Physics 225: Final Exam Formula Sheet

$$t' = \gamma(t - \beta x / c) \qquad t = \gamma(t' + \beta x' / c)$$
$$x' = \gamma(x - \beta c t) \qquad x = \gamma(x' + \beta c t')$$
$$y' = y \qquad y = y'$$
$$z' = z \qquad z = z'$$

$$\vec{F} = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v} \qquad W = \int \vec{F} \bullet d\vec{l} = \Delta E$$

$$E = mc^2 = \sqrt{(pc)^2 + (m_0 c^2)^2} \qquad KE = E - m_0 c^2$$

$$m = \gamma m_0 \qquad \vec{p} = \gamma m_0 \vec{v} \qquad \beta = \frac{pc}{E} \qquad \gamma = \frac{E}{m_0 c^2}$$

$$\beta \equiv \frac{v}{c} \qquad \gamma \equiv \frac{1}{\sqrt{1-\beta^2}}$$

$$I = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2 = (c\Delta\tau)^2$$

$$x^\mu \equiv (ct, x, y, z) \qquad d\tau = \frac{dt}{\gamma_u} \qquad \gamma_u = \frac{1}{\sqrt{1-(u/c)^2}}$$

$$\Lambda^\mu_\nu \equiv \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\eta^\mu \equiv \frac{dx^\mu}{d\tau} = \gamma_u\left(c, u_x, u_y, u_z\right)$$

$$p^\mu \equiv m_0 \eta^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right) \qquad \frac{f'}{f} = \sqrt{\frac{1-\beta}{1+\beta}} \qquad E_\gamma = hf$$

$$u'_x = \frac{u_x - v}{1 - u_x v / c^2}$$

$$u'_{y,z} = \frac{u_{y,z}/\gamma}{1 - u_x v / c^2}$$

$$\tilde{X}_L = iX_L = i\omega L$$

$$\tilde{X}_C = -iX_C = -i\frac{1}{\omega C}$$

$$\tilde{I}(t) = I_{\max} e^{i(\omega t + \alpha)}$$

$$\tilde{V}_R(t) = \tilde{I}(t) R$$

$$\tilde{V}_L(t) = \tilde{I}(t) \tilde{X}_L$$

$$\tilde{V}_C(t) = \tilde{I}(t) \tilde{X}_C$$

$$\tilde{\mathcal{E}}(t) = \tilde{I}(t)\, \tilde{Z}$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int dq\, \frac{\vec{r} - \vec{r}_q}{\left|\vec{r} - \vec{r}_q\right|^3}$$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{|\vec{r} - \vec{r}_q|}$$

where

$$dq = \rho\, dV_q \ \text{ or } \ \sigma\, dA_q \ \text{ or } \ \lambda\, dl_q$$

$$\vec{E} = -\vec{\nabla} V \qquad V(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{E} \cdot d\vec{l}$$

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}(\vec{r}) \equiv \frac{d\vec{I}}{dA_\perp} \longrightarrow I = \int_{\text{Surface}} \vec{J} \cdot d\vec{A}$$

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\int \vec{I}\, dl_q \times \frac{\vec{r} - \vec{r}_q}{\left|\vec{r} - \vec{r}_q\right|^3} \qquad \begin{cases} \vec{I}\, dl_q \\ \text{or} \\ \vec{J}\, dV_q \end{cases}$$

GGS

$$\int_a^b \vec{\nabla} f \cdot d\vec{l} = f(\vec{r}_b) - f(\vec{r}_a) \qquad \int_{\text{Area}} \left(\vec{\nabla} \times \vec{E}\right) \cdot d\vec{A} = \oint_{\partial\text{Area}} \vec{E} \cdot d\vec{l} \qquad \int_{\text{Vol}} \vec{\nabla} \cdot \vec{E}\ dV = \oint_{\partial\text{Vol}} \vec{E} \cdot d\vec{A}$$

From Physics 212

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ charged sphere (exterior) $\qquad B = \dfrac{\mu_0 I}{2\pi s}$ ∞ wire

$E = \dfrac{\lambda}{2\pi\varepsilon_0 s}$ charged ∞ line $\qquad B = \mu_0 n I$ ∞ solenoid (n turns/length)

$E = \dfrac{\sigma}{2\varepsilon_0}$ charged ∞ sheet $\qquad B = \mu_0 N I$ toroid of N turns

$$|\vec v| \equiv \sqrt{\vec v \cdot \vec v} \qquad \vec v = \sum_{i=1}^{3} (\vec v \cdot \hat r_i)\, \hat r_i \qquad d\vec l_{path} = \frac{d\vec l}{du}\, du \qquad d\vec A = \left(\frac{\partial \vec l}{\partial u} \times \frac{\partial \vec l}{\partial v}\right) du\, dv$$

$$df(x_1,\ldots,x_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\, dx_i \qquad dV = \left(\frac{\partial \vec l}{\partial u} \times \frac{\partial \vec l}{\partial v}\right) \cdot \frac{\partial \vec l}{\partial w}\, du\, dv\, dw$$

Conceptual version: $d\vec l_u \equiv \frac{\partial \vec l}{\partial u} du$ $\Rightarrow$ $d\vec l_{path} = d\vec l_u$, $d\vec A = d\vec l_u \times d\vec l_v$, $dV = (d\vec l_u \times d\vec l_v) \cdot d\vec l_w$

Taylor

$$f(x_0) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

1st order approx for $x \ll 1$:

- $(1+x)^n \approx 1 + nx$
- $\sin x \approx x$ • $\sin^{-1} x \approx x$
- $\cos x \approx 1 - \frac{x^2}{2}$ • $\cos^{-1} x \approx \frac{\pi}{2} - x$
- $\tan x \approx x$ • $\tan^{-1} x \approx x$
- $e^x \approx 1 + x$ • $\ln(1+x) \approx x$

Limit $x/a \to \infty$:

- $(x^n + a) \to x^n$ • $(x^{-n} + a) \to a$

for positive n …*"forever and a day"*

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$$\cos a\, \cos b = \tfrac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$
$$\sin a\, \cos b = \tfrac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

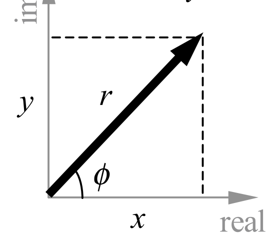
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} \qquad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

Complex Numbers

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$\tilde z = x + iy = re^{i\theta}$$



$$\tilde z^* \equiv x - iy = re^{-i\theta}$$

$$|\tilde z| \equiv \sqrt{\tilde z^* \tilde z} = r$$

(circuit formulae on previous page)

Integral Table

$$\int_0^{2\pi} \sin^2\phi\, d\phi = \int_0^{2\pi} \cos^2\phi\, d\phi = \pi \qquad \int \cos^n\theta\, \sin\theta\, d\theta = -\frac{\cos^{n+1}\theta}{n+1} \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(a^2 \pm x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 \pm x^2}} \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln\left(x + \sqrt{a^2 + x^2}\right)$$

$$\int \frac{x\, dx}{\left(a^2 \pm x^2\right)^{3/2}} = \mp \frac{1}{\sqrt{a^2 \pm x^2}} \qquad \int \frac{x\, dx}{a^2 \pm x^2} = \pm\frac{1}{2}\ln\left(a^2 \pm x^2\right) \qquad \int \frac{x\, dx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2}$$

$$\int \frac{dx}{(a \pm x)^2} = \mp \frac{1}{a \pm x} \qquad \int \frac{x\, dx}{(a \pm x)^2} = \frac{a}{a \pm x} + \frac{1}{2}\ln(a \pm x)$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) \qquad \int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| \qquad \int \frac{(x - a\cos\theta)\, \sin\theta\, d\theta}{(x^2 + a^2 - 2ax\cos\theta)^{3/2}} = \frac{1}{x^2}\frac{a - x\cos\theta}{\sqrt{x^2 + a^2 - 2ax\cos\theta}}$$