Physics 326 Midterm Exam #2 Spring 2017
Thursday Apr 20, 12:30 pm – 1:50 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **80 minutes** to work the problems.

At the beginning of the exam:
1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you **understand** what you're doing and are **not just guessing**. (If you *are* guessing, explain that, then *verify* your guess explicitly.) A good **annotated sketch** is often the best explanation of all!
3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. You should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!
5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7), but you will lose points for not simplifying answers to an **irreducible form** (e.g. $24(x^2 - y^2)/(\sqrt{9}x - \sqrt{9}y)$ is unacceptable.)

When you're done with the exam:

Turn in **EVERYTHING** : **answer booklet** and **question pages**

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

Problem 1 : Three-Mass Object

Three identical masses $m_1 = m_2 = m_3 = 1$ are connected by rigid massless rods and placed at these coordinates in the *xy*-plane : $(x,y,z) = (0,0,0)$, $(0,2,0)$, and $(\sqrt{3},1,0)$. Perform the following tasks in any order you like:

(a) Construct the object's inertia tensor **I** in the (x,y,z) coordinate system for rotations around the origin.

(b) Find the object's eigenvectors $\vec{e}_i$ for rotations through the origin. (Don't bother normalizing them.)

(c) Determine the object's principal moments (eigenvalues) for rotations around the origin.

Problem 2 : Rotational Stability of a Box

A solid box of uniformly distributed total mass M has height h, width w, and depth d. These dimensions are all different; as a result, the principal moments for rotation around the box's center-of-mass (CM) are all different: $I_1 > I_2 > I_3$ for rotations around the principal axes $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$ respectively. The box floats in empty space. At $t = 0$, an astronaut gives the box an initial rotational velocity $\vec{\omega}\big|_{t=0}$ that passes through the book's CM and is *nearly* aligned with the principal axis $\hat{e}_1$, i.e. $\vec{\omega}\big|_{t=0} \approx \omega_0\,\hat{e}_1$. The astronaut can't *quite* achieve an initial $\vec{\omega}\big|_{t=0}$ that is *perfectly* aligned with $\hat{e}_1$, but it's close: the components of $\vec{\omega}\big|_{t=0}$ along the $\hat{e}_2$ and $\hat{e}_3$ directions are extremely small compared with ω_0.

(a) Calculate the frequency of small oscillations of the small component of $\vec{\omega}$ along $\hat{e}_2$ in terms of the given values I_1, I_2, I_3, and/or ω_0. (No forces of any kind act on the object after $t = 0$.)

(b) The dimensions of the box obey $h > w > d$. Draw a quick sketch of the box showing its tall height, medium width, and small depth; using the given information $I_1 > I_2 > I_3$, indicate on your sketch the directions of the principal axes $\hat{e}_1$, $\hat{e}_2$, and $\hat{e}_3$.

Problem 3 : Differential Cross-Section

A beam of particles of mass m and initial velocity v_0 is directed toward a heavy fixed target located. The dipole moment of the target exerts a repulsive central force $F_r = +k^2 m / r^3$ on the particles where k^2 is a positive constant. Calculate the the scattering angle θ in terms of the impact parameter b and the given experimental parameters k, v_0, and/or m.

Problem 4 : The Orbit of Hera

The undiscovered planet Hera orbits around the Sun with a semi-major axis of 15 A.U. and an eccentricity of 2/3. All planets are of negligible mass compared with the mass of the Sun.

(a) Calculate the ratio of the aphelion / perihelion distances of Hera's orbit.

(b) Calculate Hera's speed at perihelion in terms of the Earth's constant orbital speed $v_\oplus$ using the assumption that the Earth's orbit is a perfect circle.