Physics 326 Midterm Exam #1 Spring 2017
Thursday Mar 2, 12:30 pm – 1:50 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **80 minutes** to work the problems.

At the beginning of the exam:

1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:

1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you **understand** what you're doing and are **not just guessing**. (If you *are* guessing, explain that, then *verify* your guess explicitly.) A good **annotated sketch** is often the best explanation of all!
3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. You should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!
5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7), but you will lose points for not simplifying answers to an **irreducible form** (e.g. $24(x^2-y^2)/(\sqrt{9}x-\sqrt{9}y)$ is unacceptable.)

When you're done with the exam:

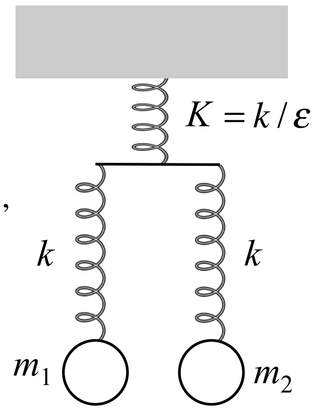
Turn in **EVERYTHING** : **answer booklet** and **question pages**

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

Problem 1 : Lecture Demo



Two identical masses $m_1 = m_2 = m$ are suspended from a horizontal massless crossbar by identical springs k. The crossbar is suspended from a fixed platform by a very stiff spring $K = k/\varepsilon$ where $\varepsilon \ll 1$. The masses only move vertically, the crossbar is always horizontal, and gravity can be ignored.

Define coordinates x_1 and x_2 to be the positions of the masses below the fixed platform, relative to their equilibrium positions. If we drop terms of order ε^2 and higher (compared with 1), the **M** and **K** matrices for this pair of coordinates are

$$\mathbf{K} = k\begin{pmatrix} 1-\varepsilon & -\varepsilon \\ -\varepsilon & 1-\varepsilon \end{pmatrix} \qquad \mathbf{M} = m\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

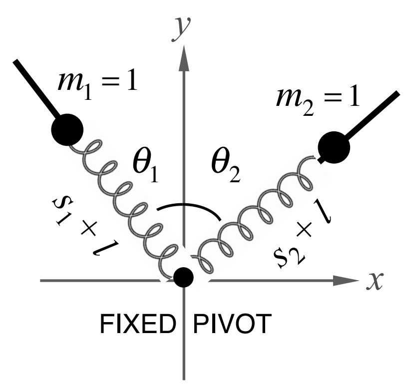
(a) Using any technique (e.g. guessing), write down the **normal coordinates** ξ_i of the system in terms of x_1 and x_2, then explicitly show that the system's **equations of motion**, as determined by the **M** and **K** matrices given above, decouple when written in terms of your ξ_i's. Finally obtain the **eigenfrequencies** of the system.

(b) Using the fact that the crossbar is massless, and dropping terms of order ε^2 or higher (compared with 1), derive the **K** matrix above. NOTE: You might want to leave this part to the end as it's the longest part.

Problem 2 : Tensor Transform

Consider a system with coordinates $\vec{q}$ and potential energy $U = \frac{1}{2}\vec{q}^{\,T}\mathbf{K}\vec{q}$. A matrix **R** transforms the coordinates $\vec{q}$ to a different "primed" coordinate system via $\vec{q}\,' = \mathbf{R}\vec{q}$. Using the fact that U is a scalar (i.e. is the same in any coordinate system), explicitly derive the transformation formula that gives the tensor $\mathbf{K}'$ in the primed coordinate system in terms of **K** and **R** (and/or their inverses and/or transposes).

Problem 3 : Beads on Rods with Pivot



Two beads, of mass $m_1 = 1$ and $m_2 = 1$, are threaded onto straight rods attached to a pivot fixed at the origin. Identical springs of unstretched length l and spring-constant k are also threaded onto the rods, connecting the pivot to the beads as shown. The pivot allows the rods to rotate within the xy-plane; one rod is sightly above the other, so they can pass over each other like the hands on a clock. Finally, a coil within the pivot continuously pushes the rods apart, with associated potential energy $U_{\text{PIVOT}} = \frac{1}{2}\alpha(\theta_1 + \theta_2)^2$.

Describe the system with 4 coordinates:

- s_1 and s_2 are the spring extensions (so the spring lengths are $s_1 + l$ and $s_2 + l$)
- θ_1 and θ_2 are the angles the rods make with the $+y$ axis, with θ_1 measuring counter-clockwise and θ_2 measuring clockwise (so $|\,\theta_1 + \theta_2\,|$ is the angle between the rods)

Calculate all the eigenfrequencies and eigenvectors for the small oscillations of this system. Express your answers in terms of the known parameters k and l (for the springs), α (for the pivot-coil), and $m_1 = m_2 = 1$. If you cannot manage to calculate all the eigenmodes, you can get partial credit by describing them.