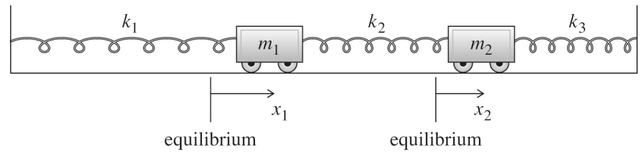
Physics 326 – Homework #1 **due in course homework box by Fri 1 pm**

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: our master rule for homework and exams is **NO WORK = NO POINTS**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof; both are posted in the same place you found this homework. Finally please write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.* ☺

Problem 1 : Ignoring the Unstretched Length



In lecture, we analyzed the 2-masses-3-springs system shown in the figure, using as our coordinates the carts' displacements from equilibrium, x_1 and x_2. When we set up the equations of motion, we made a simplifying assumption without discussing it: we assumed that when the system was at equilibrium ($x_1 = x_2 = 0$), the springs were at their unstretched lengths l_1, l_2, l_3. This assumption makes it a lot simpler to write the equations of motion! For example, we can immediately write the force of spring 1 on cart 1 as k_1x_1 instead of having to write it as k_1 (spring1_length $- l_1$); if we take the Lagrangian route, the potential energy of spring 1 is $\frac{1}{2}k_1x_1^2$ instead of ½ k_1 (spring1_length $- l_1)^2$. As it turns out, our simplifying assumption is fully justified. (Sigh of relief!) We must now prove that this is the case so that we know when we can use it in future!

(a) Let L_1, L_2, and L_3 be the lengths of the springs when the system in equilibrium ($x_1 = x_2 = 0$). The assumption made in lecture and in the textbook is that these equilibrium lengths are the same as the springs' unstretched lengths, i.e. that $L_i = l_i$. This is a most unlikely assumption ... but as we will see in part (b), it is not necessary! Use a mental image of the system in equilibrium to write down the two relations that must hold between the equilibrium lengths L_i and unstretched lengths l_i. (Hint:you should get one relation for each mass.)

(b) Write down the equations of motion for mass m_1 and mass m_2, this time <u>not</u> using the assumption that $L_i = l_i$ for any spring. Finally, use your part (a) conditions to show that these equations of motion are *exactly the same* as those you obtain by assuming $L_i = l_i$ for all springs. Very cool!

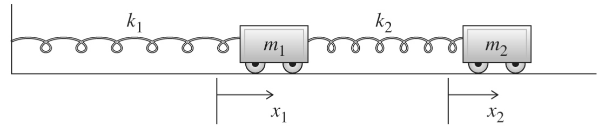
Problem 2 : 2 Masses 3 Springs

In the 2-masses-3-springs setup shown in the figure above, consider the case when the masses are equal, $m_1 = m_2 = m$, and the spring constants have these values: $(k_1, k_2, k_3) = (k, 2k, 4k)$ where k is a constant.

- Find and briefly describe (with words or sketches) the two normal mode solutions for $x_1(t)$ and $x_2(t)$.
- Write down the general solution for $x_1(t)$ and $x_2(t)$.

Problem 3 : 2 Masses 2 Springs

Consider the 2-masses-2-springs set shown in the figure at right, in the special case that the masses are equal, $m_1 = m_2 = m$, and the spring constants are $(k_1, k_2) = (3k, 2k)$.



(a) Find and describe (with words or sketches) the two normal mode solutions for $x_1(t)$ and $x_2(t)$, where x_1 and x_2 as usual refer to the carts' displacements from their equilibrium positions.

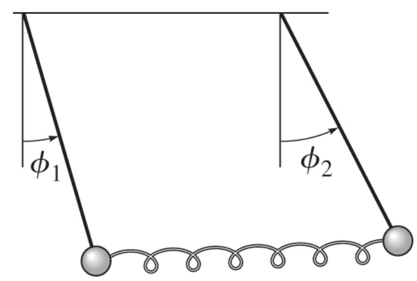
(b) Write down the general solution for $x_1(t)$ and $x_2(t)$.

(c) Determine the specific solution for these initial conditions: $\vec{x}\big|_{t=0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\dot{\vec{x}}\big|_{t=0} = \begin{pmatrix} 0 \\ v_0 \end{pmatrix}$.

(d) In Problem 1 you showed that unstretched lengths could be ignored for a line of springs connected to a pair of walls. In this problem, the springs are only connected to *one* wall. Is it still ok to ignore the unstretched lengths? The answer is again YES; to understand why, answer this question: In the notation of problem 1, what is what is the relation between l_i and L_i for this problem, where the springs are only constrained by *one wall*? If you prefer words to symbols, you can fill in the blanks in this sentence: "In this one-wall system, the unstretched lengths of the springs are necessarily ______ to the equilibrium lengths of the springs, and so can be ignored."

Problem 4 : Enter the Pendula! (at small angles, of course)

Consider two identical plane pendulums … or "pendula" if you are over-educated in the humanities. Each has length d and mass m, and they are "simple pendula", meaning that all their mass is concentrated in a small "bob" at the end. (Jargon note: if the word "pendulum" appears without further qualification, it is generally implied that it is a simple pendulum.) Simple though they may be, these pendula are arranged in an interesting configuration: as shown in the figure, their bobs are connected by a massless spring of spring constant k. The unstretched length of the spring is equal to the distance between the two supports, so the equilibrium position is at $\phi_1 = \phi_2 = 0$, with the two pendula vertical.



(a) Use the Lagrangian procedure to obtain the equations of motion for the coordinates ϕ_1 and ϕ_2 in the approximation that these <u>angles remain **small**</u> at all times.
[Taylor's hint: this means that the extension of the spring is well approximated by $d(\phi_2 - \phi_1)$.]

(b) • Find and describe the normal modes for the small oscillations of these coupled pendula.
• Write down the general solution for $\phi_1(t)$ and $\phi_2(t)$ in the small-oscillation approximation.

Problem 5 : A Triangle of Springs

Two equal masses m are constrained to move without friction, one on the positive x axis and the other on the positive y axis. They are attached to two identical springs, with the same spring constant k and the same unstretched length, whose other ends are attached to the origin. In addition, the two masses are connected to each other by a third spring of force constant k' and a different unstretched length. The springs are chosen so that the system is in equilibrium when all three springs are relaxed, i.e. when their lengths are equal to their unstretched lengths. However, the unstretched length <u>*does*</u> make an appearance in this problem: you must not ignore it here as the geometry in this problem is more complex than the 1D chain-of-springs geometry of problems 1 - 3. (This illustrates how *careful* you must be with the simplifying assumption you explored in problem 1 ➔ it is *not* always possible to completely ignore the unstretched spring lengths!)

- Find and describe the normal modes of the two masses in the approximation that the masses only make <u>**small** displacements from equilibrium</u>.
- Write down the general small-oscillation solution for the positions $x(t)$ and $y(t)$ of the two masses.