Microwave cavities

Physics 401, Spring 2017
Eugene V. Colla
Agenda

- Waves in waveguides
- Standing waves and resonance
- Setup
- Experiment with microwave cavity
- Comments on Bragg diffraction experiment
Reminder: Propagation of Plane Waves

Maxwell’s Equations

\[ \nabla \vec{D} = 0 \]
\[ \nabla \vec{B} = 0 \]
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]
\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \]

uniform plane wave traveling in z-direction \( \leftrightarrow H \perp E \)

wave equation

\[ \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \]

general form of solution

\[ E_z(z,t) = f\left( t - \frac{z}{v} \right) + g\left( t + \frac{z}{v} \right) \]

propagation speed

\[ v = \frac{1}{\sqrt{\varepsilon \mu}} \]
\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \]

E vs H

\[ E_x = E_0 e^{i(\omega t - kz)} \]
\[ H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \]
\[ E_x = ZH_y \]
Wave Propagation in Wave Guides

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t - k_z)} \]
Standing Waves in Cavities

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t - kz)} \]

\[ E_y = E_0 \sin k_x x \cdot e^{i(\omega t + kz)} \]

= \[ L = n^*\lambda/2 \]
Standing Waves in Cavities

\[ E_y = E_y(z) \]

\[ E_y = E_y(x \text{ or } z) \]
Resonances for transverse Electric Waves

\[ \omega_{mnp}^2 = v_0^2 \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 + \left( \frac{p\pi}{c} \right)^2 \right] \]

- Phase velocity

**TE\textsubscript{101} mode**: \( m=1, n=0, p=1 \)

\[ \omega_{101}^2 = v_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{1}{c} \right)^2 \right] \]
Equivalent Circuit

coupling loop

cavity

coaxial wave guide

outer conductor

inner conductor

line

$R$

$C$

$L$

$C$

$L$

$R$

Impedance of wave guide

$Z_0$

4/3/2017
**Coupling between Wave Guide and Cavity**

- **Impedance of wave guide**
- **coupling loop**
- **coaxial wave guide**
- **cavity**
- **outer conductor**
- **inner conductor**

**Equations:**

\[
Q_L = \frac{\omega L}{R + Z_0}
\]

\[
Q_L = \frac{\omega L}{Z_0 \left(1 + \frac{R}{Z_0}\right)} = \frac{Q_0}{1 + \beta},
\]

\(\beta\): coupling coefficient

**Maximum power transfer:**
\(Z_0 = R \rightarrow \beta = 1\)

\[\Rightarrow Q_L = \frac{1}{2} Q_0,\]

**\(Q_0\)** - quality factor without external load
Microwaves in Cavities. Overview of the Experiment.
Microwaves in Cavities. The Setup of the Experiment.
Experiment. Wavelength measurement.

Use detector to find distance between minimums in the slotted line (wave guide)
Use detector to find distance between minimums in the slotted line (wave guide). Distance between consequent minima correspond $\lambda/2$. 

Experiment. Wavelength measurement.
Use plunger to change the dimension of the cavity in z-direction and search for maxima in power stored using the cavity detector. Identify TE\textsubscript{101} and TE\textsubscript{102}.
Experiment. Cavity resonance.

\[ \omega_{102}^2 = v_0^2 \pi^2 \left[ \left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2 \right] \]

\[ f_{102} = \frac{v_0}{2} \sqrt{\left( \frac{1}{a} \right)^2 + \left( \frac{2}{c} \right)^2} \]

<table>
<thead>
<tr>
<th>TE_{102}</th>
<th>I (mA)</th>
<th>l (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>c_0=13.88</td>
<td>12</td>
<td>13.9</td>
</tr>
<tr>
<td>c_1=13.86</td>
<td>10</td>
<td>13.8</td>
</tr>
<tr>
<td>c_2=13.92</td>
<td>8</td>
<td>13.7</td>
</tr>
</tbody>
</table>

\[ Q = \frac{f_0}{\Delta f} \sim 450 \]
By moving the plunger we changing the resonance frequency of the cavity.
Experiment. Cavity resonance. Oscillator tuning.
1. Oscilloscope should run in X-Y mode
2. To plot the I(f) dependence you have to download both Ch1 and Ch2 data
3. Use triangular waveform as a voltage applied to modulation input of the oscillator
4. Use a proper time scale setting on the scope which could estimated from scanning frequency
5. Apply the calibration equation to calculate the frequency of the oscillator from the modulation voltage

\[ f = 0.03706 \cdot V_{\text{mod}} + 2.9349 \]
Experiment. Cavity resonance. Oscillator tuning.

Voltage tunable oscillator ZX95-3250a-S+ from Mini-Circuits®
FM Calibration for microwave oscillator

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**Equation**

\[ y = \text{Intercept} + B1x + B2x^2 \]

**Weight**

No Weighting

**Residual Sum of Squares**

8.0148E-5

**Adj. R-Square**

0.99977

<table>
<thead>
<tr>
<th>frequency</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>2.91E74</td>
<td>8.09E-05</td>
</tr>
<tr>
<td>B1</td>
<td>0.03E-08</td>
<td>3.43E-05</td>
</tr>
<tr>
<td>B2</td>
<td>-4.41E-5</td>
<td>3.20E-05</td>
</tr>
</tbody>
</table>
Experiment. Cavity resonance.

Q = 340

Q = 133

TE101
By changing of the coupling between oscillator and cavity we can control the quality factor of the cavity resonance but in the same time we changing the power delivered to the cavity.
Experiment. Coupling: Detecting of the Magnetic field.

While in resonance: turn orientation of the input loop from the vertical direction in 10° steps to 360°. Read cavity detector.
Experimental result.
Fitted to
$A|\cos(\alpha + \phi)|^n + A_0$

Experiment. Coupling: Detecting of the Magnetic field.
Presence of dielectric reduces length of cavity at a given resonance frequency $\omega_0$.
This effect grows with the electric field strength $E_y$.

1. Without dielectric the cavity length at resonance is $c_0$.
2. Place dielectric into cavity and move in 0.5cm steps, $l_i$.
3. At each place tune plunger to resonance and record $c_i$.
4. Plot $\Delta c_i = |c_0 - c_i|$ versus $l_i$: this measures now $E_y$ vs $l_i$.
Electric Field Distribution.

**electric field distribution**

**TE\textsubscript{102} mode**

<table>
<thead>
<tr>
<th>cavity return (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>dielectric position z (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>15</td>
</tr>
</tbody>
</table>

**Courtesy of P. Debevec**
Quality factor (TE\textsubscript{101} mode) of unloaded cavity can be calculated as:

$$Q_0 = \frac{abc(a^2 + c^2)}{\delta \left[ 2b(a^3 + c^3) + ac(a^2 + c^2) \right]}$$

$\delta$ is the skin depth at frequency $\omega_0$

$$\delta = \sqrt{\frac{2\rho}{\mu \omega}}$$

$\rho$ – resistivity of the cavity material

$\mu = \mu_r \mu_0 \approx \mu_0 = 4\pi \times 10^{-7}$
Calculation of the Quality factor of the Unloaded Cavity

For red brass \( \rho = 6 \times 10^{-8} \Omega \text{m} \)
\( \mu \approx 4\pi \times 10^{-7} \)
\( \delta = 2.25 \times 10^{-6} \text{m} \)
\( a = 7.22 \text{cm}, \ b = 3.42 \text{ cm}, \ c = 6.91 \text{cm} \) (TE\(_{101}\))

\[ \delta = \sqrt{2 \rho / \mu \omega} \]

\[ Q_0 = \frac{abc \left( a^2 + c^2 \right)}{\delta \left[ 2b \left( a^3 + c^3 \right) + ac \left( a^2 + c^2 \right) \right]} \]

\( Q_0 \approx 7700 \)
Bragg diffraction.

\[ n\lambda = 2d \sin \theta \]  

Bragg's Law

\[ \theta' = 90^\circ - \theta \]
Bragg diffraction. Results.*

Matthew Stupca
Longxiang Zhang

I (µA)

Θ (degree)
Bragg diffraction. Possible origin of the \( \sim 10^\circ \) peak

Second order reflection

Lloyd's mirror effect