6 Basic Pieces : math that tends to get lost somewhere

Piece #0: **Vector Magnitudes & Dot Products**

❿ $|\vec{v}| = \sqrt{\vec{v}\cdot\vec{v}}$

Tools for Dot Products, esp. of vector sums: ● **distribute** the dot ● **geometry** → $A\ B\cos\theta$ ● **orthonormal components** → $\sum_i A_i B_i$ & Pythagoras

Piece #1: **Expressing Vectors in Component Form**

❶ $$\vec{v} = \sum_{i=1}^{3} (\vec{v}\cdot\hat{r}_i)\,\hat{r}_i$$

for any complete, orthonormal set of unit vectors $\hat{r}_1, \hat{r}_2, \hat{r}_3$

Piece #2: **Playing with Differentials**

❷ $\dfrac{df}{dx} = g$ is equivalent to $df = g\,dx$

i.e.: You can algebraically manipulate differentials df like finite numbers Δf (rearrange, regroup, etc), then mentally return them to d's & derivatives at the end.

Tactic for changing variables in differential equations:
multiply by d<blah> / d<blah>, then **regroup differentials** into new derivatives.

Piece #3: **The 3D Chain Rule**

❸ $$df(x_1,\ldots,x_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\, dx_i$$

where the variables $x_1,\ldots,x_n$ are all *independent*

In words: The change df that occurs in a function f when the independent coordinates x_i on which it depends are changed by amounts dx_i is just sum of each change dx_i times the relevant slope $\partial f / \partial x_i$.

Piece #4: **Functions Do Not Commute With Operators**

❹ e.g. $\dfrac{d}{dx} f \neq f \dfrac{d}{dx}$ & $\vec{\nabla} f \neq f\vec{\nabla}$

In both examples, the object on the left is just a function: a derivative of f. The object on the right is an operator: it will act on whatever comes next.

Piece #5: **1D Derivatives "Push Through" Dot- and Cross-products**

❺ e.g. $\dfrac{\partial}{\partial q}\left(\vec{A}\cdot\vec{B}\right) = \dfrac{\partial\vec{A}}{\partial q}\cdot\vec{B} + \vec{A}\cdot\dfrac{\partial\vec{B}}{\partial q}$ & $\left(\vec{A}\dfrac{\partial}{\partial q}\right)\times\vec{B} = \vec{A}\times\left(\dfrac{\partial\vec{B}}{\partial q}\right)$

for any vector fields $\vec{A}$ and $\vec{B}$ that depend on any variable q.

Not obvious? Try it in Cartesian coordinates … you'll see. ☺ Since the final result makes no reference to any coordinate system, your Cartesian proof will be perfectly general.

N.C.R. Makins, 225 Recap