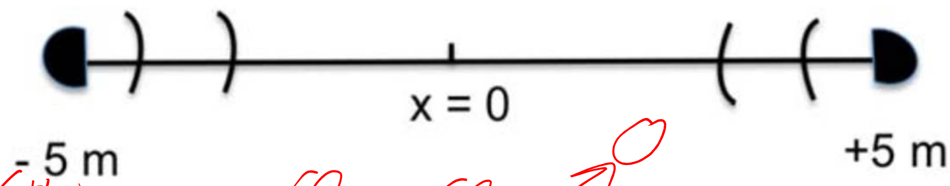
PHYS 214 Exam Spring 2017 Midterm

1. Two identical loudspeakers produce sound of equal intensity and frequency = 1200 Hz. The sound waves travel at a speed of 340 m/s. The speakers are driven in phase and are located at x = - 5 m and x = + 5 m. A listener at x = 0 hears a total sound intensity I_{Total} If the listener moves to the left by a distance *d*, further from one speaker and closer to the other, the total intensity that she hears drops to $I_{Total}/2$.
What is the minimum value of *d* ?

a. 0.567 m
b. 0.142 m
c. 0.283 m
d. 0.0708 m
e. 0.0354 m



$I_{Tot} = 4I_1\cos^2\left(\frac{\phi}{2}\right)$

$\lambda f = v$

$\lambda = \frac{v}{f}$

$\phi = \phi_{offset} + \phi_{distance}$

0

$= 360\frac{\Delta x}{\lambda}$

$360\left(\frac{L_2 - L_1}{\lambda}\right)$

$\phi = 90°$

$\phi_2 - \phi_1 = 90° = 360\left(\frac{(5+d)-(5-d)}{\lambda}\right)$

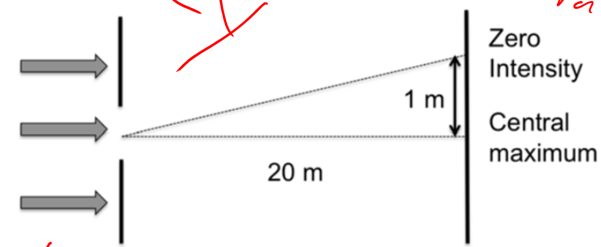
$= 360\frac{2d}{\lambda}$

$\frac{2d}{\lambda} = \frac{1}{4}$

$d = \lambda/8 = \frac{v}{8f}$

2. Microwaves of frequency 5×10^{10} Hz are incident on a circular hole that is located 20 meters from a wall, as shown below. At the wall, the microwave intensity is zero at a radius of 1 m from the central maximum.

$\theta_{rad} = \frac{1}{20}$



What is the diameter of the hole?

a. 0.29 m
b. 0.71 m
c. 0.52 m
d. 0.12 m
e. 0.15 m

Slit

$y = L\tan\theta \sim L\theta$

$\lambda f = c$

$\lambda = \frac{c}{f}$

Slit

$\sin\theta_{min} = \frac{\lambda}{a}$

$\delta = \frac{a}{2}\sin\theta \approx \frac{a}{2}\theta = \frac{\lambda}{2}$

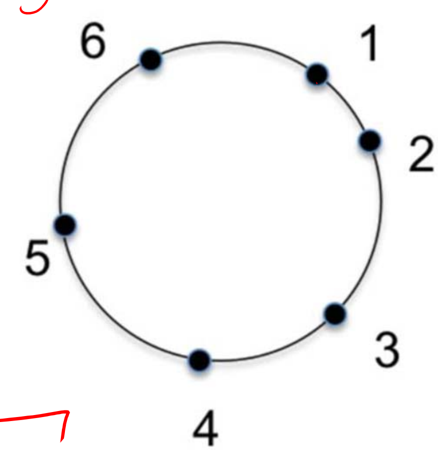
Circular opening

$\sin\theta_{min} = \frac{1.22\lambda}{D}$ ~ Diameter

$D = \frac{1.22\lambda L}{y} = \frac{1.22cL}{fy} \Leftarrow \frac{1.22\lambda}{D} = \theta = \frac{y}{L}$

3. Six loudspeakers are located on a circle. Assume that the sound wave from each speaker has a $\pi/2$ phase shift (in radians) relative to the previous one. That is, $\phi_1 = 0$, $\phi_2 = \pi/2$, $\phi_3 = \pi$, etc. Speakers 1-5 each separately generate sound at the center of the circle with intensity of 1 W/m^2. Speaker 6 generates a sound intensity of 5 W/m^2.

$A_1 = \sqrt{1\ \text{W/m}^2}$ $\quad A_6 = \sqrt{5}$



What is the total sound intensity at the center of the circle in W/m$_2$?

Hint: Use phasors.

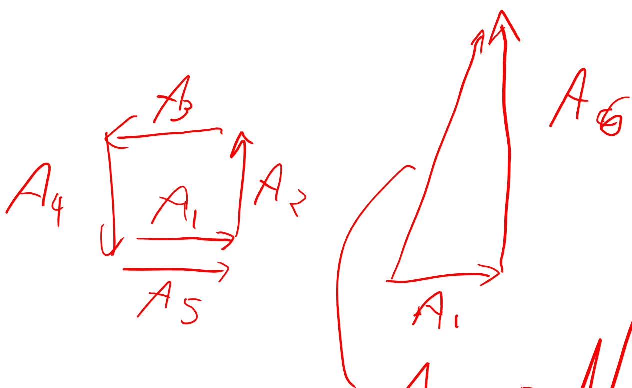
a. 10

b. 2.5

c. 10

d. 0

e. 6



$A_{TOT} = \sqrt{A_1^2 + A_6^2}$

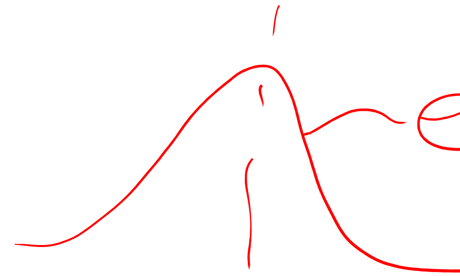
$I_{TOT} = A_1^2 + A_6^2 = I_1 + I_6 = 6\ \text{W/m}^2$

Pulse moving toward $x = -\infty$

4. A water-wave disturbance travelling along the surface of a lake takes the form, $h(x,t) = 0.2\exp[-(2x+5t)^2/3]$.
Here, x has units of meters and t has units of seconds. Suppose, instead, that a generator produces sinusoidal water waves of frequency 8 Hz on the same lake.
What would be the wavelength of these water waves?

a. 0.438 m
b. 0.312 m
c. 0.625 m
d. 0.5 m
e. 0.104 m

k ω



e^{-x^2}

$x=0$

$\sin(x \pm vt)$

$\sin(kx \pm \omega t)$

$$\left(\frac{2\pi}{\lambda}x - \cancel{f}\,2\pi f t\right) = \frac{2\pi}{\lambda}\left(x - \cancel{2\pi}\,\lambda f t\right)$$

$$= k(x - vt)$$

$$\lambda = \frac{v}{f} = \frac{5/2}{8}$$

$$2\pi\lambda\ \frac{f}{2\pi} = v = \frac{\omega}{k} = \frac{5}{2}$$

Consider the wavefunction shown below, relating to an excited eigenstate of an electron in an infinite 1D quantum well. The 1D well has a length L = 0.5 nm.

5. If the electron decays to a lower energy level, what is the ***longest*** wavelength of photon that may be emitted?

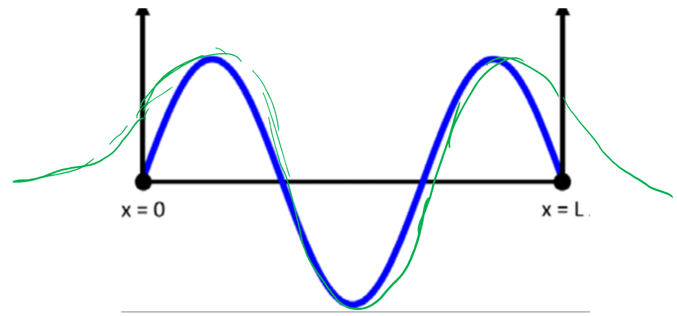
a. 206 nm

b. 91.6 nm

c. 103 nm

d. 165 nm

e. 26.2 nm



$n=3$

$E_{photon} = hf = \frac{hc}{\lambda}$

longest $\lambda \Rightarrow$ smallest E_{photon}

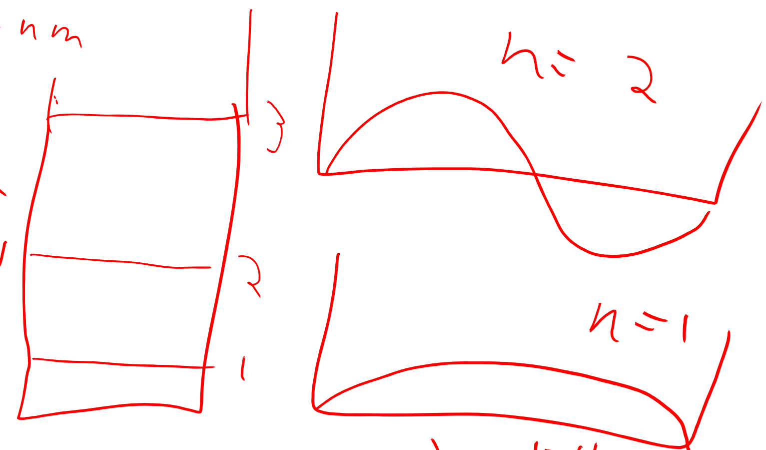
$\lambda = \frac{hc}{E_{photon}} = \frac{1240\,eV\text{-}nm}{E_{photon}}$

$E_{photon} = E_3 - E_2 = 3^2 E_1 - 2^2 E_1$

$= 5E_1$

$= 5\frac{h^2}{8mL^2} = 5\left(\frac{h^2}{2m}\right)\frac{1}{4L^2}$

$1.505\,eV\text{-}nm^2 = 7.5\,eV \Rightarrow \lambda = \frac{1240}{7.5}$



6. Imagine that particle in the well is a **muon** instead of an electron, where a muon has the same charge as an electron but a rest mass that is 207 times greater. Which of the following relationships would describe the relative energies of the electron's second excited state ($E_{el}^{n=3}$) and the muon's second-excited state ($E_{mu}^{n=3}$)?

a. $E_{el}^{n=3} > E_{mu}^{n=3}$
b. $E_{el}^{n=3} < E_{mu}^{n=3}$
c. $E_{el}^{n=3} = E_{mu}^{n=3}$

$$E_n = \frac{h^2}{8mL^2} n^2$$

$$E \propto \frac{1}{m}$$

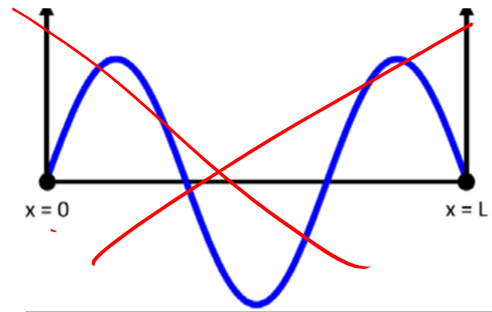
$$\frac{E_{el}}{E_{muon}} = \frac{m_{muon}}{m_{electron}} > 1$$

7. Consider now a muon in the ground state (n = 1). A measurement of its position determines that it is within the region between x = 0 and x = 0.05 nm. If you subsequently performed a measurement of the muon's speed, what is roughly (to an order of magnitude) the maximum value that you would expect to observe?

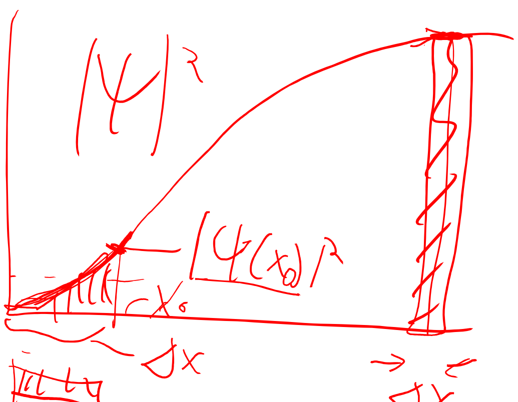
a. ~10^6 m/s

b. ~0 m/s

c. ~10^4 m/s

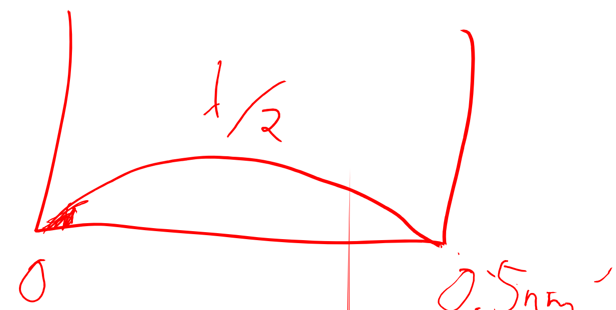


$$\Delta x \, \underset{m\Delta v}{\Delta p} \geq \hbar = h/2\pi$$



$$\Delta v = \frac{\hbar}{\Delta x\, m}$$

$$= \frac{6.6\times10^{-34}\ \text{J-s}/2\pi}{0.05\times10^{-9}\ \text{m}\,(207\, m_e)} \qquad m_e = 9.1\times10^{-31}\ \text{kg}$$



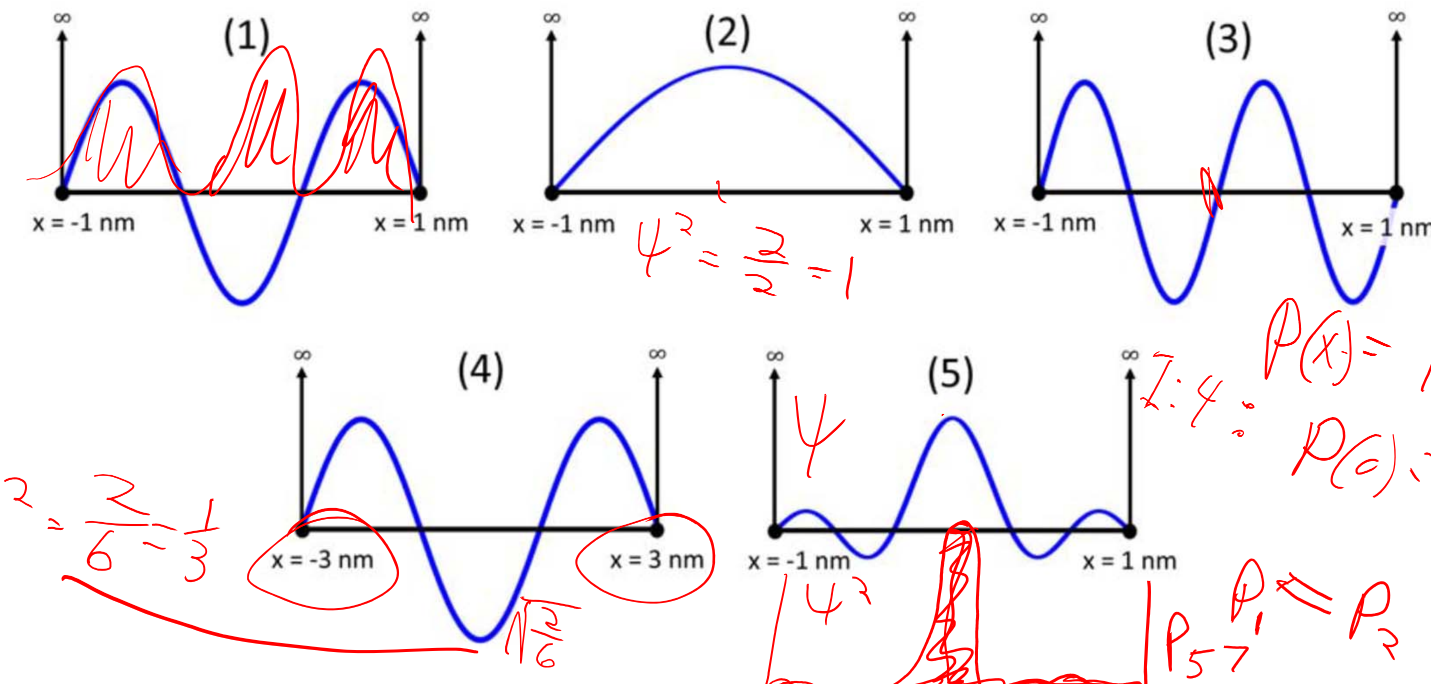
$$\psi(x) = \sqrt{\tfrac{2}{L}}\sin(kx) \qquad \approx 11{,}000\ \text{m/s}$$

Consider the quantum state wavefunctions shown below (labeled #1-#5).
Note that the first 4 relate to energy eigenstates, while the final plot is for a superposition of energy eigenstates.
Also note that the axes (both horizontal and vertical) of the various plots are not necessarily scaled the same.

Normalization fcn

1:4:

$\sqrt{\frac{2}{L}}$



$\psi^2 = \frac{2}{2} = 1$

$P(x) = |\psi(x)|^2$

1:4:

$P(0): \frac{2}{L}$

$|\psi(x=0)|^2 = \frac{2}{6} = \frac{1}{3}$

$\sqrt{\frac{2}{6}}$

ψ

ψ^2

$P_5 > P_1 = P_2 > P_3(x=0) = 0$

8. What is the correct ordering of the above wavefunctions, according to their probability density P(0) to be found at the point x = 0?

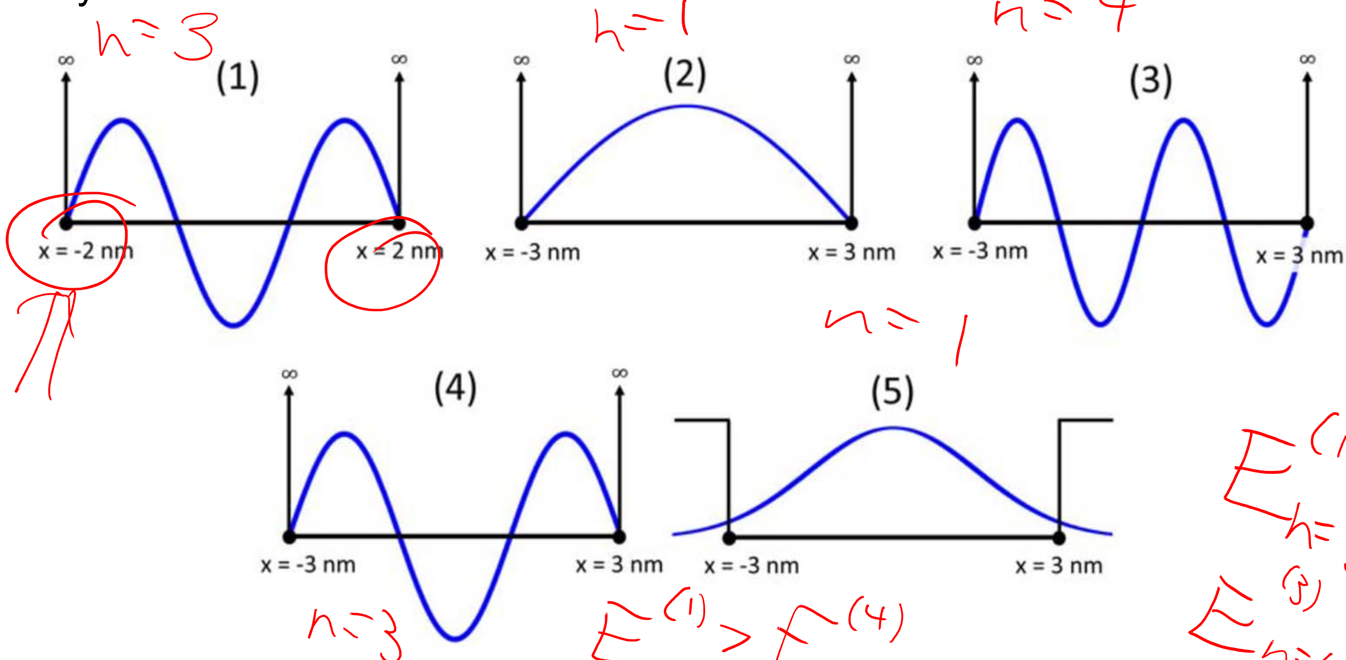
a. $P_5(0) > P_1(0) = P_2(0) > P_4(0) > P_3(0)$
b. $P_2(0) > P_1(0) > P_4(0) > P_5(0) > P_3(0)$
c. $P_5(0) > P_2(0) > P_3(0) > P_4(0) > P_1(0)$

$P_5 > P_1 = P_2 > P_4 > P_3 = 0$

Compare $P(-0.01 < x < 0.01)$ for (1) & (2):

P(ground) > P(n=3 state) > P(n=2)

Consider the quantum eigenstate wavefunctions shown below (labeled #1-#5). Note that the axes (both horizontal and vertical) of the various plots are not necessarily scaled the same.



Infinite

$E_n = \frac{h^2}{8mL^2} n^2$

Finite

E_n lower

$E^{(1)}_{n=3,\,4nm} \propto \frac{3^2}{4^2} = 0.56$

$E^{(3)}_{n=4,\,6nm} \propto \frac{4^2}{6^2} = 0.44$

$E^{(1)} > E^{(4)}$

9) What is the correct ordering of the above wavefunctions according to their energy E?

a. $E_1 > E_3 > E_4 > E_5 > E_2$

b. $E_3 > E_4 > E_5 > E_2 > E_1$

c. $E_5 > E_3 > E_1 > E_4 > E_2$

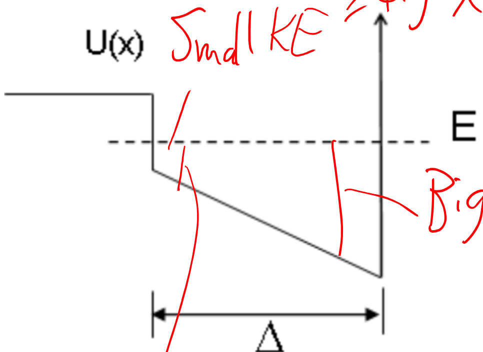
d. $E_1 > E_3 > E_4 > E_2 > E_5$

e. $E_3 > E_1 > E_4 > E_2 > E_5$

$E^{(1)}_{n=3,\,4nm} > E^{(3)}_{n=4} > E^{(4)}_{n=3} > E^{(2)}_{n=1} > E^{(5)}_{n=1,\,finite}$

10. Consider a particle in the 5th excited state (n = 6) of the potential well shown below. The numbering is such that n=1 is the ground state. Which of the following best represents the probability distribution of the particle? (The horizontal axis corresponds to probabilty = 0.)

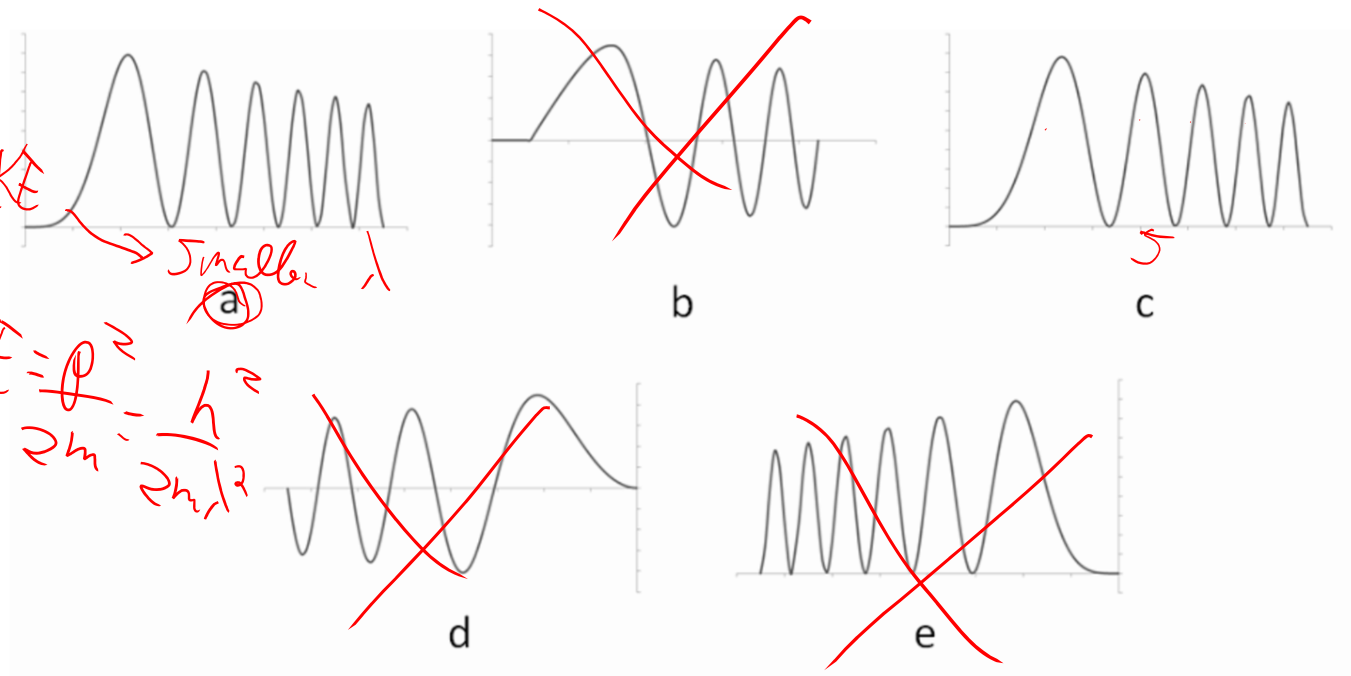
$|\psi|^2$



Small KE = big λ

Bigger KE

$KE = E - U$



Smaller λ

5

$$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$$

A screen with multiple slits is fully illuminated by a red laser (wavelength 700 nm) and the following interference pattern is observed on a screen 3 m away.

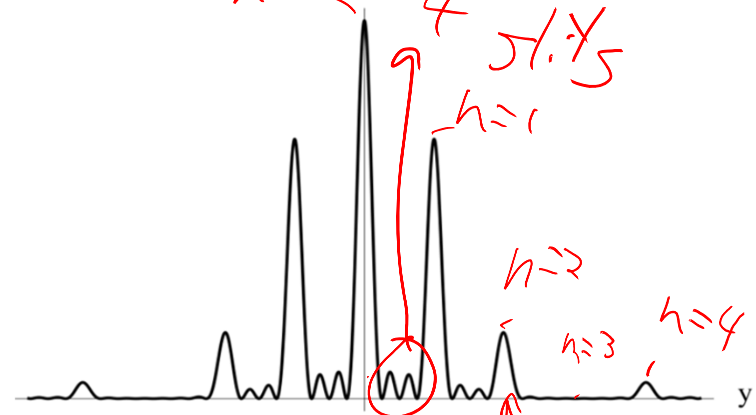
$L = 3$ m

11. How many slits are there on the screen?

a. 4

b. 5

c. 3



2 slits

3 slits

12. Consider now the second principal maximum of the intensity. At the point of this maximum, what is the path difference for the waves coming from the first slit and the third slit of the diffraction grating? Assume the wavelength is λ.

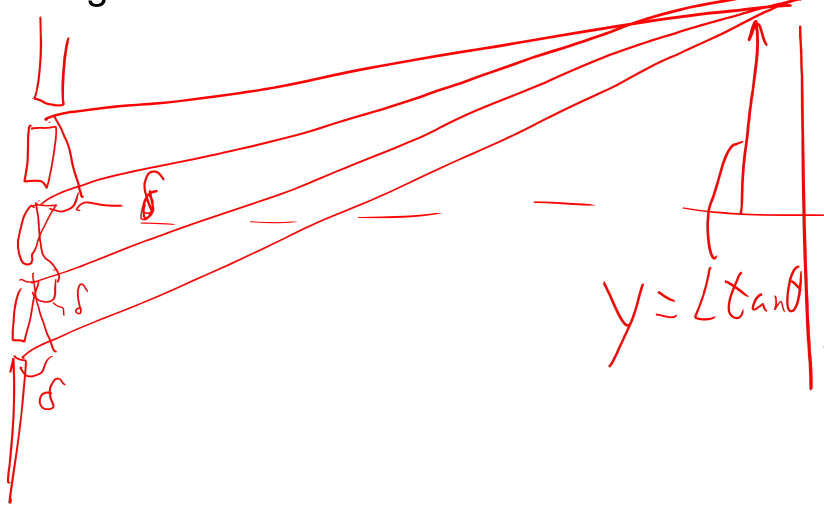
a. 4 λ

b. λ /2

c. 2 λ

Path length difference between slit 1 & 3 $= 2\delta$

$= 4\lambda$



0 Prin. Max

$\delta = 0$

1st Princ Max

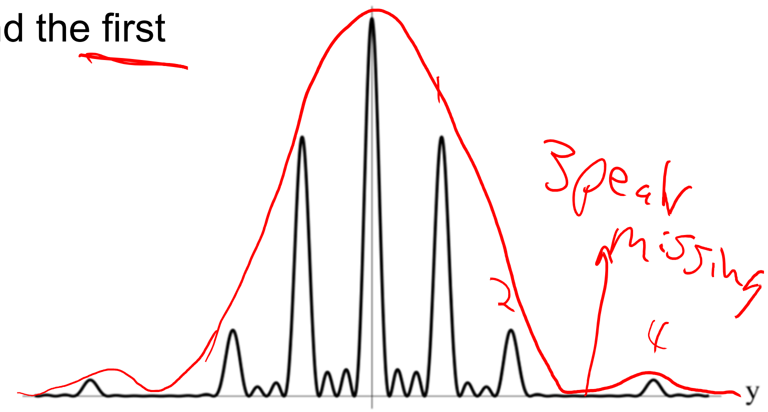
$\delta = \lambda$

2nd Prin Max

$\delta = 2\lambda$

13. Suppose now the distance between the zero principal maximum and the first principal maximum on the screen is 0.001 m. Calculate the distance between the slits of the diffraction grating.

a. 0.0021 m
b. 0.0105 m
c. 0.042 m
d. 0.021 m
e. 7×10^{-4} m



$\delta = \lambda = d\sin\theta \sim d\theta$

$y = L\tan\theta \sim L\theta$

$\delta = \lambda = d\frac{y}{L}$

$d = \frac{\lambda L}{y} = \frac{700\,nm\ 3\,m}{0.001\,m} = 0.0021\,m$

2.1 mm

What is slit width?

3rd missing principal max sits on diffraction minimum

$\sin\theta_n = \frac{n\lambda}{d}$

$\sin\theta_{min} = \frac{\lambda}{a} = \frac{3\lambda}{d} \Rightarrow a = \frac{d}{3} = 0.7\,mm$

Starshot

Traveling to interstellar space is difficult, in large part because it's very expensive to carry along all the necessary fuel. An alternative is to put an optical reflector on the back of the space ship, and then push it using laser beams from the earth. Initially, scientists (including Steven Hawking) envision using very small "nanocraft", each the size of a postage stamp (mass = 1 gram) and each equipped with a thin reflective circular sail that is 4 m in diameter. The purpose of the sail is to reflect all of the collected photons from the lasers back to the source, which doubles the momentum transfer to the space ship.

$F = ma \quad a = F/m \approx 2\,\text{Power}\,\frac{d^2}{D^2} / mc$

14. The original laser beam will be very large (~1 km in diameter). Initially neglecting spreading of the beam (valid when a nanocraft is close to earth), estimate the acceleration that the nanocraft will feel, assuming the total laser power over the entire 1-km diameter is a sustained 100 GigaWatts (1 GigaWatt = 10^9 W). Don't forget to account for the finite size of the sail, or the fact that it is 100% reflective.

a. 330,000 m/s^2
b. 10.7 m/s^2
c. 3.2 m/s^2
d. 0.23 m/s^2
e. 670,000 m/s^2

$F = \frac{\Delta p}{\Delta t} = \underbrace{2}_{\text{reflection}} \underbrace{p_{photon}}_{h/\lambda}\, \#\text{phot}/s$ # that hit sail

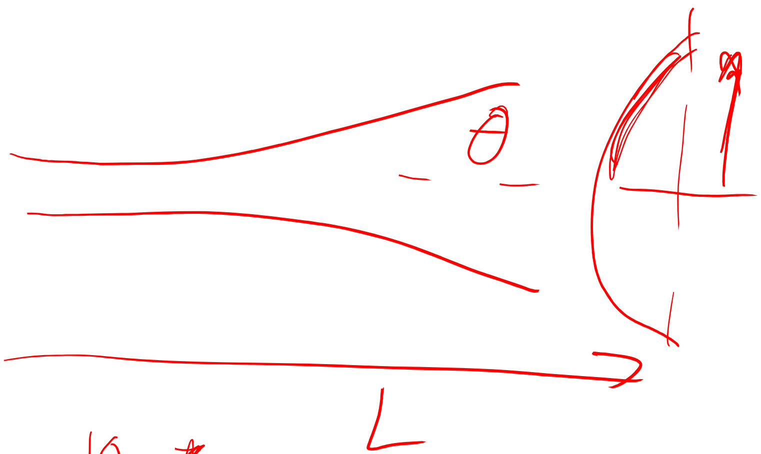
$\#\text{phot}/s = \#\text{phot}/s\ \frac{A_{sail}}{A_{beam}} = \#\text{phot}/s\ \frac{d^2}{D^2}$

$\text{Power} = \underbrace{E_{photon}}_{hc/\lambda}\, \#\text{phot}/s \qquad \#\text{phot}/s = \frac{\text{Power}}{hc/\lambda}$

$\#\text{phot}/s = \frac{\text{Power}\,\lambda}{hc} \frac{d^2}{D^2}$

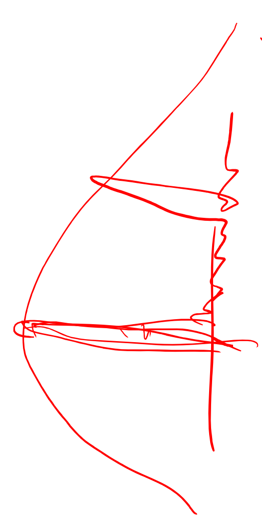
$F = 2\frac{\cancel{h}}{\cancel{\lambda}} \frac{\text{Power}}{\cancel{h}c} \cancel{\lambda} \frac{d^2}{D^2} = \frac{2\,\text{Power}}{c} \frac{d^2}{D^2}$

How far do we need to go before laser beam diameter is 10x [10 km] ?



$w \approx L\theta = L\, 1.22\lambda/D$

$D' = 2w = 2.44 \frac{\lambda L}{D} = 10D$



$L = \frac{10D^2}{2.44\lambda} \approx \frac{10\; 10^6 m^2}{2.44\; 1\mu m}$

$= \frac{10^7}{2.4\times10^{-6}} \approx 10^{13} m$

15. Of course when the ship gets far enough away, the diffraction of the beam will reduce the intensity of the laser radiation hitting the sail. Which of the following would reduce the diffraction spreading of the beam?
1. Use a shorter wavelength for the 100 GW laser.
2. Use a longer wavelength for the 100 GW laser.
3. Decrease the size of the laser beam at the source.

$\theta \approx \frac{\lambda}{D}$

a. 2 and 3 would both work
b. 2
c. 3
d. 1 and 3 would both work
e. 1

16. Assuming that the wavelength of the laser beam is 400 nm and that the "sail" is made of gold (which has a work function of 5.1 eV) what is the kinetic energy of electrons that may be released from the gold by the photoelectric effect?

a. 1.0 eV

b. No electrons are released.

c. 3.1 eV

$$E_{photon} = \phi + KE_{max}$$

$$= \frac{1240 \text{ eV-nm}}{400} \sim 3 \text{ eV} < 5.1 \text{ eV}$$

No emitted electrons

If $E_{photon} > \phi$

$$\frac{1}{2}mv^2 = E_{photon} - \phi$$

17. A beam of electrons (initially nearly at rest) is accelerated through an electostatic potential of 5000 V, resulting in electrons that have a kinetic energy of 5.0 ± 0.1 keV. These electrons are then directed onto a nanofabricated structure with two small slits separated by a distance of 10 nm. An imaging detector is placed 2 m away from the slits.

Assuming that only one electron passes through the slits at a time, what is the separation of adjacent interference maxima on the screen?

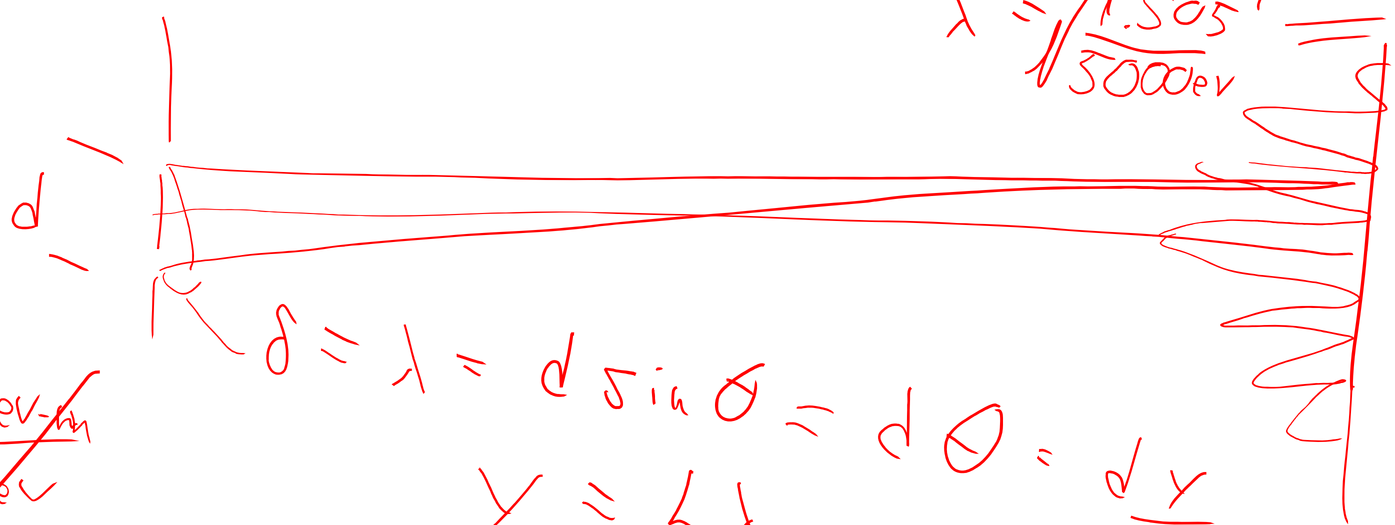
a. 5 cm

b. There is no interference if only one electron passes the slits at a time.

c. 3.5 mm

$KE = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{1.505 eV\text{-}nm^2}{\lambda^2}$

$\lambda = \sqrt{\frac{1.505}{5000 eV}} = 0.0173 nm$



$\Delta y = \theta L$

~~$\lambda = \frac{1240 eV\text{-}nm}{5000 eV} \Rightarrow 0.25 nm$~~

$y = 0.05 m$

5 cm

$\delta = \lambda = d \sin\theta = d\theta = d\frac{y}{L}$

$y = \frac{L\lambda}{d}$

$y = 0.0035 m$

18. A beam of photons is now directed onto the same pair of slits and generates the *same* interference probability distribution interference pattern for the arrival probability distribution on the screen. What is the energy of the photons?

a. 24.5 eV
b. 71.5 keV
c. 5 keV

Same pattern $\rightarrow$ Same λ

$\lambda_{elec} = 0.0173\ nm$

$E_{photon} = \frac{hc}{\lambda} = \frac{1240\ eV\text{-}nm}{0.0173\ nm}$

19. The top slit is now slightly charged to slow the electrons going through that slit to 90% of the speed of the electrons going through the other slit. What will happen to the overall pattern of the electrons on the screen, assuming the total number of electrons reaching the screen from each slit is not changed?

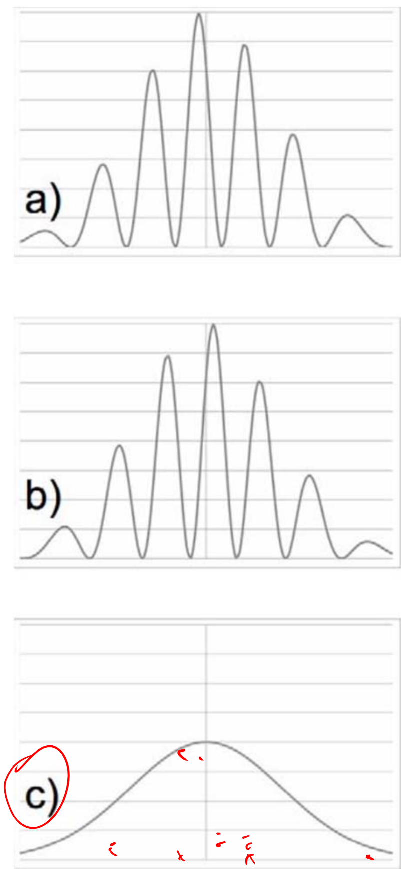


Figure c: There will no longer be interference fringes, but the overall envelope will be slightly wider.

Figure b: The entire pattern will shift up slightly, i.e., the central principal maximum will occur at a (small) positive angle; the top of the screen corresponds to the right side of the diagram.

Figure a: The entire pattern will shift down slightly, i.e., the central principal maximum will occur at a (small) negative angle; the top of the screen corresponds to the right side of the diagram.

Interference requires indistinguishable processes!

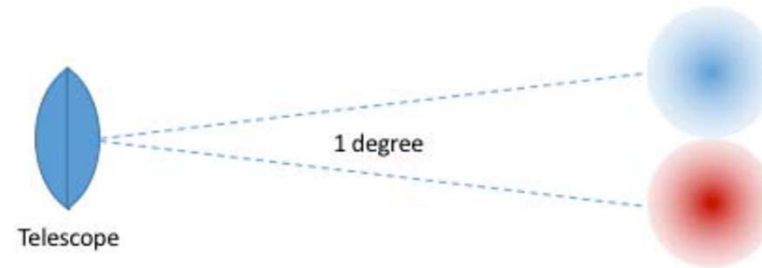
Electrons that were slowed

smaller $p \Rightarrow$ larger λ

larger pattern

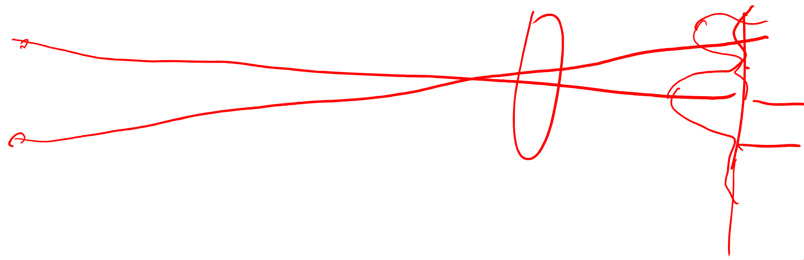
(10% wider)

The Cosmic Microwave Background (CMB) is a primordial glow of electromagnetic radiation, which comes to us with almost equal intensity from every direction in the sky. The CMB shows a characteristic pattern of very faint bright and dim patches, with a typical patch separation of one degree. We would like to build a telescope that can resolve these bright and dim patches, i.e. distinguish two sources in the sky separated by 1 degree.



20. Suppose that we build our telescope to measure the CMB at a frequency of 120 GHz. What diameter must the telescope's (circular) aperture have in order to just barely resolve these features?

a. 0.0025 m
b. 5.73 m
c. 0.175 m
d. 0.00205 m
e. 0.349 m



$$\theta_c = \underbrace{\frac{1.22\lambda}{D}}_{\text{radians}} < 1^\circ$$

$$\lambda = \frac{c}{f}$$

$$\sin\theta_c = \frac{1.22\lambda}{D}$$

$$D > \frac{1.22\lambda}{1^\circ}\,\frac{360^\circ}{2\pi}$$

$$= 0.175\ \text{m}$$

21. Suppose that this telescope collects an average power flow of 0.15 pW, again at a frequency of 120 GHz. On average, how many photons does it collect each second?

a. 800 per second
b. 3.1 × 10^{19} per second
c. 3.1 × 10^{17} per second
d. 1.9 × 10^{21} per second
e. 1.9 × 10^{9} per second

$$\text{Power} = \underbrace{E_{photon}}_{hf}\left(\frac{\#\,phot}{s}\right)$$

$$\frac{\#\,phot}{s} = \frac{0.15\times10^{-12}\ \frac{J}{s}}{6.6\times10^{-34}\ J\cdot s\ \ 120\times10^{9}\ s^{-1}}$$

$$= 1.9\times10^{9}\ \frac{1}{s}$$

A spectrometer is used to analyze the light coming from a distant source. The spectrometer has a 9-mm wide diffraction grating with 3×10^3 lines. Light is normally incident on the grating.

22. The third-order principle maximum is observed at an angle of 45º from normal incidence. What is the wavelength of the light coming from the source? Assume the **entire** grating is illuminated.

a. 109 nm
b. 1014 nm
c. 707 nm
d. 105 nm
e. 100 nm

$$d = \frac{9\,mm}{3000} = \frac{9000\,\mu m}{3000} = 3\,\mu m$$

$$\delta = 3\lambda = d\sin\theta_3 \qquad \theta \sim \frac{3\lambda}{d}$$

45° Not a small angle:

$$d\sin\theta_3 = 3\lambda$$

$$\lambda = \frac{3\,\mu m}{3}\sin 45^\circ = 0.707\,\mu m$$

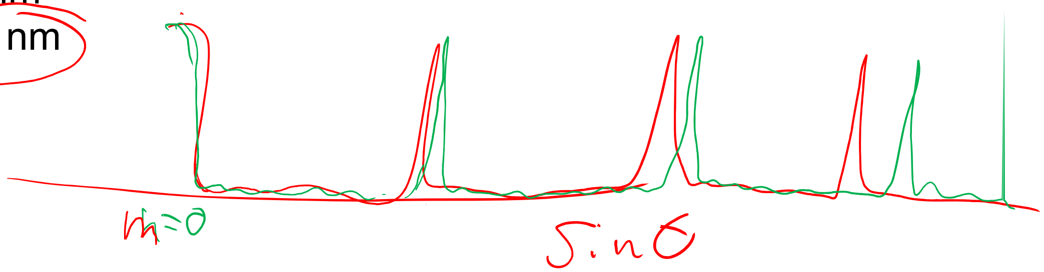
$$\lambda = \frac{d\theta}{3} = \frac{3\,\mu m}{3}45^\circ = \cancel{45\,\mu m}$$

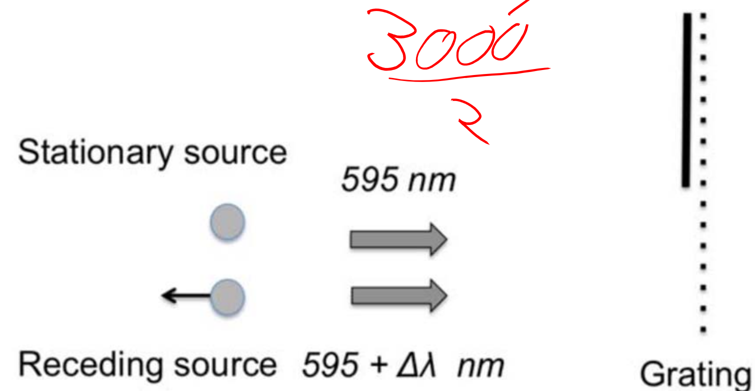
$$\frac{3\,\mu m}{3}\frac{\pi}{4} = 0.78\,\mu m$$

23. Light from two distant sources now reaches the spectrometer. The stationary source emits light of wavelength 595 nm. Due to the Doppler shift, the receding source emits a longer wavelength, 595 + Δλ nm. What is the smallest value of Δλ that can be detected by this spectrometer, now assuming that only **half** the number of lines in the grating are illuminated? (Hint: what is the highest order that can be used?)

$\frac{3000}{2}$

a. 0.32 nm
b. 0.4 nm
c. 0.08 nm





$$\delta = d\underbrace{\sin\theta_m}_{\le 1} = m\lambda$$

$$m \le d/\lambda = 3\mu m / 0.595\mu m = 5.04 \Rightarrow 5$$

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

$$v = \frac{0.08}{595} c \rightarrow 40.3 km/s$$

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{Nm}$$

$$\Delta\lambda \ge \frac{\lambda}{Nm} = \frac{595 nm}{\frac{3000}{2} \cdot 5} = 0.08 nm$$

The LIGO interferometer is used to detect the very, very small compression and expansion of space as a gravity wave passes by. It is essentially a Michelson interferometer, each of whose arms L_1 and L_2 is 4 km long. The input is a 200-W laser at a wavelength of 1064 nm. Assume that at the start the interferometer arms are adjusted so that all the light exits to the detector shown.

$L_1 = L_2$ gives the full 200 W coming to the detector.

24. What is the minimum increase in L_2 required so that only 100 W goes to the detector?

a. 266 nm

b. 133 nm

c. 532 nm

$L_1 = L_2$

$\Delta x = \frac{\delta}{2} = \frac{\lambda}{8}$

$\boxed{I_{Tot} = 4 I_1 \cos^2 \frac{\phi}{2}}$

$I_1 = \frac{I_{laser}}{4} = I_2$

$\boxed{I_{Tot} = I_{laser} \cos^2 \phi/2}$

$I_{laser}(1 - \cos^2 \phi/2)$

$= I_{laser} \sin^2 \frac{\phi}{2}$

$P_{Tot} = P_{laser} \cos^2 \frac{\phi}{2}$

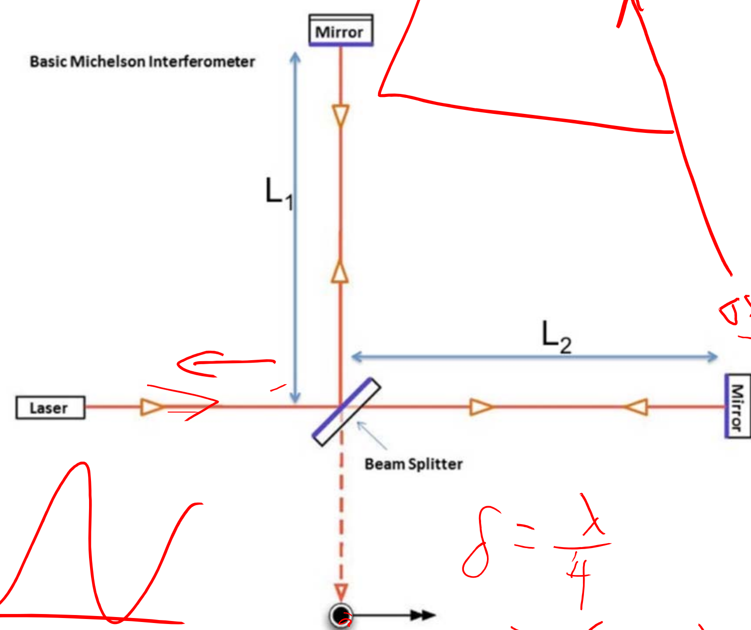
$\frac{1}{2}$

$I_{back\ to\ laser} = I_{laser} - I_{Tot}$

$\cos \frac{\phi}{2} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\phi}{2} = 45^\circ$

$\phi = 90^\circ = \frac{360\,\delta}{\lambda}$

200W



$\delta = \frac{\lambda}{4}$

$= 2(L_2 - L_1)$

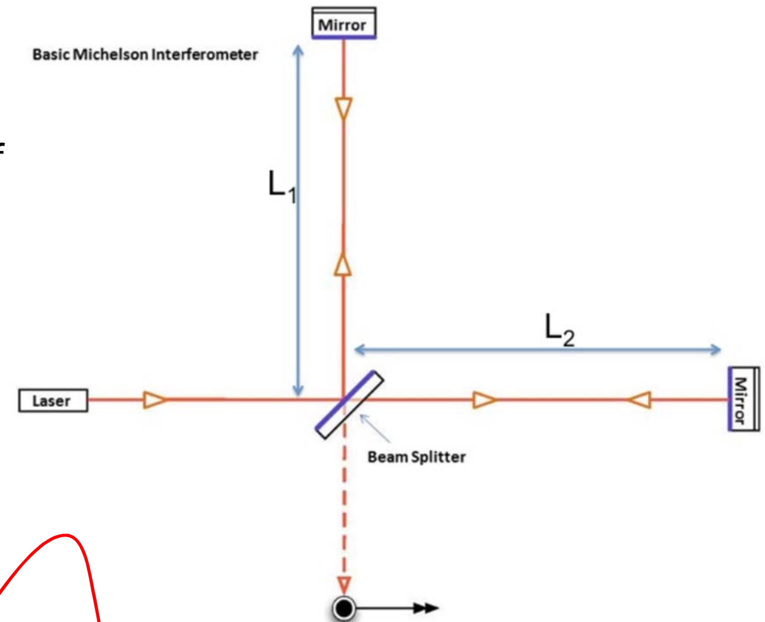
$= 2\Delta x$

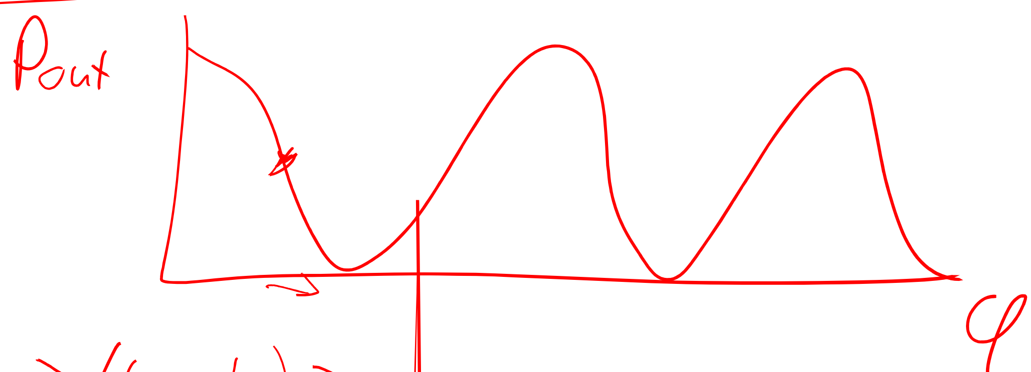
25. Assume that L_2 has been increased by the amount calculated in the previous question, so 100 W is reaching the detector. The LIGO interferometer arms are kept under high vacuum. If a small amount of air now leaks into the system (filling both arms equally), what will initially happen to the power on the detector? Neglect any absorption of the light by the air.

a. The power will increase slightly.

b. The power will stay the same.

c. The power will decrease slightly.





$$\varphi = \frac{2(L_2 - L_1)360}{\lambda}$$

$$\lambda f = v = \frac{c}{n} \Rightarrow \lambda = \frac{c}{f n} = \frac{\lambda_0}{n}$$

$n\uparrow \quad \lambda\downarrow \quad \varphi\uparrow \quad$ Power $\downarrow$