Phys 325  Homework 1A  
Name: __________________________

Due: Thursday Jan 24, 2019 by 1pm (lecture or 325 box)

1. **(10 points)** A force \( F(t) = 20 t^3 \) acts on a particle of unit mass. At time \( t = 2 \) the particle is at \( x = 3 \) with velocity 4.
   - What is \( x(t) \) for later times \( t \)?

2. **(10 points)** A particle of mass \( m \) is observed to be moving along the \( x \)-axis with a velocity that varies with the displacement \( x \) according to \( v(x) = V_0 \exp(\beta x) \), where \( \beta \) and \( V_0 \) are constants.
   - What are the physical dimensions of \( V_0 \) and \( \beta \)?
   - Find the force \( F(x) \) acting on the particle as a function of \( x \) and the given constants.
   - Check your expression for \( F(x) \) for dimensional consistency.

3. **(15 points)** A particle of mass \( m \) moves in 1-d in a potential \( U(x) = A/x^2 - B/x \) with \( A \) and \( B \) positive.
   - Sketch a plot of \( U(x) \) for the case \( A=B=1 \).
   - In terms of the assumed positive \( A \) and \( B \), construct the corresponding force \( F(x) = -dU/dx \). Sketch \( F(x) \) for the case \( A=B=1 \).
   - In terms of the assumed positive \( A \) and \( B \), find the equilibrium point \( x_{eq} \) and determine (by examining \( U'' \) there) whether or not motion near that point is stable.
   - The particle starts at position \( x_o > 0 \) and speed \( v_o \). Find the inequality relating \( A, B, m, x_o \) and \( v_o \) necessary and sufficient to assure that the particle's subsequent motion is bounded and periodic.

4. **(15 points)** A particle of mass \( m \) moves in 1-d in a conservative force field with force \( F(x) = -Ax \exp(x^2/a^2) \) where \( a \) has units of length.
   - What is the associated \( U(x) \)?
   - Show that \( x=0 \) is an equilibrium point.
   - Find the period \( 2\pi/\omega \) of oscillations assuming that deviations from equilibrium are small (\( |x| \ll a \)). You may wish to use the methods of p 13-16 of the lecture notes.