Physics 326 Midterm Exam #1 Spring 2016
Thursday Feb 18, 12:30 pm – 1:50 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **80 minutes** to work the problems.

At the beginning of the exam:
1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you **understand** what you're doing and are **not just guessing**. (If you *are* guessing, explain that, then *verify* your guess explicitly.) A good **annotated sketch** is often the best explanation of all!
3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. You should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!
5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7), but you will lose points for not simplifying answers to an **irreducible form** (e.g. $24(x^2 - y^2)/(\sqrt{9}x - \sqrt{9}y)$ is unacceptable.)

When you're done with the exam:

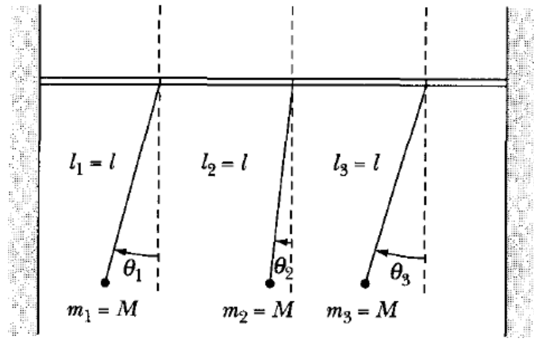
Turn in **EVERYTHING** : **answer booklet** and **question pages**

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

Problem 1 : Coupled Pendula



Three identical pendula, each consisting of a massless rod attached to a mass M, are connected to a horizontal bar as shown. The bar is slightly flexible and this couples the motions of the pendula. (When one pendulum moves, the others are nudged a bit.)
The total kinetic and potential energies of the system for small oscillations are:

$$T = \frac{1}{2}\left(\dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2\right)$$

$$U = \frac{1}{2}\left(\theta_1^2 + \theta_2^2 + \theta_3^2 - 2\varepsilon\theta_1\theta_2 - 2\varepsilon\theta_2\theta_3 - 2\varepsilon\theta_3\theta_1\right)$$

where ε is a constant. The eigenfrequencies of this system are $\omega^2 = (1-2\varepsilon), (1+\varepsilon), (1+\varepsilon)$. Calculate eigenvectors corresponding to these three frequencies, making sure that they are orthogonal to each other.

Problem 2 : Quickies

In both questions, repeated indices are implicitly summed over (Einstein convention).

(a) Consider three matrices **A**, **B**, and **C**. A fourth matrix **D** is given by the relation $D_{ij} = C_{ab}\, A_{ai}\, B_{bj}$. Express this relation in pure matrix form, i.e. using matrix multiplication and no indices.

(b) A vector $\vec{r}$ in normal 3D space can be written as $\vec{r} = x_i\, \hat{x}_i$ using the Cartesian basis vectors $\{\hat{x}_1, \hat{x}_2, \hat{x}_3\}$.
A new set of basis vectors is constructed:

$$\hat{u}_1 = \frac{1}{2}\hat{x}_1 - \frac{\sqrt{3}}{2}\hat{x}_3\,, \qquad \hat{u}_2 = \hat{x}_2\,, \qquad \hat{u}_3 = \frac{\sqrt{3}}{2}\hat{x}_1 + \frac{1}{2}\hat{x}_3$$

In this new basis, the same vector is written as $\vec{r} = u_i\, \hat{u}_i$. The components u_i in the new basis are related to the components x_i in the old basis by a transformation matrix **R** :

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \mathbf{R} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Find the inverse transformation matrix $\mathbf{R}^{-1}$.

Problem 3 : Reverse Engineering

A helpful TA emails you a coupled-oscillator problem with solution … but the evil email server mangles most of the pages! Here's what we know: the problem has two generalized coordinates, x_1 and x_2, and the eigenfrequencies, normalized eigenvectors, and mass matrix are:

$$\omega_S^2 = 4/3 \qquad \hat{a}_S = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 2 \end{pmatrix},\ \hat{a}_F = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \qquad \mathbf{M} = \frac{1}{4}\begin{pmatrix} 4 & 1 \\ 1 & 1 \end{pmatrix}$$
$$\omega_F^2 = 4$$

We can use this information to recover missing parts of the problem.

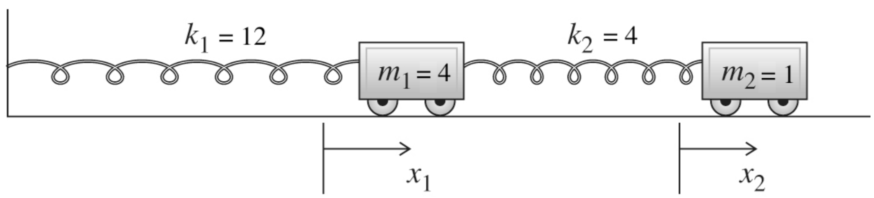
(a) Find the solution $\vec{x}(t)$ for these <u>initial conditions</u>: $\vec{x}\big|_{t=0} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\dot{\vec{x}}\big|_{t=0} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

Hint: Do NOT switch to ξ-space. For this small 2-DOF problem, that would be shooting a fly with a tank.

(b) What is the <u>normal coordinate</u> ξ_F for the fast mode in terms of x_1 and x_2?

(c) What is the spring matrix $\mathbf{K}^\xi$ in <u>ξ-space</u> ?

Problem 4 : Damped 2m2s



Two carts and two springs are connected as shown, with the x coordinates set to zero at the carts' equilibrium positions as usual. The masses and spring constants are:

$m_1 = 4,\ \ m_2 = 1,\ \ k_1 = 12,\ \ k_2 = 4$ (shown in figure)

Each cart also experiences a velocity-dependent <u>drag force</u>:

- cart 1 experiences drag force $F_x = -8\dot{x}_1$
- cart 2 experiences drag force $F_x = -2\dot{x}_2$

The general solution for the carts' motion is a superposition of a slow and a fast mode:

$$\vec{x}(t) = \vec{a}_S\, f_S(t) + \vec{a}_F\, f_F(t)\,, \qquad \text{where } \vec{a}_S \ \&\ \vec{a}_F \text{ are constant vectors.}$$

Find <u>either</u> of the time-dependent functions $f_S(t)$ or $f_F(t)$.

(i.e. you do not need to get both of them ... and do **NOT** solve for the eigenvectors $\vec{a}_S$ or $\vec{a}_F$)

Hint 1: Do NOT attempt a ξ-space solution or you will waste time needlessly.
Hint 2: You should get a determinant of the form $a^2 - b^2$, which factorizes into $(a-b)(a+b)$.