

Physics 326 Final Exam Spring 2016  
Wednesday May 11, 8:00 am – 11:00 am

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **3 hours** to work the problems.

*At the beginning of the exam:*

- 1) Write your **name** and **netid** on your answer booklet(s).
- 2) Turn your **cell phone off**.
- 3) Put away all calculators, phones, computers, notes, and books.

*During the exam:*

- 1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
- 2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!
- 3) **All question parts on this exam are independent:** you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
- 4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. **IMPORTANT:** If you think you've made a mistake but can't find it, **explain what you think is wrong** → you may well get partial credit for noticing your error!
- 5) It is fine to leave answers as **radicals or irreducible fractions** (e.g.  $10\sqrt{3}$  or  $5/7$ ).

Remember: There are many **Math Tables** provided → do **Use Them!**

*When you're done with the exam:*

Turn in only your **answer booklet**. (Keep the exam questions).

**Academic Integrity:**

*The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.*

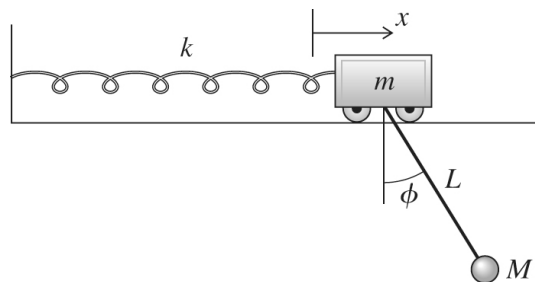
*Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.*

**You must derive any expression that is not on the formula sheet.**

You may of course use from memory any formulae from a previous class, e.g. 325, 225, 211.

### Problem 1 : Cart, Spring, and Pendulum

A simple pendulum consists of a massless rod of length  $L$  and a point mass  $M$  at its bottom end. The top end of the pendulum is attached to a cart of mass  $m$  that can move horizontally (no friction). A spring of spring-constant  $k$  connects the cart to a vertical wall on the left. For simplicity, take the values  $m = M = L = g = 1$  and  $k = 2$  (all in appropriate units).



Calculate the general solution for  $x(t)$  and  $\phi(t)$ , where both variables are measured with respect to their equilibrium positions. Consider small oscillations only, i.e. small variations of  $x$  and  $\phi$  from their equilibrium positions.

### Problem 2 : Vulcan

The undiscovered planet Vulcan orbits around the Sun with a semi-major axis,  $a$ , of 25 A.U. and an eccentricity of  $2/3$ . If you need them, use the excellent approximations that  $\pi^2 = 10$ , that the orbit of the Earth is circular, and that all planets are of negligible mass compared with the mass of the sun. Also see the Earth Data provided on the formula sheet.

- (a) Calculate the ratio of the aphelion / perihelion distances of Vulcan's orbit.
- (b) When Vulcan is at perihelion, an alien vessel uses its super-beam to deliver a single impulse to the planet and force it onto a circular orbit. Calculate the change  $v_{\text{after}} - v_{\text{before}}$  in Vulcan's speed caused by this impulse. For full points, calculate your result in meters / second.

### Problem 3 : Assault on the Death Star

The ultra-massive Death Star exerts a central force  $\vec{F}(r) = -\frac{\gamma}{r^3}$  on any mass in its vicinity. The center of the

Death star will serve as the origin of our standard 2D-polar coordinate system  $(r, \phi)$ . A rebel fighter launches a missile of mass  $m$  in the  $-x$  direction from far, far away ( $r = \infty$ ) with initial speed  $v_0$ , in such a way that the missile's angular momentum with respect to the origin is  $L$  and  $L$  is sufficiently large that the missile is NOT captured by the Death Star.

Calculate the path  $r(\phi)$  of the missile in terms of the given parameters, and determine the condition on  $L$  that allows it to escape.

**Problem 4 : Rotational Stability**

A book has three principal axes  $\hat{e}_1, \hat{e}_2, \hat{e}_3$  with associated moments of inertia  $\{I_1, I_2, I_3\} = \{4, 2, 1\}$  in appropriate units. In free space (no forces, no torques), the book is spun so that its initial rotation vector  $\vec{\omega}$  is nearly along the axis  $\hat{e}_1$ , i.e.  $\omega_1 \approx \omega$ , but not quite  $\rightarrow \vec{\omega}$  also has very small components along the  $\hat{e}_2$  and  $\hat{e}_3$  directions. Calculate the small-oscillation frequency of the  $\omega_2$  component in terms of the initial value  $\omega_1$ .

**Problem 5 : Harmonics**

A string of uniform mass-per-unit-length  $\rho$  and tension  $T$  is stretched between two fixed endpoints at  $(x,y) = (-D, 0)$  and  $(+D, 0)$ . Transverse displacements of the string are described by the differential equation

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{\rho} \frac{\partial^2 y}{\partial x^2}$$

At time  $t = 0$ , the string is held at rest in the following “square wave” shape:

$$y(x,0) = h(x) \quad \text{where} \quad \begin{aligned} h(x) &= a \text{ for } x < 0 \\ h(x) &= 0 \text{ for } x > 0 \end{aligned}$$

- What is the speed-squared  $c^2$  of wave propagation along this string? (You can use this  $c$  in the remainder of your answers.)
- Calculate  $y(x,t)$  for the first harmonic (aka the “fundamental”) and the second harmonic. Do not calculate the amplitudes of these harmonics, just their dependence on  $x$  and  $t$ .
- Calculate the amplitude of the second harmonic.

**Problem 6 : The GR Lagrangian**

Show explicitly that the GR Lagrangian  $L_{gr} = -mc^2 d\tau / dt$  and the Schwarzschild metric for  $d\tau$  reproduce the familiar Lagrangian  $L = T - U$  in the Newtonian Limit of weak gravity and slow speeds. (It’s up to you to know what we mean by weak gravity and slow speeds.)