Qualitative Studies with Microwaves

Physics 401, Fall 2018
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Qualitative Studies with Microwaves

The main goals of the Lab:

✓ Refreshing the memory about the electromagnetic waves propagation

✓ Microwaves. Generating and detecting of the microwaves

✓ Microwaves optics experiments

This is two weeks Lab
Microwaves place in the electromagnetic spectrum

The microwave range includes ultra-high frequency (UHF) (0.3–3 GHz), super high frequency (SHF) (3–30 GHz), and extremely high frequency (EHF) (30–300 GHz) signals.
Application of the microwaves

- Microwave oven (2.45GHz)
- Communication (0.8-2.69GHz)
- Satellite TV (4-18GHz)
- Radar (up to 110GHz)
- Motion detector (10.4GHz)
- Weather radar (8-12GHz)
- GPS 1.17-1.575 GHz

*by courtesy Wikipedia
Maxwell equations

\[ \nabla \vec{D} = \rho \quad (1) \]

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (3) \]

\[ \nabla \vec{B} = 0 \quad (2) \]

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (4) \]

If \( \rho = 0 \) and \( \vec{J} = 0 \) and taking into account that \( \vec{D} = \varepsilon \vec{E} \) and \( \vec{B} = \mu \vec{H} \), (1) and (4) can be rewritten as

\[ \nabla \vec{D} = \varepsilon \left[ \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \right] = \mathbf{0} \]

\[ \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \]
Now assuming that plane wave propagate in z direction and what leads to $E_y = E_z = 0$ and $H_x = H_z = 0$

Now (3) and (4) could be simplified as

$$\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t} \quad (5)$$

$$\frac{\partial H_y}{\partial z} = -\varepsilon \frac{\partial E_y}{\partial t} \quad (6)$$

where $\mu = \mu_0 \mu_r$ and $\varepsilon = \varepsilon_0 \varepsilon_r$

$\mu_0$ is the free space permeability, $\varepsilon_0$ is the free space permittivity.

$\mu_r$ is permeability of a specific medium, $\varepsilon_r$ is permittivity of a specific medium.
Combining (5) and (6) (see Lab write-up for more details) we finally can get the equations of propagation of the plane wave:

\[
\frac{\partial^2 E_x}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 E_x}{\partial t^2} \tag{7}
\]
\[
\frac{\partial^2 H_y}{\partial z^2} = \frac{1}{v^2} \frac{\partial^2 H_y}{\partial t^2} \tag{8}
\]

where \( v = \frac{1}{\sqrt{\varepsilon \mu}} \)

\[
E_x = E_{x0} \cos(\omega t - kx)
\]
\[
H_y = H_{y0} \cos(\omega t - kx)
\]

Solution for (7) and (8) can found as

\[ H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \quad \text{or} \quad E_x = Z H_y \]

where \( Z = \sqrt{\frac{\mu}{\varepsilon}} \) known as characteristic impedance of medium

\( k \) is wave vector and is defined as

\[ k = \frac{2\pi}{\lambda} \quad \text{or} \quad k = \frac{\omega}{v} \]

For free space \( (\varepsilon_r=1 \text{ and } \mu_r=1) \)

\[ Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ohms} \]
Plane wave

\[ E_x = E_{x0} \cos(\omega t - kx) \]
\[ H_y = H_{y0} \cos(\omega t - kx) \]

\[ v = \frac{1}{\sqrt{\varepsilon \mu}} \quad H_y = \sqrt{\frac{\varepsilon}{\mu}} E_x \]

\[ Z = \sqrt{\frac{\mu}{\varepsilon}} \quad E_x = Z H_y \quad k = \frac{2\pi}{\lambda} \quad \text{or} \quad k = \frac{\omega}{v} \]

\[ Z_{fs} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \approx 377 \text{ohms} \]

For free space (\( \varepsilon_r = 1 \) and \( \mu_r = 1 \))

*by courtesy Wikipedia*
Generating of the microwaves

Vacuum tubes: klystron, magnetron, traveling wave tube

Solid state devices: FET, tunneling diodes, Gunn diodes

Tunable frequency from 9 to 10GHz; maximum output power 20mW

Microwave oven magnetron; typical power 0.7-1.5kW

Heated cathode as electron source
Klystron. A piece of history.

Russell Harrison
Varian (April 24, 1898 – July 28, 1959)

Sigurd Fergus
Varian (May 4, 1901 – October 18, 1961)

Varian Brothers...Klystron Tube (1940)
Generating of the microwaves. Klystron.

**Advantages:**
- well defined frequencies,
- high power output

**Reflection klystron**

High power klystron used in Canberra Deep Space Communications Complex (courtesy of Wikipedia)

**Single transit klystron**
GENERAL CHARACTERISTICS
Frequency Range 8,500 to 9,660 Mc
Cathode Oxide-coated, indirectly heated
Heater Voltage 6.3 Volts
Heater Current 0.44 Amperes
Experimental setup. Main components.

- Klystron
- Frequency meter
- Attenuator
- Detector
- Horn
- Digital Volt Meter or Oscilloscope
- Microwave Transmitter Arm
- Microwave Receiver Arm
- Detector
- Termination
Experimental setup. Main components.

- **Attenuator**
- **Klystron**
- **Frequency meter**
- **Detector**
Detecting of the microwaves

Taylor expansion for exp function will give:

\[ \exp(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \]

\[ I = I_0 \left[ \exp \left( \frac{eV}{kT} \right) - 1 \right] \]

\[ I \propto aV + bV^2 + \ldots \]

If \( V = V_0 \sin \omega t \):

\[ b \cdot \frac{V_0^2}{2} \left( 1 - \cos 2\omega t \right) \]

And finally:

\[ I_{DC} \propto b \frac{V_0^2}{2} + \ldots \]
Detecting of the microwaves

HSCH-9161
HSCH-9162
GaAs Detector Diode

\[ f_c \approx 200\text{GHz} \]
**Experiments: Michelson interferometer**

- **Mirror A**
- **Mirror B**
- **Transmitter**
- **Receiver**
- **Beam splitter**
- **L<sub>R</sub>, L<sub>B</sub> optical paths (OP) for "red" and "blue" rays**

**Condition for constructive interference**

\[
2 \left| L_R - L_B \right| = k \lambda
\]

**L<sub>R</sub>, L<sub>B</sub>** - optical paths (OP) for "red" and "blue" rays

**OP = n*L<sub>G</sub>**

**n – refraction index;**

**L<sub>G</sub> – geometrical length**

- **Albert Abraham Michelson**
  (1852 - 1931)
- **The Nobel Prize in Physics 1907**
Experiments:
Michelson interferometer

Physics 403 Lab Michelson interferometer setup
Experiments: Double slit Interference. T. Young 1801

For constructive Interference
\( \Delta r = n\lambda \) or \( d\sin\theta = n\lambda \)

The measured envelope of the diffraction pattern can be defined as:

\[
|\psi_{ss}|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 \left[ (kd \sin(\theta/2)) \right]
\]

where \( x = kb \sin(\theta/2) \) and \( k = \frac{2\pi}{\lambda} \) is wave vector of the plane wave.
Experiments: Double slit interference

Physics 401 Lab setup and example of the data

distance between slits=7.0 cm, slit width=1.7 cm

$\lambda=3.141\text{cm}$
Experiments: Double slit interference. Fitting

\[ |\psi_{ss}|^2 = |\psi_0|^2 \left( \frac{\sin x}{x} \right)^2 \times \cos^2 \left[ (kd \sin(\theta / 2)) \right] \]

\[ x = kb \sin(\theta / 2) \]

Model  | Two slit (User)
--- | ---
Equation  | \[ y = I_0 \left( \frac{\sin(K_1 \sin(\pi x/360+f))}{K_1 \sin(\pi x/360+f)} \right)^2 \cos^2 \left( K_2 \sin \left( \frac{\pi x}{360} + f \right) \right) + I_{00} \]
Reduced Chi-Sqr  | 94.62111
Adj. R-Square  | 0.96659

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_0 )</td>
<td>190.6014</td>
<td>3.042882</td>
</tr>
<tr>
<td>( K_1 )</td>
<td>4.384042</td>
<td>0.074754</td>
</tr>
<tr>
<td>( K_2 )</td>
<td>13.51332</td>
<td>0.052244</td>
</tr>
<tr>
<td>( f )</td>
<td>-0.01525</td>
<td>7.19E-04</td>
</tr>
<tr>
<td>( I_{00} )</td>
<td>9.572049</td>
<td>1.440409</td>
</tr>
</tbody>
</table>

Here in fitting expression:

\[ I_0 = |\psi_0|^2; \]
\[ K_1 = kb; \]
\[ K_2 = kd \]
Difference of the wave paths of “red” and “blue” rays is:

\[
\Delta S = \sqrt{h^2 + d_1^2} + \sqrt{h^2 + d_2^2} - (d_1 + d_2)
\]

For constructive interference

\[
\Delta S = n\lambda
\]
**Total internal reflection experiment. Snell’s law**

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

**Snell’s law**

Equation for critical angle:

\[ n_1 \sin \theta_c = n_2 \sin 90^\circ \]

\[ \theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right) \]
Total internal reflection experiment

Transmitter

Turntable

Lucite prism

Receiver

n1(lucite)

n2(air)

Experimental setup and the example of the data
Microwave polarization

Transmitter

Polarizer

Receiver

Metallic grid

Etienne-Louis Malus
1775 – 1812

Malus law

\[ E = E_0 \cos \theta \]

\[ I = I_0 \cos^2 \theta \]
Microwave polarization

I = I₀ \cos^2 \theta

Transmitter

Rotatable receiver

Polarizer

Experimental data

Y = 0.4003 + 2.45233 X
Bragg diffraction

Interference of the EM waves reflected from the crystalline layers

\[ n\lambda = 2d \sin \theta \]

Bragg’s Law

The Nobel Prize in Physics 1915

"for their services in the analysis of crystal structure by means of X-rays"

Sir William Henry Bragg 1862-1942

William Lawrence Bragg 1890-1971
Bragg diffraction

Different orientations of the crystal

(100)  (110)  (210)
In our experiment $\lambda \sim 3\text{cm}$; For cubic symmetry the angles of Bragg peaks can be calculated from:

$$\left( \frac{\lambda}{2d} \right)^2 = \frac{\sin^2 \theta}{h^2 + k^2 + l^2}$$

where $h, k, l$ are the Miller Indices. For crystal with $d=5\text{cm}$ and $\lambda=3\text{cm}$ the 3 first Bragg peaks for (100) orientation can be found at angles: \(~17.5^\circ; 36.9^\circ\text{ and } 64.2^\circ\)
Bragg diffraction

Receiver

Rotating arm

Transmitter

Turntable

Crystal

illinois.edu
Bragg diffraction. Results.*

I (μA)

Θ (degree)

*courtesy of Matthew Stupca
Bragg diffraction. X-rays.

\[ \lambda \approx 0.01 \div 10 \text{nm} \]

X-ray tube

*courtesy of Wikipedia*
Bragg diffraction. X-rays.

**X-ray K-series spectral line wavelengths (nm) for some common target materials**

<table>
<thead>
<tr>
<th>Target</th>
<th>Kβ₁</th>
<th>Kβ₂</th>
<th>Kα₁</th>
<th>Kα₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe</td>
<td>0.17566</td>
<td>0.17442</td>
<td>0.193604</td>
<td>0.193998</td>
</tr>
<tr>
<td>Co</td>
<td>0.162079</td>
<td>0.160891</td>
<td>0.178897</td>
<td>0.179285</td>
</tr>
<tr>
<td>Ni</td>
<td>0.15001</td>
<td>0.14886</td>
<td>0.165791</td>
<td>0.166175</td>
</tr>
<tr>
<td>Cu</td>
<td>0.139222</td>
<td>0.138109</td>
<td>0.154056</td>
<td>0.154439</td>
</tr>
<tr>
<td>Zr</td>
<td>0.70173</td>
<td>0.68993</td>
<td>0.78593</td>
<td>0.79015</td>
</tr>
<tr>
<td>Mo</td>
<td>0.63229</td>
<td>0.62099</td>
<td>0.70930</td>
<td>0.71359</td>
</tr>
</tbody>
</table>


*Courtesy of Matthew Stupca*
Bragg diffraction. X-rays.

Study of structural and photoluminescent properties in barium titanate nanocrystals synthesized by hydrothermal process

Ming-Sheng Zhang\textsuperscript{a,*}, Zhen Yin\textsuperscript{a}, Qiang Chen\textsuperscript{a}, Weifeng Zhang\textsuperscript{b}, Wanchun Chen\textsuperscript{c}

\textsuperscript{a}courtesy of Matthew Stupca
Comments and suggestions

1. Klystron is very hot and the high voltage (~300V) is applied to repeller.

2. You have to do 6 (!) experiment in one Lab session – take care about time management. The most time consuming experiment is the “Bragg diffraction”.

3. Do not put on the tables any extra stuff – this will cause extra reflections of microwaves and could result in smearing of the data.

4. This is two weeks experiment but the equipment for the week 2 will be different. Please finish all week 1 measurements until the end of this week

Good luck!