1. **(25 points) cf Lecture Notes but with friction.** A bead slides along a rough rigid rod with a coefficient of dry friction $\mu_s = \mu_d > 0$. The rod is rotating at a constant rate $d\phi/dt = \omega$. *Neglect gravity.*

- Find the differential equation that governs the radial motion $r(t)$.
- You will need the formula for acceleration in polar coordinates:
  $$\ddot{a} = (\dot{r}(t) - r(t)\dot{\phi}^2)\hat{r} + (r(t)\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi}$$
- You will also need to recognize that the total force on the bead is simply a normal force $N\hat{\phi}$ plus a friction force $-\mu_s|N| \text{ sign}(\dot{r}) \dot{r}\hat{r}$ that acts in the radial direction and opposite to the speed $\dot{r}$. You don't know $N$ *apriori* but you do have an equation for it terms of the bead's motion.
- You should get a linear constant coefficient homogeneous ODE of the form
  $$Ar^2 + Br + Cr = 0$$
  where $A$ and $B$ and $C$ are given in terms of the system parameters $m$, $\mu$ and $\omega$, and in which there aren't any non-analytic factors like absolute values or sign functions. (you may wish to use the identity $\text{sign}(x) |x| = x$)

- Find two basis solutions of this differential equation by trying $r(t) = \exp(\lambda t)$ and determining the two values $\lambda_1$ and $\lambda_2$ that satisfy it.
- Form a linear combination of those two basis solutions
  $$r(t) = a \exp(\lambda_1 t) + b \exp(\lambda_2 t)$$
  and then construct the values of $a$ and $b$, and hence find $r(t)$, for the case of initial conditions such that at $t = 0$, $r = r_0$, and $dr/dt = 0$.

2. **(15 points) A particle of mass $m$ slides (both sideways and radially) on a smooth frictionless horizontal table.** It is attached to a cord that is being pulled downwards at a prescribed constant speed $v$ by a force $T$. ( $T$ may be varying.)

- Use $F=ma$ in polar coordinates to derive an expression for the tension $T$ of the cord pulling on the particle ($T$ will depend on the particle's coordinates $r$ and $\phi$ and how they may be changing)
- Show that the particle's polar coordinates $\{r(t), \phi(t)\}$ satisfy $r^2 d\phi/dt = \text{constant}$. Hint: the only horizontal force on the particle is $T$ and it acts purely in the inward radial direction. Also $dr/dt$ is known, constant and equal to $-v$. 