

Physics 326 Midterm Exam #2 Spring 2016  
Thursday Apr 14, 12:30 pm – 1:50 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **80 minutes** to work the problems.

*At the beginning of the exam:*

- 1) Write your **name** and **netid** on your answer booklet(s).
- 2) Turn your **cell phone off**.
- 3) Put away all calculators, phones, computers, notes, and books.

*During the exam:*

- 1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
- 2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you **understand** what you're doing and are **not just guessing**. (If you *are* guessing, explain that, then *verify* your guess explicitly.) A good **annotated sketch** is often the best explanation of all!
- 3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. You should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
- 4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. **IMPORTANT**: If you think you've made a mistake but can't find it, **explain what you think is wrong** → you may well get partial credit for noticing your error!
- 5) It is fine to leave answers as **radicals or irreducible fractions** (e.g.  $10\sqrt{3}$  or  $5/7$ ), but you will lose points for not simplifying answers to an **irreducible form** (e.g.  $24(x^2 - y^2)/(\sqrt{9}x - \sqrt{9}y)$  is unacceptable.)

*When you're done with the exam:*

Turn in the **answer booklet** only; keep the question pages and formula sheets.

**Academic Integrity:**

*The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.*

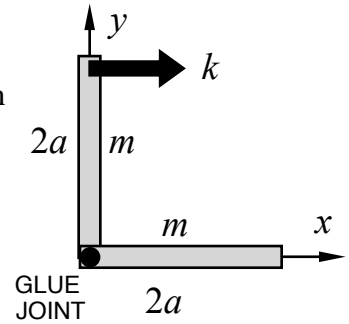
*Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.*

**Problem 1 : Two Rods Glued at 90°**

Two thin rods of mass  $m$  and length  $2a$  are glued together at their ends so they form a 90° angle (an “L” shape). The glued rods are placed in the  $xy$ -plane as shows with the glue joint at the origin. The object’s CM position relative to the origin is

$\vec{R} = \frac{a}{2}(\hat{x} + \hat{y})$  (so you don’t have to calculate it).

The inertia tensor of the object **relative to the CM** is  $\mathbf{I}' = \begin{pmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 10 \end{pmatrix}$ .



(a) What are the **principal axes** of the object for rotation **around the CM**? You can either perform a calculation or explain your answer in some other way.

(b) At time  $t = 0$ , an impulse  $k\hat{x}$  is applied to the object at the point  $(x, y, z) = (0, 2a, 0)$ .

Calculate the velocity  $\vec{v}_0$  **of the glue joint** at the instant  $t = 0+$  immediately after the impulse is applied.

**Problem 2 : Two Masses Connected by a Spring**

Two point masses  $m_1 = m$  and  $m_2 = 3m$  are connected by a massless spring with potential energy  $U(r) = kr^2/2$ , where  $r$  is the length of the spring. The system is given a total energy  $E$  and a tiny angular momentum  $L$  such that  $L^2 = (3mE^2\varepsilon)/(2k)$ , where the dimensionless quantity  $\varepsilon \ll 1$ . The given quantities are thus  $m, k, E$ , and  $\varepsilon$ .

(a) Calculate the minimum and maximum separations-squared,  $r_{\min}^2$  and  $r_{\max}^2$ , between the masses to leading non-vanishing order in the small quantity  $\varepsilon$ .

(b) When the separation between the masses is 20 cm, what is the distance of mass 2 from the system’s CM?

**Problem 3 : A Perfectly Thrown Football**

As we saw, star quarterback Drew Brees throws an American football so that its **spin**  $\omega_3$  is 600 rpm = 10 revolutions per second, while its  $\hat{e}_3$  vector (the long axis of the football) **wobbles** (precesses) so that it completes 3 wobbles for every 5 spins in the frame of the stadium. To achieve this perfect spiral, Drew spins the football so that the angle  $\alpha$  between  $\vec{\omega}$  and  $\hat{e}_3$  is 0.04 radians ( $= 2.3^\circ$ ).

The football has moments of inertia  $I_1 = I_2$  and  $I_3$  whose ratio is  $I_3/I_1 = (3/5)(1 - \varepsilon)$  where  $\varepsilon \ll 1$ .

→ What value of  $\varepsilon$  is needed to produce the observed motion of the football?

NOTE: You need only calculate  $\varepsilon$  to two significant digits, so be sure to approximate your work to leading non-vanishing order in the small quantity  $\varepsilon$ .