

Physics 326 Final Exam Spring 2017
Monday May 8, 1:30 pm – 4:30 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **3 hours** to work the problems.

At the beginning of the exam:

- 1) Write your **name** and **netid** on your answer booklet(s).
- 2) Turn your **cell phone off**.
- 3) Put away all calculators, phones, computers, notes, and books.

During the exam:

- 1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
- 2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!
- 3) **All question parts on this exam are independent:** you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
- 4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. **IMPORTANT:** If you think you've made a mistake but can't find it, **explain what you think is wrong** → you may well get partial credit for noticing your error!
- 5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or $5/7$).

Remember: There are many **Math Tables** provided → do **Use Them!**

When you're done with the exam:

Turn in only your **answer booklet**. (Keep the exam questions).

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

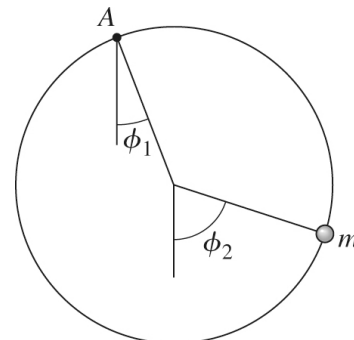
Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

You must derive any expression that is not on the formula sheet.

You may of course use from memory any formulae from a previous class, e.g. 325, 225, 211.

Problem 1 : Bead on a Swinging Hoop

A bead of mass m is threaded on a frictionless circular wire hoop of radius R and mass $M = \eta m$ where η is an unknown dimensionless constant. The hoop is suspended at the point A and is free to swing in its own vertical plane as shown in the figure. Uniform gravity g points downward.



Using the angles ϕ_1 and ϕ_2 as generalized coordinates, the \mathbf{M} and \mathbf{K} matrices for this system are

$$\mathbf{M} = mR^2 \begin{pmatrix} 5 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{K} = mgR \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$$

(a) For small oscillations of this system, find the normal frequencies and normal modes, and write down the general solution for $\phi_1(t)$ and $\phi_2(t)$.

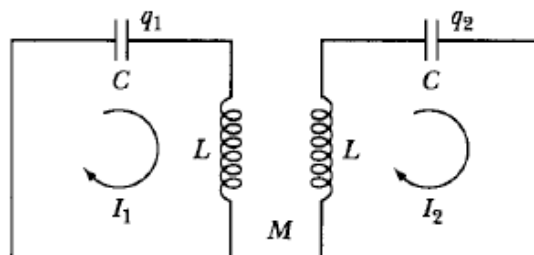
(b) Explicitly derive all entries of the given matrices \mathbf{M} and \mathbf{K} , and in doing so, figure out the unknown constant η .

Problem 2 : Coupled AC Circuits with Resistance

The two AC circuits shown at right are coupled together via the mutual inductance M between their nearby inductors. The wires with which the circuits are built have an intrinsic resistance R . The Kirckhoff equations for this system are

$$Q_1 + R\dot{Q}_1 + LC\ddot{Q}_1 + MC\ddot{Q}_2 = 0 \quad \text{and}$$

$$Q_2 + R\dot{Q}_2 + LC\ddot{Q}_2 + MC\ddot{Q}_1 = 0$$



The general solution for the charges on the capacitors, $Q_1(t)$ and $Q_2(t)$, is a superposition of two normal modes. Find and sketch the time-dependence of each normal mode, using these values : $C = L = 1$, $R = 2$, and $M = 4$.

Problem 3 : Captured

A particle of mass m starts out at infinity with velocity v_0 in the $+x$ direction. The Death Star is at the origin. The Death Star's mass M is infinitely greater than the particle's mass m . The Death Star exerts a force on the particle, described by the potential energy $U(r) = -K/(3r^3)$ where K is a positive constant. Calculate the capture cross section for the particle in terms of the given parameters.

Problem 4 : Pound-Rebka Experiment

In 1960, Pound and Rebka sent gamma rays (high-energy photons) from the top of a high tower to the bottom, where they were detected. The gamma rays were emitted from a sample of unstable nuclei at the top of the tower, then detected (absorbed) by a sample of the same material at the bottom of the tower. The trick to the experiment is this : the sample at the bottom could only absorb gamma rays with the *same frequency* as those emitted by the sample at the top. In order to enable this absorption, the experimenters introduced a small relative speed $\beta = v/c$ between the emitting sample and the detector.

(a) The experimenters found that a relative speed of $v = 7.5 \times 10^{-7}$ m/s was needed for efficient detection of the gamma rays. How high was their tower? Take the gravitational acceleration to be $g = 10$ m/s² and recall that

the formula for the relativistic Doppler shift is $\frac{f'}{f} = \sqrt{\frac{1-\beta}{1+\beta}}$.

(b) What was the *direction* of the relative motion between source and detector needed to make the experiment work, i.e. was the source *approaching* the detector or *receding* from it when absorption occurred? Be sure to explain your answer briefly but clearly.

Problem 5 : Football in Space

An American football has a total mass $M = 1$ that is evenly distributed across its surface. The football's principal moments for rotation around its center of mass are 5 and 3 (in appropriate units), with one of those values being duplicated (you figure out which one) to provide all three principal moments.

(a) At time $t = 0$ the football is placed so that its center-of-mass is at the origin and its axis of symmetry runs along the z axis. The football is in free space with no net external forces acting on it ... however an external torque is acting on it, so as to make the football rotate with a constant angular velocity $\vec{\omega} = \gamma(\hat{x} + 2\hat{y})$ where γ is a positive constant. Calculate the torque vector, $\vec{\tau}$, that is required to accomplish this at time $t = 0$.

(b) The football is once more placed at rest with its CM at the origin ... but this time, its axis of symmetry is lined up along the direction $\hat{x} + \sqrt{3}\hat{z}$. Calculate the inertia tensor \mathbf{I} of the football in the (x, y, z) coordinate system when it is in this new orientation, again for rotations around the origin=CM.

Problem 6 : Alice, Bob, and a Black Hole

Alice takes her spaceship to do some exploring near a slowly-rotating black hole X1, which has mass M and angular momentum J around the \hat{z} axis. The spacetime around a slowly rotating black hole is given by the Kerr metric, which is different from the Schwarzschild metric by one additional term:

$$d\tau_{\text{Kerr}}^2 = d\tau_{\text{Schwarzschild}}^2 + \frac{4J}{r^2} \sin^2 \theta (r d\phi)(dt)$$

Alice's brother Bob sits far, far away (i.e. at infinity) in an interstellar space station, awaiting signals from Alice. **NOTE:** All given values are in **natural units**; your answers should be too.

(a) Alice puts her ship in a circular orbit above the equator of X1 (i.e. her orbit is at polar angle $\theta = 90^\circ$). She measures the circumference of the orbit as she goes, and finds it to be C using her onboard instruments. Alice contacts Bob by sending him two radio signals, one emitted when she started her orbit and the other emitted when she completed it. According to Alice's onboard clock, a time interval T_A elapsed between the emission of the two signals. Far away on the interstellar space station, Bob receives the two signals. Calculate the time interval T_B between the reception of the two signals, according to Bob's station clocks.

(b) Alice now moves her ship radially inward toward the center of X1. Her onboard instruments tell her that she has moved a small radial distance of Δd . Alice now puts the ship into a circular orbit at the new radius. She again measures the circumference of her orbit by traveling around X1 in a circle at $\theta = 90^\circ$, and finds it to be C' . Calculate the difference $\Delta C = C' - C$. You may use the fact that $\Delta C \ll C$ since Alice hasn't changed her orbit very much.

(c) The additional term in the Kerr metric above is given in natural units. Translate it into SI units.

Problem 7 : North Poles

The earth has a slight equatorial bulge that makes its principal moments *nearly* equal, except for a fractional difference of k , i.e. one or two of them (you decide) is larger than the other(s) by a factor of $(1+k)$ where k is a small positive constant ($k \ll 1$). In addition, the Earth's *geometric* north pole is a distance d away from the Earth's *rotational* north pole, where d is measured along the Earth's surface and is *much* smaller than the Earth's radius R ($d \ll R$).

(a) How long does it take for the Earth's rotational North Pole to complete one full circle around its geometric North Pole? Be sure to specify the units of your answer, and approximate your answer to leading non-vanishing order in any small quantities that appear.

(b) What is the angle α between the Earth's axis of symmetry and its angular momentum vector \vec{L} ?