Last Name:________________First Name_________________NetID_______________
Discussion Section:________Discussion TA Name:______________________________

Instructions—
Turn off your cell phone and put it away.
Keep your calculator on your own desk. Calculators may not be shared.
This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil; do **not** use a mechanical pencil or a pen. Fill in completely (until there is no white space visible) the circle for each intended input – both on the identification side of your answer sheet and on the side on which you mark your answers. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner.

2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.

3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter "I" and the numeral "1" and for the letter "O" and the numeral "0". **Do not** mark the hyphen circle at the bottom of any of these columns.

4. You may find the version of this Exam Booklet at the top of page 2. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**

5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.

6. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION block.

7. On the **SECTION** *line*, print your **DISCUSSION SECTION**. (You need not fill in the COURSE or INSTRUCTOR lines.)

8. Sign **(DO NOT PRINT)** your name on the **STUDENT SIGNATURE** *line*.

Before starting work, check to make sure that your test booklet is complete. You should have 11 ***numbered pages*** *plus two Formula Sheets at the end.*

Academic Integrity—**Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.**

This Exam Booklet is Version A. Mark the **A** circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of **108** points, composed of two types of questions.

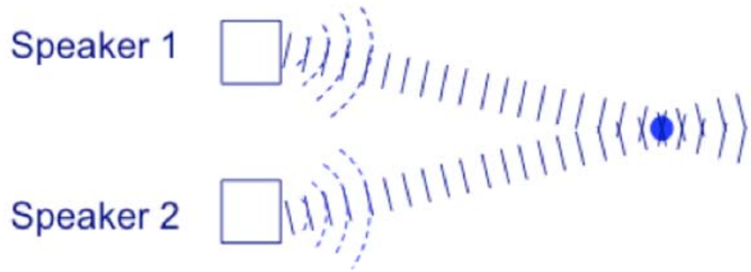
MC5: *multiple-choice-five-answer questions, each worth 6 points.*
Partial credit will be granted as follows.

(a) If you mark only one answer and it is the correct answer,
you earn **6** points.
(b) If you mark *two* answers, one of which is the correct answer,
you earn **3** points.
(c) If you mark *three* answers, one of which is the correct answer,
you earn **2** points.
(d) If you mark no answers, or more than *three*, you earn 0 points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*
No partial credit.

(a) If you mark only one answer and it is the correct answer,
you earn **3** points.
(b) If you mark a wrong answer or no answers, you earn **0** points.

The next three questions pertain to the situation described below.



Suppose that you are listening to music playing from two sound speakers. One is emitting a harmonic wave $y_1 \propto cos(kx - \omega t)$ and the second is emitting a harmonic wave $y_2 \propto cos(kx - \omega t - \pi/3)$. Assume both waves produce an intensity I_1 when they are on individually.

1) What is the intensity at a point equidistant from both speakers when both speakers are on?

a. $3I_1$
b. $2I_1$
c. $4I_1$
d. I_1
e. 0

2) Suppose that Speaker 1 is now pushed directly away from the listener by 1% of the wavelength of the sound. How does the intensity at the listener change?

a. Decreases
b. Stays the same
c. Increases

3) Now we move the listener to a location where no sound at all is heard when both speakers are on. We then reduce the intensity from Speaker 2 to 1/9 that from Speaker 1: $I_2 = I_1/9$. What is the new total intensity at the listener?

a. $9I_1/4$
b. $4I_1/9$
c. $I_1/2$
d. 0
e. $I_1/4$

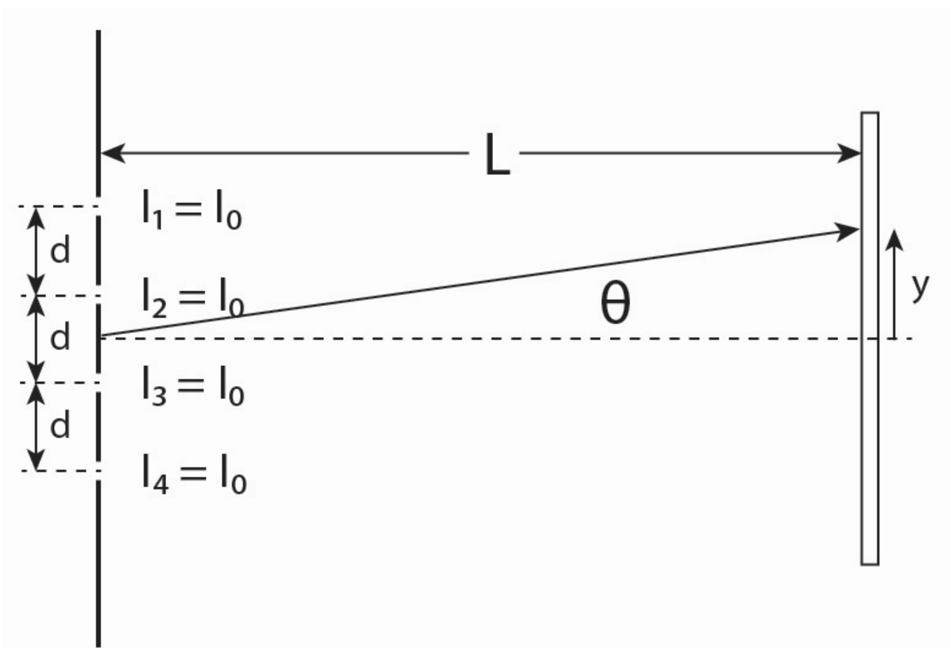
A wave is described by the following equation:
$y = A \sin[(3m^{-1})x-(900s^{-1}t)] - A \cos[(3m^{-1})x-(900s^{-1}t)]$.

4) Which of these statements is correct?

a. The wave is a standing wave with amplitude A.
b. The wave is moving to the Left at the speed of 300 ms^{-1} with an amplitude $2A^2$.
c. The wave is moving to the Right at the speed of 300 ms^{-1} with an amplitude $\sqrt{2}$ A.
d. The wave is moving to the Right at the speed of 0.00333 ms^{-1} with an amplitude 2A.
e. The wave is moving to the Right at the speed of 30 ms^{-1} with an amplitude $\sqrt{2}$ A.

The next three questions pertain to the situation described below.

Consider the 4-slit interference experiment illustrated below, on which a laser with wavelength λ shines through 4 equally spaced slits and the transmitted light is observed on a distant screen. NOTE: the distance to the screen L is very, very large (picture not drawn to scale).

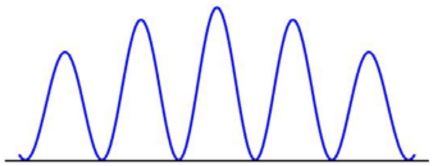


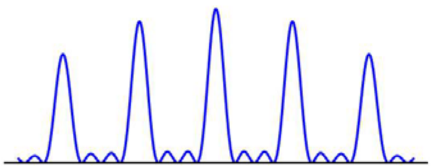
The slits are all of the same size, such that the light intensity at the center of the screen (y=0) coming from any one slit, when all other slits are blocked, is found to be I_0.

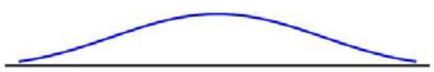
5) What is the intensity measured at the center of the screen, i.e. at y=0?

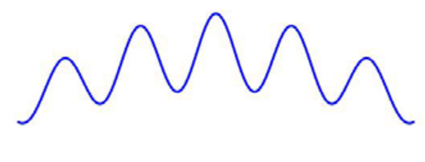
a. 9 I_0
b. 4 I_0
c. 16 I_0

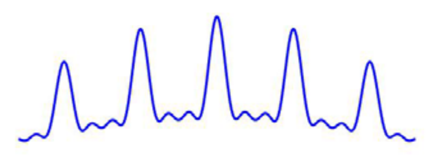
6) Assume the incident light is unpolarized. Which of the following patterns would be observed if a horizontal polarizer covers slits 1 and 3, while a vertical polarizer covers slits 2 and 4?

(a)


(b)


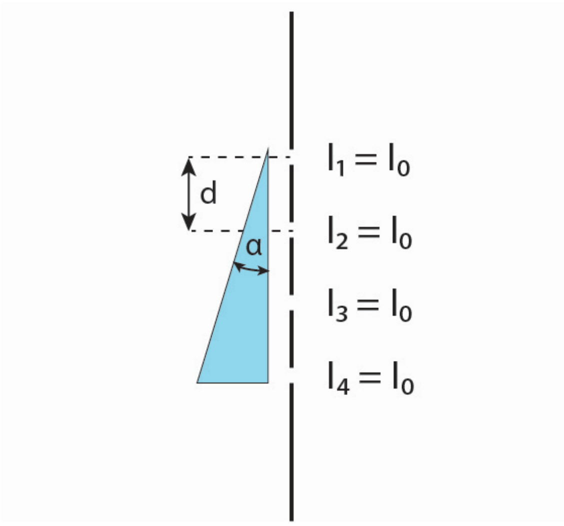
(c)


(d)


(e)


a. Graph (b)
b. Graph (a)
c. Graph (d)
d. Graph (c)
e. Graph (e)

7) Imagine instead that the light remains unpolarized, but a very thin wedge of material with an index of refraction **n** is placed in front of the slits as shown below. Before the slits, this wedge results in an optical path length difference of $\Delta = (\mathbf{n}-1)\, d \tan(\alpha)$ between adjacent slits.

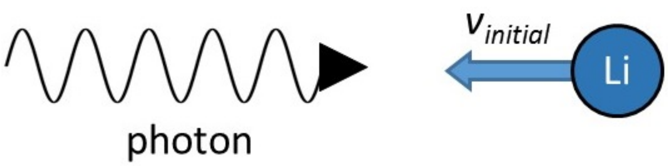


If $d \tan(\alpha) = \lambda/2$, what is the minimum value of **n** such that an interference minimum appears at y=0?

a. n = 2
b. n = 1
c. n = 1.5

The next two questions pertain to the situation described below.

Consider a lithium atom (^{7}Li, mass of 7 amu ≈ 1.16 x 10^{-26} kg) traveling with an initial velocity of $v_{initial}$ = 200 m/s. As shown below, we shine in laser photons that directly oppose the atom's motion, and which have the exact photon energy (1.85 eV) needed to resonantly excite the atom to a higher electonic energy level.



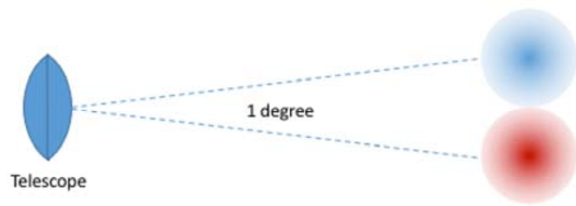
8) At what velocity would the lithium atom have a de Broglie wavelength equal to the wavelength of the laser photons?

a. 85.06 mm/s
b. 42.53 mm/s
c. 170.12 mm/s

9) Every time the atom is excited, it slows down due to the transfer of one photon momentum (the atom then de-excites, emitting a photon in a random direction -- the momentum from these events cancel out on average). To slow the atom from $v_{initial}$ = 200 m/s to a complete stop (v ~ 0 m/s), roughly how many photon excitations must take place?

a. 1175.62
b. 14773.22
c. 2351.23

The next three questions pertain to the situation described below.



The Cosmic Microwave Background (CMB) is a primordial glow of electromagnetic radiation, which comes to us with almost equal intensity from every direction in the sky. The CMB shows a characteristic pattern of very faint bright and dim patches, with a typical patch separation of one degree. We would like to build a telescope that can resolve these bright and dim patches, i.e. distinguish two sources in the sky separated by 1 degree.

10) Suppose that we build our telescope to measure the CMB at a frequency of 120 GHz. What diameter must the telescope's (circular) aperture have in order to just barely resolve these features?

a. 0.349 m
b. 0.0025 m
c. 0.00205 m
d. 0.175 m
e. 5.73 m

11) The CMB emits electromagnetic radiation at a broad range of frequencies (it has a "blackbody spectrum", which some of you will learn about in Physics 213). Suppose that we instead build a telescope to observe it at 90 GHz with the same angular resolution. How would the diameter of this new telescope compare to the one described in the previous problem?

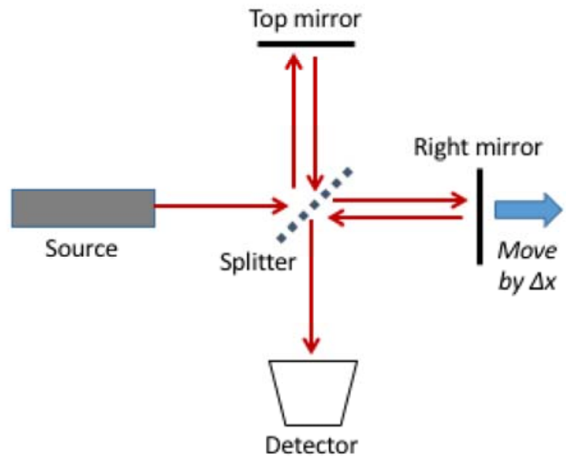
a. The new aperture would be bigger
b. They would be the same size
c. The new aperture would be smaller

12) Suppose that this telescope collects an average power flow of 0.15 pW, again at a frequency of 120 GHz. On average, how many photons does it collect each second?

a. 1.9×10^{21} per second
b. 3.1×10^{19} per second
c. 800 per second
d. 1.9×10^{9} per second
e. 8×10^{-23} per second

The next three questions pertain to the situation described below.

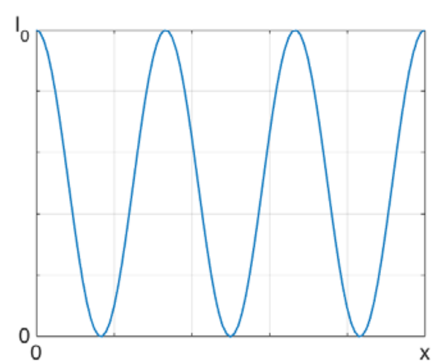
Consider the Michelson interferometer shown. Electromagnetic radiation from a source enters from the left, is divided by a balanced (50:50) beam splitter, traverses the two arms, and finally is recombined and directed to the intensity detector at bottom. The right mirror is movable, so that we can make the right arm longer or shorter than the top arm. Such an adjustable interferometer can be used to measure the wavelength of the input radiation, and is also known as a "Fourier Transform Spectrometer".



13) Suppose that we illuminate the input port on the left with a far-infrared laser of wavelength 900 micrometers and intensity 4 W/m ^{2}. Suppose that we have initially positioned the right mirror so that we obtain maximum intensity at the detector, i.e. all light from the source is directed to the detector. Now suppose that we move the right mirror to the right by 0.1 mm. What intensity will I now observe at the detector?

a. 2.35 W/m^2
b. 0.695 W/m^2
c. 0.121 W/m^2

14) Suppose now that we replace the laser with an unknown, single-frequency light source and adjust the position of the right mirror to record a maximum in intensity. As we slowly lengthen the right arm by a total of x=3 mm the detector signal goes from minimum to maximum again three times (see figure). What is the wavelength of the unknown light source?



a. 1 mm
b. 9 mm
c. 2 mm

15) Suppose that we use a second light source and see twice as many fringes during the same change in arm length. What does that imply about the two light sources?

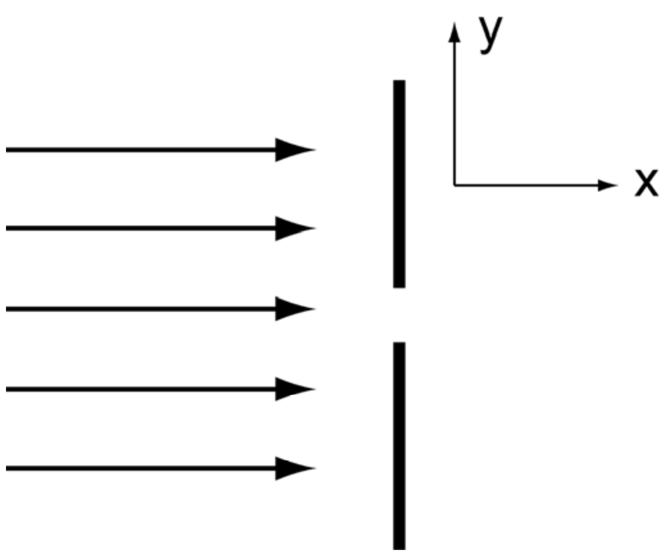
a. The second source has a shorter wavelength than the first
b. The second source has a longer wavelength than the first
c. Impossible to determine from this information

Asuume that the absolute value of the kinetic energy of the electron in the Hydrogen atom equals the absolute value of the total energy of the ground state. The ground state energy is -13.6 eV. Use Heisenberg uncertainty to estimate the radius of the atom.

16) The estimated approximate radius is:

a. 0.5 nm
b. 5 nm
c. 10 nm
d. 0.05 nm
e. 1 nm

17) Electrons are accelerated to 1400 eV and made incident on an aperture with a 100−nm diameter. Estimate the minimum range of transverse (y-component) velocities that the electrons possess after passing through the aperture.



a. 10^4 m/s
b. 10^6 m/s
c. 10^3 m/s
d. 10 m/s
e. 0 (any electrons that actually pass through the opening do not acquire any transverse momentum.)

The next three questions pertain to the situation described below.

The wavefunction of an electron occupying the $n = 3$ energy eigenstate of a 10 nm wide one-dimensional infinitely-deep quantum well is given by $\psi(x) = A\sin(kx)$, where A is the overall amplitude. The quantum well extends from $x = 0$ nm to $x = 10$ nm.

18) What is the energy of this state?

a. 3.00 eV
b. 1.602 eV
c. 10^{-8} eV
d. 0.531 eV
e. 33.9 meV

19) The overall amplitude, A, is given by

a. 1.00 $m^{-1/2}$
b. 1.414 x 10^4 $m^{-1/2}$
c. 25 $m^{-1/2}$
d. 5 x 10^{-4} $m^{-1/2}$
e. 10^4 $m^{-1/2}$

20) Compare the probability of finding the electron at x = 3.33 nm with the probability of finding the electron at x = 5.0 nm. P(3.33 nm)/P(5 nm) equals

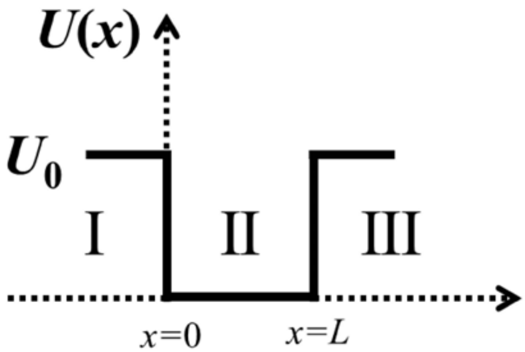
a. 1
b. .707
c. 0

21) Two infinitely deep one-dimensional quantum wells have widths L_1 and L_2. What is the ratio L_1/L_2 that causes the energy levels to be different by a factor of two?

a. 4
b. 1.414
c. 2

The next three questions pertain to the situation described below.

A one-dimensional finite quantum well extends from $x = 0$ to $x = L$. The potential inside the well is zero and outside the well it is $U_0 > 0$. It contains an electron which is in its ground state.



22) Compare the energy of the ground state of the electron confined in this well with the energy of an electron confined in an infinitely deep quantum well of the same width:

E_1(finite well)/E_1(infinite well) is

a. equal to 1
b. greater than 1
c. less than 1

23) Which statement about the ground state wavefunction is true?

a. The first derivative of the wavefunction discontinuously jumps at x = 0 and also at x = L.
b. Both the wavefunction itself and its first derivative are continuous at x = 0 and x = L.
c. The value of the wave function discontinuously jumps at x = 0 and also at x = L.

24) In region III it is determined that the wavefunction is given by $\psi(x) = Ae^{-x/b}$, where b=2x10^{-10} m. The particle is an electron and its mass is given in the formula sheet. Use Schroedinger's equation to calculate the energy difference $U_0 - E_1$ =

a. 0.662 eV
b. 1.351 eV
c. 0.389 eV
d. 0.027 eV
e. 0.945 eV

Physics 214 Common Formulae

SI Prefixes

Power	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^0		
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Physical Data and Conversion Constants

speed of light	$c = 2.998 \times 10^8$ m/s
Planck constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.135 \times 10^{-15}$ eV·s
Planck constant / 2π	$\hbar = 1.054 \times 10^{-34}$ J·s $= 0.658 \times 10^{-15}$ eV·s
electron charge	$e = 1.602 \times 10^{-19}$ C
energy conversion	1 eV $= 1.602 \times 10^{-19}$ J
conversion constant	$hc = 1240$ eV·nm$= 1.986\times 10^{-25}$ J-m
useful combination	$h^2/2m_e = 1.505$ eV nm^2
Bohr radius	$a_o = (4\pi\varepsilon_o)\hbar^2 / m_e e^2 = 0.05292$ nm
Rydberg energy	$hcR_\infty = m_e e^4 / 2(4\pi\varepsilon_o)^2 \hbar^2 = 13.606$ eV
Coulomb constant	$\kappa = 1/(4\pi\varepsilon_o) = 8.99 \times 10^9$ N·m^2 / C^2
Avagadro constant	$N_A = 6.022 \times 10^{23}$ / mole
electron mass	$m_e = 9.109 \times 10^{-31}$ kg $= 0.511$ MeV/c^2
proton mass	$m_p = 1.673 \times 10^{-27}$ kg $= 938.3$ MeV/c^2
neutron mass	$m_n = 1.675 \times 10^{-27}$ kg $= 939.6$ MeV/c^2
hydrogen atom mass	$m_H = 1.674 \times 10^{-27}$ kg
Electron magnetic moment	$\mu_\varepsilon = 9.2848 \times 10^{-24}$ J/T $= 5.795 \times 10^{-5}$ eV/T
Proton magnetic moment	$\mu_p = 1.4106 \times 10^{-26}$ J/T $= 8.804 \times 10^{-8}$ eV/T

Trigonometric identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\sin\theta + \sin\phi = 2\sin\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$

$$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$$

$$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$A_1\sin(\omega t+\phi_1) + A_2\sin(\omega t+\phi_2) = A_3\sin(\omega t+\phi_3)$$

$A^2 + B^2 + 2AB\cos\phi = C^2$ (ϕ here is the *external* angle)

Waves, Superposition

$$k \equiv \frac{2\pi}{\lambda} \quad \omega \equiv 2\pi f \quad T \equiv \frac{1}{f} \quad v = \lambda f = \frac{\omega}{k}$$

General relation for I and A: $I \propto A^2$, $A = A_1 + A_2 + \ldots$

Two sources: $I_{max} = |A_1 + A_2|^2$, $I_{min} = |A_1 - A_2|^2$

Two sources, same I_1: $I = 4I_1\cos^2(\phi/2)$ where $\phi = 2\pi\delta/\lambda$

Interference: Slits, holes, etc.

Far-field path-length difference: $\delta \equiv r_1 - r_2 \approx d\sin\theta$

Phase difference: $\dfrac{\phi}{2\pi} \equiv \dfrac{\delta}{\lambda} = \dfrac{d\sin\theta}{\lambda} \approx \dfrac{d\,\theta}{\lambda} \approx \dfrac{d}{\lambda}\dfrac{y}{L}$ if θ small

Principal maxima: $d\sin\theta_{max} = \pm m\lambda \quad m = 0, 1, 2, \ldots$

N slit: $I_N = I_1\left\{\dfrac{\sin(N\phi/2)}{\sin(\phi/2)}\right\}^2$ where $\phi = 2\pi d\sin\theta/\lambda$

Single slit: $\delta_a = a\sin\theta \quad a\sin\theta_{min} = \pm m\lambda$ with $m = 1, 2, 3\ldots$

$$\frac{\beta}{2\pi} \equiv \frac{\delta_a}{\lambda} = \frac{a\sin\theta}{\lambda} \approx \frac{a\,\theta}{\lambda} \approx \frac{a}{\lambda}\frac{y}{L}$$

Single slit: $I_1 = I_0\left\{\dfrac{\sin(\beta/2)}{\beta/2}\right\}^2$ with $\beta = 2\pi a\sin\theta/\lambda$

slit: $\theta_0 \approx \lambda/a$ *or* hole: $\theta_0 \approx 1.22\lambda/D \approx \alpha_c$

Approx. grating resolution: $\dfrac{\Delta\lambda}{\lambda} \geq \dfrac{1}{Nm}$

Quantum laws, facts….

UNIVERSAL: $p = \hbar k = h/\lambda \qquad E = hf = \hbar\omega$

Light: $E = hf = \hbar\omega = hc/\lambda = pc$

Slow particle: $KE = mv^2/2 = p^2/2m = h^2/2m\lambda^2$

Photoelectric effect: $KE_{max} = eV_{stop} = hf - \Phi$

UNIVERSAL: $\Delta x\,\Delta p_x \geq \hbar \qquad \Delta E\,\Delta t \geq \hbar$

$$\psi^*(x)\psi(x) \equiv |\psi(x)|^2$$

$$P_{ab} = \int_a^b |\psi(x)|^2\,dx, \quad a \leq x \leq b$$

(Slow) particle in fixed potential U:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + U(x)\psi(x) = i\hbar\frac{\partial\psi(x,t)}{\partial t}$$

Physics 214 Common Formulae

Quantum stationary states (energy eigenstates):

$$\Psi(x,t)=\psi(x)e^{-i\omega t} \quad \text{where } E=\hbar\omega$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+U(x)\psi(x)=\hbar\omega\,\psi(x)=E\,\psi(x)$$

In 1-D box: $n\lambda=2L$ where $n=1,2...$

$$\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right) \quad \text{for} \quad 0\le x\le L$$

$$E_n=\frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2=\left(\frac{h^2}{8mL^2}\right)n^2=E_1n^2 \quad \text{(*last part*)}$$

Box, 3-D:

$$\psi(x,y,z)=\sqrt{\frac{8}{abc}}\sin\left(\frac{n_1\pi}{a}x\right)\sin\left(\frac{n_2\pi}{b}y\right)\sin\left(\frac{n_3\pi}{c}z\right)$$

$$E(n_1,n_2,n_3)=\frac{h^2}{8m}\left(\frac{n_1^2}{a^2}+\frac{n_2^2}{b^2}+\frac{n_3^2}{c^2}\right)$$

Simple Harmonic Oscillator (SHO):

$$E_n=\left(n+\tfrac{1}{2}\right)\hbar\omega \quad \text{where} \quad n=0,1,2...$$

$$\omega=\sqrt{k/m}$$

Free slow particle with definite p:

$$\Psi(x,t)=Ae^{i(kx-\omega t)} \; with \; \hbar\omega=\hbar^2k^2/2m$$

H-like atom

potential $U(r)=-\frac{\kappa Ze^2}{r}$

$$E_n=\frac{-1}{4\pi\varepsilon_o}\frac{(Ze)^2}{2a_o}\frac{1}{n^2}=-\frac{1}{(4\pi\varepsilon_o)^2}\frac{me^4Z^2}{2\hbar^2n^2}$$

$$=-13.606\,\text{eV}\,\frac{Z^2}{n^2}$$

Ground state: $\psi_{1s}(r,\theta,\phi)=\frac{1}{\sqrt{\pi a_o^3}}e^{-r/a_o}$

Radial density for s-state: $P(r)\,dr=4\pi r^2\,|\psi(r)|^2\,dr$

Form of *n, l, m* eigenstate:

$$\psi_{n\ell m}(r,\theta,\phi)=R_{n\ell}(r)\,Y_{\ell m}(\theta,\phi)$$

$$Y_{00}=\frac{1}{\sqrt{4\pi}}, \quad Y_{10}=\sqrt{\frac{3}{4\pi}}\cos\theta,$$

$$Y_{1\pm1}=\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\varphi}$$

Tunneling

$$T\approx Ge^{-2KL} \quad \text{where} \quad G=16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$$

$$K=\sqrt{\frac{2m}{\hbar^2}(U_0-E)}=2\pi\sqrt{\frac{2m}{h^2}(U_0-E)}$$

Angular momentum and magnetism

Orbital: $L_z=m\hbar$ where $m=0,\pm1,\pm2,...\pm\ell$

$L^2=\ell(\ell+1)\hbar^2$ where $\ell=0,1,2,...$

Spin: $S_z=m_s\hbar$ where $m_s=\pm 1/2$

Magnetic energy: $U=-\vec{\mu}\cdot\vec{B}$

Force: $F_z=\mu_z\frac{dB_z}{dz}$ where $\mu_z\approx-\frac{e}{m_e}S_z$

Atomic orbital filling