1. (10 points) For each of the following force functions, determine whether or not it represents a conservative force. If conservative, find the potential energy function $U(x,y,z)$

a) $\vec{F} = x^3 \hat{i} + y^{-2} \hat{j} + z^4 \hat{k}$
b) $\vec{F} = xy^3 \hat{i} + x^2 \hat{j} + z^4 \hat{k}$
c) $\vec{F} = \exp\{z\} \hat{i} + 2y \hat{j} + x \exp\{z\} \hat{k}$
d) $\vec{F} = 2x \cos(\alpha z) \hat{i} + z^2 \alpha \sin(\alpha y) \hat{j} + (2z \sin(\alpha y) - x^2 \alpha \sin(\alpha z)) \hat{k}$

2. (10 points) A particle of mass $m$ moves in 3-d in response to a force field

$$\vec{F}(x) = Ax^3 \hat{i} + Ay^3 \hat{j} + 3Axz^2 \hat{k}$$

Its speed is $v_o$ when it passes through the origin at $(0,0,0)$.

- Later on it is found to pass through position $\vec{r} = -\hat{i} + 2\hat{j} + 3\hat{k}$. Use conservation of energy to find its speed there.

3. (20 points) (Adapted from Taylor Pb 3.11) A rocket launches vertically from the earth with initial total mass $m_o$ burning fuel at a constant rate $k$ (kg/sec) (so that $m(t) = m_o - kt$) with exhaust velocity $u$. It starts at speed $v_o = 0$.

The governing ODE is

$$m(t)\ddot{v} = ku - m(t)g$$

- Solve this ODE for $v(t)$ (You can use separation of variables)

- For the case $m_o = 2 \times 10^6$ kg and $k = 10^4$ kg/sec and $u = 3000$ m/sec (roughly corresponding to the space shuttle), find its speed at $t = 100$ seconds. What was its initial acceleration? What is its acceleration at $t = 100$ seconds?

- What is the value of your expression for the speed at $t = 200$ seconds? What is wrong with applying your analysis at $t = 200$?

- Describe qualitatively what happens if (like the Saturn V) the burn rate $k$ were smaller, such that the right side of the ODE were (initially) negative? The above differential equation says that the acceleration would be initially downwards! So where is the error?