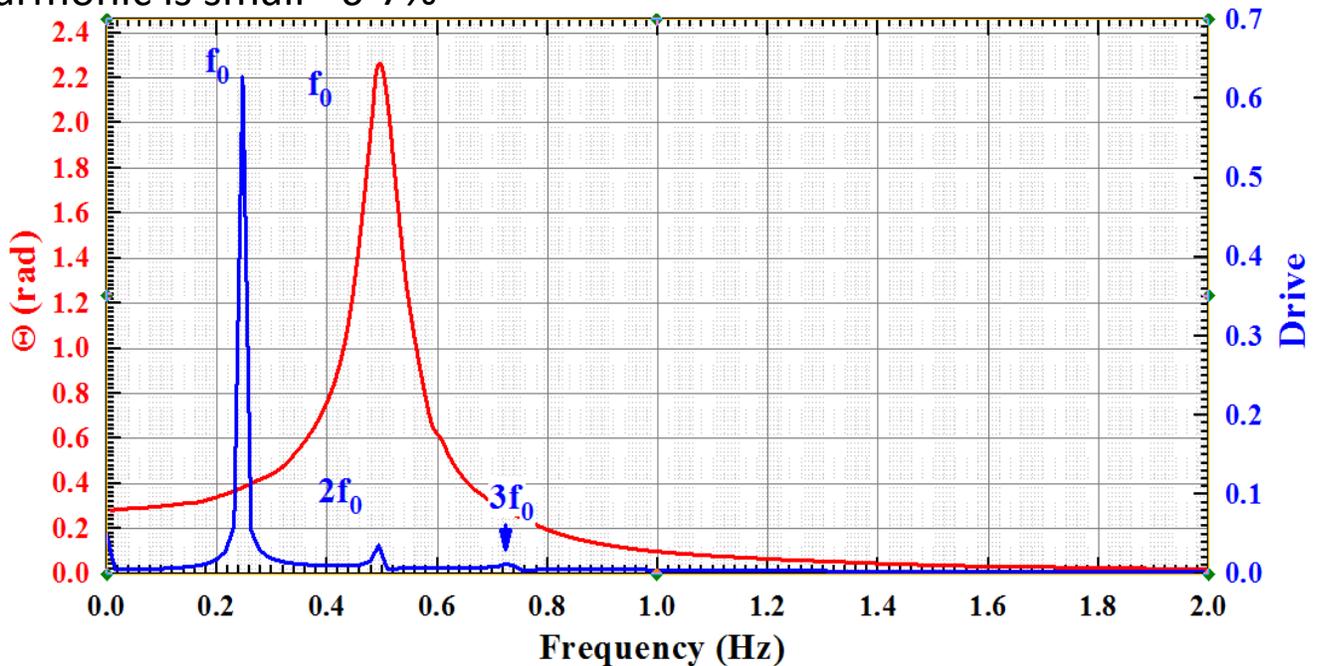
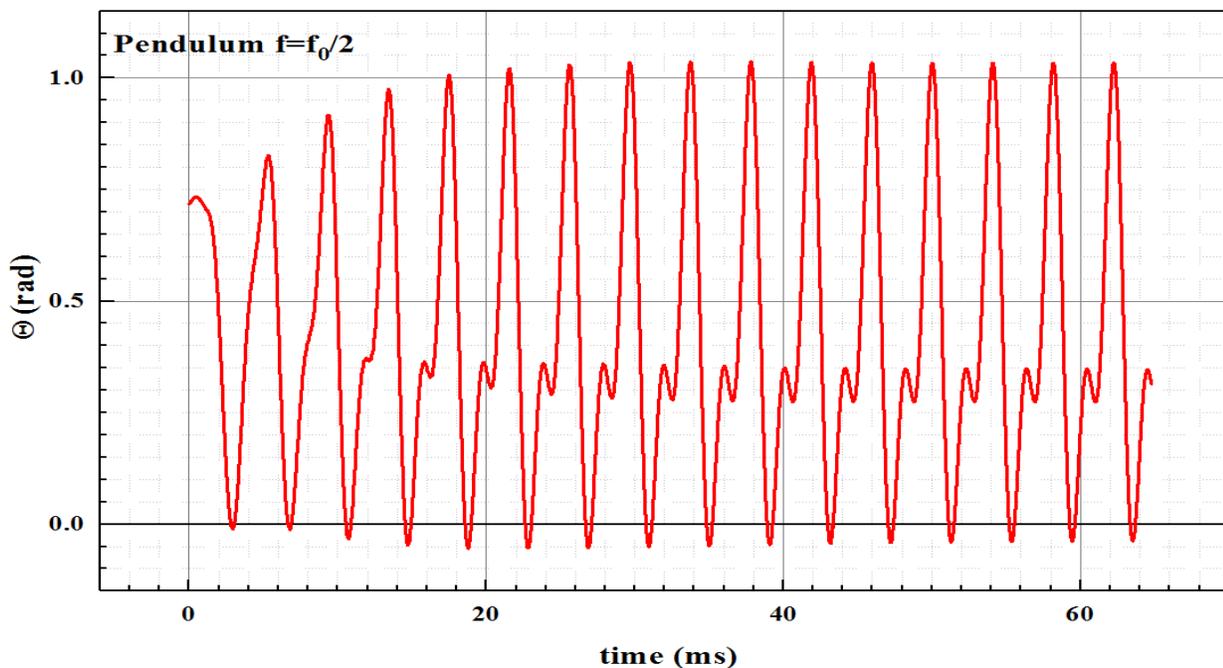


Fig.6. The frequency response of the pendulum on the fundamental driving frequency being in the vicinity of the pendulum resonance. There is no significant contribution coming from  $2f$  and  $3f$  harmonics.

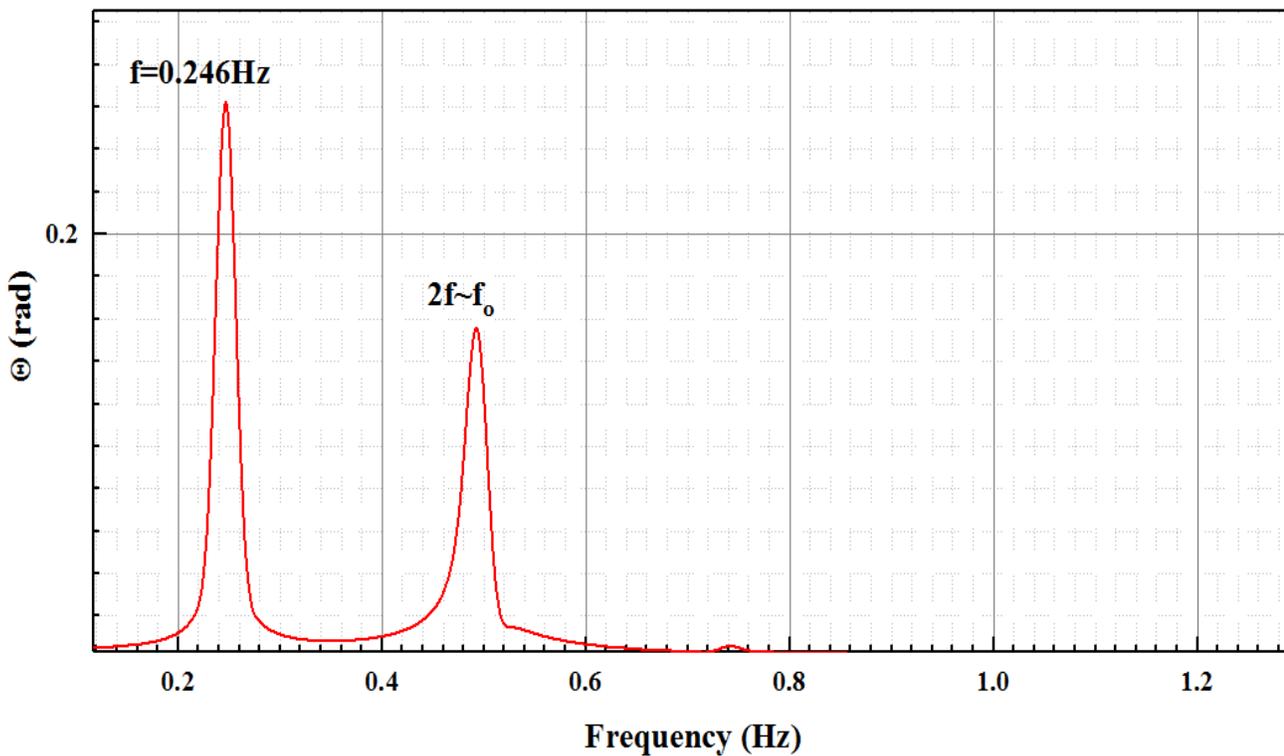
Now the driving frequency is set as  $f_0/2$  and it means that the second harmonics of the drive will correspond the resonance frequency of the pendulum and because of this it will create significant contribution to the pendulum motion no matter that the amplitude of the second harmonic is small  $\sim 6-7\%$



**Fig.7. Pendulum resonance curve (red) and driving signal spectrum (blue) in the case when fundamental frequency of the drive is one half of the pendulum resonance frequency**



**Fig.8. Pendulum response on the  $f_0/2$  frequency**



**Fig.9. Frequency spectrum corresponding the pendulum motion driven at frequency  $f_0/2$ , where  $f_0$  is the resonance frequency of the pendulum. Now the second harmonic of the drive comes exactly in resonance with pendulum and as a result contributes to the net response comparable to fundamental frequency.**

These distortions of the pendulum motion have different nature than beats and correspond to the steady state solution in the case of the multi frequency drive and will not disappear in time like a transient effects.