$$|\vec{v}| \equiv \sqrt{\vec{v}\cdot\vec{v}} \qquad \vec{v} = \sum_{i=1}^{3} (\vec{v}\cdot\hat{r}_i)\,\hat{r}_i \qquad d\vec{l}_{path} = \frac{d\vec{l}}{du}\,du \qquad d\vec{A} = \left(\frac{\partial \vec{l}}{\partial u}\times\frac{\partial \vec{l}}{\partial v}\right)du\,dv$$

$$df(x_1,\ldots,x_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\,dx_i \qquad dV = \left(\frac{\partial \vec{l}}{\partial u}\times\frac{\partial \vec{l}}{\partial v}\right)\cdot\frac{\partial \vec{l}}{\partial w}\,du\,dv\,dw$$

Conceptual version:

$$d\vec{l}_u \equiv \frac{\partial \vec{l}}{\partial u}\,du$$

$$d\vec{l}_{path} = d\vec{l}_u$$

$$d\vec{A} = d\vec{l}_u \times d\vec{l}_v$$

$$dV = (d\vec{l}_u \times d\vec{l}_v)\cdot d\vec{l}_w$$

Taylor

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

1st order approx for $x \ll 1$:

- $(1+x)^n \approx 1 + nx$
- $\sin x \approx x$
- $\sin^{-1} x \approx x$
- $\cos x \approx 1 - \frac{x^2}{2}$
- $\cos^{-1} x \approx \frac{\pi}{2} - x$
- $\tan x \approx x$
- $\tan^{-1} x \approx x$
- $e^x \approx 1 + x$
- $\ln(1+x) \approx x$

	0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$$\sin a\,\sin b = \tfrac{1}{2}\left[\cos(a-b) - \cos(a+b)\right]$$

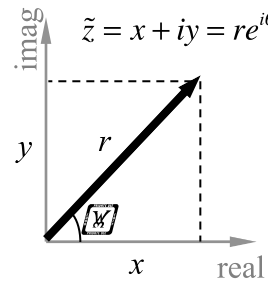
$$\cos a\,\cos b = \tfrac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

$$\sin a\,\cos b = \tfrac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

Complex Numbers

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\tilde{z} = x + iy = re^{i\theta}$$



$$\tilde{z}^* \equiv x - iy = re^{-i\theta}$$

$$|\,\tilde{z}\,| \equiv \sqrt{\tilde{z}^*\tilde{z}} = r$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} \qquad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin(a+b) = \sin a\cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a\cos b - \sin a \sin b$$

Integral Table

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{x\ dx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2}$$

$$\int_0^{2\pi} \sin^2\phi\ d\phi = \int_0^{2\pi} \cos^2\phi\ d\phi = \pi$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x\ dx}{(a \pm x)^2} = \frac{a}{a \pm x} + \ln\left(a \pm x\right)$$

$$\int \sin^2\phi\ d\phi = \frac{\phi}{2} - \frac{\sin(2\phi)}{4}$$

$$\int \frac{dx}{(a \pm x)^2} = \mp\frac{1}{a \pm x}$$

$$\int \frac{x\ dx}{a^2 \pm x^2} = \pm\frac{1}{2}\ln\left(a^2 \pm x^2\right)$$

$$\int \cos^2\phi\ d\phi = \frac{\phi}{2} + \frac{\sin(2\phi)}{4}$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x\ dx}{\left(a^2 \pm x^2\right)^{3/2}} = \mp\frac{1}{\sqrt{a^2 \pm x^2}}$$

$$\int \sin^3\theta\ d\theta = \frac{\cos^3\theta}{3} - \cos\theta$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$$

$$\int \cos^3\theta\ d\theta = \sin\theta - \frac{\sin^3\theta}{3}$$

$$\int \ln(ax)\ dx = x\ln(ax) - x$$

$$\int \cos^n\theta\ \sin\theta\ d\theta = -\frac{\cos^{n+1}\theta}{n+1}$$

$$\int \frac{dx}{(a^2 \pm x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 \pm x^2}}$$

$$\int \frac{\ln(ax)}{x}\ dx = \frac{1}{2}\left[\ln(ax)\right]^2$$

$$\int \sqrt{a^2 - x^2}\ dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$\int \frac{x^2}{\sqrt{a^2 - x^2}}\ dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$\int \sqrt{x^2 \pm a^2}\ dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \frac{(x - a\cos\theta)\sin\theta\ d\theta}{(x^2 + a^2 - 2ax\cos\theta)^{3/2}} = \frac{1}{x^2}\frac{a - x\cos\theta}{\sqrt{x^2 + a^2 - 2ax\cos\theta}}$$