1. A particle of mass \( m \) moves in 2-d subject to a force field \( \mathbf{F} = -A \exp(-\alpha r) \hat{r} \). Thus it is attracted to the origin with a strength that diminishes with distance.
   Why do we call it a central force?
   Why do we call it conservative?

2. Give an expression for the potential energy as a function of position, \( U(r) \) such that \( \vec{F} = -\vec{\nabla} U \)
   Choose the constant of integration such that \( U = 0 \) at \( r=\infty \).

   We write \( \mathbf{v} \) in polar coordinates: \( \mathbf{v} = \dot{r} \hat{r} + r \dot{\phi} \hat{\phi} \)

3. Give an expression for the total energy \( E \) (as a function of \( \dot{r} \) and \( \dot{\phi} \) and \( r \))

4. Use our knowledge that the angular momentum \( \mathbf{L} = \mathbf{r} \times \mathbf{p} \) with magnitude \( L = m r^2 \frac{d\phi}{dt} \) is a constant (which follows from \( F_\phi = 0 = m \{ r \ddot{\phi} + 2r \dot{\phi} \} \)) and eliminate \( \frac{d\phi}{dt} \) from \( E \) to find an expression for \( E \) as a function of \( \dot{r} \) and \( r \) and \( L \).

5. For purposes of illustration, let us henceforth take \( m=1, \alpha = 1, A = 3, L = 1 \). Sketch the effective potential \( U_{\text{eff}}(r) \) that appears in \( E = \frac{1}{2} m \dot{r}^2 + U_{\text{eff}}(r) \). (A graphing calculator is helpful here.)
   For \( E=-0.25 \), identify the turning points, where \( \dot{r} = 0 \), in your sketch.
   Find the energy \( E \) of a circular orbit with unit angular momentum \( L \), i.e., one that has \( r=\text{constant} \).

6. The particle starts at \( x=1, y = 0, \frac{dx}{dt} = 0.5, \frac{dy}{dt} = 1 \). Construct \( \mathbf{L} = \mathbf{r} \times m \mathbf{v} \) and show this has \( L = 1 \). Compute the total energy and show that it is negative, and that therefore the ensuing trajectory is bound, i.e., that \( r \) never goes to \( \infty \).

7. Find the maximum \( r \) and the minimum \( r \) of this orbit (you will need to solve a transcendental algebraic equation numerically, a graphing calculator provides an easy way to do this.)

8. Now let us reconsider the above, but with a weaker attractive force: \( A = \frac{1}{2} \). Still take \( m=1, \alpha = 1 \). Take the same initial conditions: \( x=1, y = 0, \frac{dx}{dt} = 0.5, \frac{dy}{dt} = 1 \). What is \( L \)? What is \( E \)? Show that the ensuing trajectory is not bound; it goes to \( \infty \).