INTERFERENCE

\[ \delta = r_1 - r_2 = d \sin \theta. \]  
Equation for path length difference. \( r_1 - r_2 \) is completely general. Use \( \delta \sin \theta \) only when the two sources are far away from the observation point.

\[ \frac{\phi}{2\pi} = \frac{\delta}{\lambda} \]  
is completely general whenever you have waves from two sources interfering.

\[ \frac{\phi}{2\pi} = \frac{d \sin \theta}{\lambda} = \frac{d \theta}{\lambda} = \frac{d}{\lambda} \frac{y}{L} \]  
applies to interference from multiple slits. \( \phi \) is the phase difference between waves from successive slits at the point of observation. \( d \) is the slit separation. \( \lambda \) is the wavelength. \( \theta \) is the position on the screen measured as an angle. \( y \) is the position on the screen measured as a distance. \( L \) is the distance from the slits to the screen.

\[ \sin \theta = \pm \frac{n \lambda}{d} \]  
applies to interference from multiple slits. \( \theta \) is the angular position of the \( n \)th order peak. Note that: \( \sin \theta = \theta = \pm \frac{n \lambda}{d} \) for small angles and that \( \Delta \theta = \frac{\lambda}{d} \) where \( \Delta \theta \) is the angular separation between successive peaks.

\[ I = 4I_0 \cos^2 \left( \frac{\phi}{2} \right) \]  
applies only to the superposition of 2 waves.

DIFFRACTION

\[ \delta_s = a \sin \theta \]  
applies to diffraction. \( \delta_s \) is the path length difference between the top and bottom of the slit of width \( a \).

\[ \beta = \ldots \]  
applies to diffraction. Here \( \beta \) is the phase difference between the waves coming from the top and the bottom of the slit.

\[ \sin \theta = \pm \frac{m \lambda}{a} \]  
applies to diffraction. \( \theta \) is the angular position of the \( m \)th order \textbf{minimum} caused by diffraction.
INTERFERENCE PLUS DIFFRACTION

\[ I_i = I_0 \left\{ \frac{\sin(\beta / 2)}{\beta / 2} \right\}^2 \] gives the shape of the diffraction pattern (the envelope).

\[ I_N = I_1 \left\{ \frac{\sin(N\phi / 2)}{\sin(\phi / 2)} \right\}^2 \] gives the shape of the interference pattern (the peaks). \( N \) is the number of slits.

Note that: \[ I = I_0 \left\{ \frac{\sin(\beta / 2)}{\beta / 2} \right\} \left\{ \frac{\sin(N\phi / 2)}{\sin(\phi / 2)} \right\}^2 \] gives the total intensity pattern.

RESOLUTION OF LENSES, GRATINGS, ETC

\[ \theta_0 = \frac{\lambda}{a} \] is the minimum angular separation of two objects resolvable through a 1D slit of width \( a \).

\[ \theta_0 = 1.22 \frac{\lambda}{D} \] is the minimum angular separation of two objects resolvable through a lens or circular aperture of diameter \( D \). \( \alpha \) can also be taken to mean the minimum resolvable angle.

\[ \frac{\Delta \lambda_{\text{min}}}{\lambda} = \frac{1}{Nm} \] applies to resolution of two interference peaks through a diffraction grating. \( \Delta \lambda \) is the minimum resolvable wavelength difference. \( N \) is the number of slits. \( m \) is the order of the peak.
ENERGY & MOMENTUM

\[ KE_{\text{max}} = eV_{\text{stop}} = hf - \Phi = h(f - f_0) \] applies to the photoelectric effect. The maximum kinetic electrons coming off the metal is \( KE_{\text{max}} \). \( V_{\text{stop}} \) is the stopping voltage. \( hf \) is the energy of the photon. \( \Phi \) is the work function of the metal.

Note: Multiplying any voltage \( V \) by electric charge \( e \) gives energy in eV numerically equal to the voltage.

For example: If \( V = 69 \text{ volts} \), then \( eV = e(69 \text{ volts}) = 69 \text{ eV} \).

\[ KE = \frac{1}{2} m v^2 = \frac{b^2}{2m} \] gives the kinetic energy for any massive particle. Note that a photon is not a massive particle.

\[ E = pc = hf = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \] gives the energy of a photon. For \( \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda} \) use nanometers for wavelength.

\( \lambda = b / p \) applies to both massive particles and photons.

\[ KE = \frac{p^2}{2m} = \frac{b^2}{2m\lambda^2} \] gives the kinetic energy of any massive particle. \( KE = \frac{1.505 \text{ eV} \cdot \text{nm}^2}{\lambda^2} \) is for electrons.

SCHRODINGERS EQUATION

\[-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + U(x) \Psi = i\hbar \frac{\partial \Psi}{\partial t} \] is the time dependent schrodinger equation. Here capital psi \( \Psi \) is a function of \( x \) and \( t \).

\( \Psi(x,t) = \psi(x) e^{-i\omega t} \) is the time dependent solution to the schrodinger equation. Lowercase psi \( \psi(x) \) is solution to the time independent schrodinger equation.

\[-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + U(x) \psi = E\psi \] is the time independent schrodinger equation. \( E \) is the energy of the particle.

\[ \psi^*(x)\psi(x) = |\psi(x)|^2 \] is the probability density function, it gives the probability per unit length that the particle can be found at \( x \). The * denotes complex conjugate.
\[ P_{ab} = \int_a^b |\psi(x)|^2 \, dx \] gives the probability that the particle can be found between \( x=a \) and \( x=b \).

\[ \Psi(x,t) = Ae^{(ikx-\omega t)} \] is the solution to the Schrödinger equation for a free particle (the potential energy \( U(x) \) is zero).

Note that \[ \hbar \omega = \frac{\hbar^2 k^2}{2m} = E, \] the energy of the particle.

\[ \Delta x \Delta p \geq \hbar \] is the Heisenberg uncertainty principle. The uncertainty in momentum multiplied by the uncertainty in position must be greater than or equal to \( \hbar \).

\[ \psi_n(x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n\pi}{L} x \right) \] gives the \( n \)th state wavefunction for a particle in an infinite square well of length \( L \).

\[ E_n = \frac{\hbar^2}{2m} \left( \frac{n\pi}{L} \right)^2 = \left( \frac{\hbar^2}{8mL^2} \right) n^2 = E_1 n^2 \] gives the \( n \)th state energy for a particle in an infinite square well of length \( L \).

\[ n\lambda = 2L \] gives the \( n \)th state wavelength of the wavefunction for a particle in an infinite square well of length \( L \).
THE FINAL STUFF

\[ T \sim e^{-2KL} \]

where \( K^2 = \frac{2m}{h^2}(U_o - E) \)

\( T \) is the probability that a particle of energy \( E \) can tunnel through a potential energy barrier of length \( L \) and height \( U_o \).

\[ t_o = \frac{b}{2(E_2 - E_1)} \]

This equation gives the half-period of the time-dependent wavefunction that results from a superposition of two stationary states.

\[ U(r) = -\frac{\kappa e^2}{r} \]

The “coulomb potential”, or in other words, the potential that an electron in a hydrogen atom “feels”. \( e \) is the electric charge (and here we assume there is a single proton; otherwise it would be \( e(Ze) \)). \( r \) is the distance to the nucleus. \( \kappa = 1/4\pi\varepsilon_0 \) is a constant.

\[ \psi(x, y, z) = \sqrt{\frac{8}{abc}} \sin\left(\frac{n_1\pi}{a} x\right) \sin\left(\frac{n_2\pi}{b} y\right) \sin\left(\frac{n_3\pi}{c} z\right) \]

This is the wavefunction for a particle in an 3-dimensional infinite square well of lengths \( a, b, c \), in the \( x, y, z \) directions respectively. \( n_1, n_2, \) and \( n_3 \) are independent of each other, but must be >1.

\[ E(n_1, n_2, n_3) = \frac{b^2}{8m} \left(\frac{n_1^2}{a^2} + \frac{n_2^2}{b^2} + \frac{n_3^2}{c^2}\right) \]

Allowed energies for the particle in 3D infinite square well.

\[ \psi_{1s}(r, \theta, \phi) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \]

The ground state wavefunction for the electron in the hydrogen atom. \( a_0 \) is the bohr radius.

\[ E_s = \frac{-13.6 \text{ eV}}{n^2} \]: Energy levels for the hydrogen atom.
\[ E_n = -13.6 \text{ eV } \frac{Z^2}{n^2} \]

Energy levels for an electron subject to \( Z \) positive charges. Note that \( Z=1 \) gives the hydrogen equation.

\[ P(r) \, dr = 4\pi r^2 |\psi(r)|^2 \, dr \]

You probably won’t have to use this equation. What it means it that \( P(r) \) probability per unit of radial distance is equal to \( 4\pi r^2 |\psi(r)|^2 \). To find probability over a whole range of \( r \), integrate with respect to \( r \).

\[ \psi_n(r, \theta, \phi) = R_n(r) Y_l^m(\theta, \phi) \]

General form of hydrogen wavefunctions. \( R \) is the radial wavefunction and \( Y \) is the spherical harmonic. They are independent of each other.

\[ L^2 = l(l + 1) \hbar^2 \]

Very important. \( L \) is total angular momentum. \( l \) is the familiar quantum number.

\[ L_z = m_\ell \hbar \]

Also important. Angular momentum in z direction is proportional to \( m \) quantum number.

\[ Y_{00}, Y_{1\pm 1}, Y_{10} = \ldots \]

The spherical harmonics for \( l=0 \) and \( l=1 \).

\[ U = -\mu B \]

Potential energy of a particle in a magnetic field is equal to magnetic moment times field strength.

\[ F_z = -\mu_z \frac{dB}{dz} : \text{Follows directly from above. Take derivate w.r.t } z. \]

\[ \mu_z = -\frac{e}{m_e} S_z \]

Magnetic moment of electron. \( e \) is electric charge. \( m_e \) is mass. \( S_z \) is the spin of the electron in the z direction.

\[ E_n = \left( n + \frac{1}{2} \right) \hbar \omega : \text{energy levels of the harmonic oscillator.} \]

\[ S^2 = s(s + 1) \hbar^2 : s \text{ is the spin angular momentum. } \]

\[ S_z = m_\ell \hbar : S_z \text{ is the } z \text{ component of the spin angular momentum. } m_\ell \text{ is another quantum number related to } \ell. \]