

• Coordinates : $\vec{r} \equiv \vec{r}_1 - \vec{r}_2$, $M\vec{R} = m_1\vec{r}_1 + m_2\vec{r}_2$... $\vec{r}_1 = \vec{R} + \frac{m_2}{M}\vec{r}$, $\vec{r}_2 = \vec{R} - \frac{m_1}{M}\vec{r}$

• L-Equation : $\dot{\phi} = \frac{L}{\mu r^2}$

• E-Equation : $E = \frac{1}{2}\mu\dot{r}^2 + U + \frac{L^2}{2\mu r^2}$

• Reduced Mass : $\mu = \frac{m_1 m_2}{M}$

• Force Equation : $\mu\ddot{r} = F(r) + F_{cf}(r)$

• Effective Potential : $U^* \equiv U + U_{cf}$

• Centrifugal force & PE : $\vec{F}_{cf} = \frac{L^2}{\mu r^3}\hat{r}$, $U_{cf} = \frac{L^2}{2\mu r^2}$

PATH EQUⁿ

instead of $r(t), \phi(t) \rightarrow$ get shapes of orbits $r(\phi)$

⊕ Trick: $u(\phi) \equiv \frac{1}{r(\phi)}$... change $\frac{d}{dt} \rightarrow \frac{d}{d\phi}$

$$r \equiv \frac{1}{u} \rightarrow \frac{dr}{dt} = \frac{dr}{du} \cdot \frac{du}{d\phi} \cdot \frac{d\phi}{dt} = -\frac{1}{u^2} u' \dot{\phi}$$

and $\dot{\phi} = \frac{L}{\mu r^2} = \frac{L u^2}{\mu} \Rightarrow \boxed{\dot{r} = -\frac{L u'}{\mu}}$ needed for I.C.'s

$$\rightarrow \ddot{r} = \frac{d}{dt}(\dot{r}) = \frac{d\dot{r}}{du'} \cdot \frac{du'}{d\phi} \cdot \frac{d\phi}{dt} \Rightarrow \ddot{r} = -\frac{L^2}{\mu^2} u'' u^2 //$$

Plug into

⊕ F_{equ} \rightarrow (usu easier to solve!) $\mu\ddot{r} = F(r) + \frac{L^2}{\mu r^3}$

path equ: for $u(\phi)$ $\boxed{u'' + u = -\frac{\mu F(1/u)}{L^2 u^2}}$