

Last Name: _____ First Name _____ NetID _____
Discussion Section: _____ Discussion TA Name: _____

Instructions—

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil. Do not use a mechanical pencil or pen. Darken each circle completely, but stay within the boundary. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner. Be especially careful that your mark covers the **center** of its circle.
2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.
3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter “I” and the numeral “1” and for the letter “O” and the numeral “0”. **Do not** mark the hyphen circle at the bottom of any of these columns.
4. You may find the version of this Exam Booklet at the top of page 2. Mark the version circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**
5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.
6. Print your UIN# in the STUDENT NUMBER designated spaces and mark the corresponding circles. You need not write in or mark the circles in the SECTION block.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. (You need not fill in the COURSE or INSTRUCTOR lines.)
8. Sign (**DO NOT PRINT**) your name on the **STUDENT SIGNATURE line**.
Before starting work, check to make sure that your test booklet is complete. You should have 10 numbered pages plus a Formula Sheet at the end.

*Academic Integrity—***Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including expulsion.**

This Exam Booklet is Version A. Mark the **A** circle in the TEST FORM box near the middle of your answer sheet. **DO THIS NOW!**

Exam Grading Policy—

The exam is worth a total of 120 points, composed of two types of questions.

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

**Did you bubble in your name, exam version and network-ID?
Check to make sure you have bubbled in all your answers.**

The next four problems are related.

Consider a sealed container with a volume of 1 m^3 filled with 1×10^{23} helium atoms and 2×10^{23} molecules of nitrogen. Recall that helium is a monatomic gas and nitrogen is a diatomic gas. Initially the temperature of the gas mixture is 1000 K.

1. Find the pressure inside the container.

- a. 8980 Pa
- b. 8290 Pa
- c. 4140 Pa

2. Find the ratio of the total molecular rotational energy to the total translational energy (molecular and atomic) inside the container. Assume that equipartition applies.

- a. 0.333
- b. 0.444
- c. 0.666
- d. 0.777
- e. 1.333

3. We next raise the temperature of the gas in the container from 1000 K to 1500 K. How much heat must the gas in the container absorb to do this? Note the volume is held constant. Assume that equipartition applies.

- a. 1120 J
- b. 2240 J
- c. 4490 J

4. Calculate the ratio of the r.m.s. velocity of the helium atoms to the r.m.s. velocity of the nitrogen molecules. The temperature is not specified, and do not necessarily assume that equipartition applies to the rotational or vibrational degrees of freedom of the N_2 .

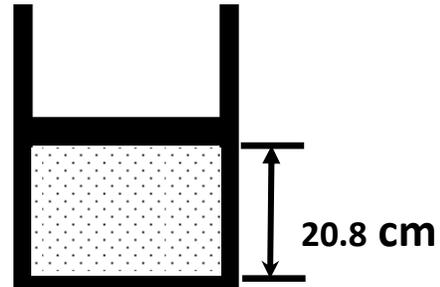
- a. $v_{\text{rms,He}}/v_{\text{rms,N}_2} = 0.378$
- b. $v_{\text{rms,He}}/v_{\text{rms,N}_2} = 1.00$
- c. $v_{\text{rms,He}}/v_{\text{rms,N}_2} = 2.65$
- d. $v_{\text{rms,He}}/v_{\text{rms,N}_2} = 7$
- e. The answer depends on temperature since at high temperature vibrational modes can be excited.

The next four problems are related.

Consider 10 moles of nitrogen within a cylinder with a cross sectional area of 1 m^2 . Assume that the cover (“cap”) of the cylinder, is a disk which can slide frictionlessly within the cylinder. Assume that the cap makes a “leak-proof” fit so that 10 moles of nitrogen always exist within the cylinder, independent of the cap height. The gas in the cylinder has pressure $120 \times 10^3 \text{ Pa}$ (hint: what does that say about the external pressure?).

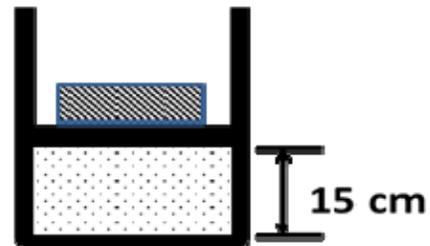
5. The initial height of the cylinder is 20.8 cm; find the temperature of the gas.

- a. 473.3 K
- b. 600.6 K
- c. 350.3 K
- d. 300.2 K
- e. 377.6 K



6. An additional 2000-kg mass is placed on the cap, which causes the cylinder to finally contract to a final equilibrium height of 15 cm. Compute the new temperature of the gas contained within the shorter cylinder. Assume the cylinder is in contact with an appropriate thermal reservoir, i.e., that heat can flow in or out of the gas. Assume $g = 9.8 \text{ m/s}^2$.

- a. 150.3 K
- b. 175.2 K
- c. 251.9 K
- d. 225.1 K
- e. 269.6 K



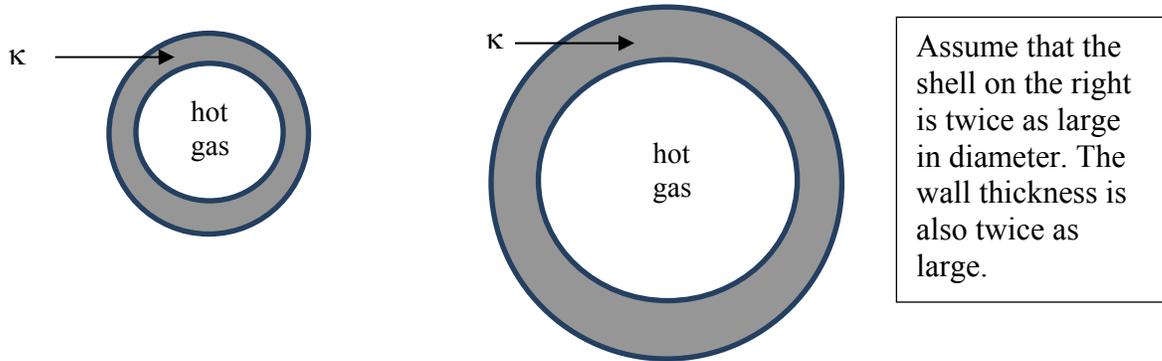
7. Which is true for the Q of the gas during this process when the cap finally comes to rest at it's new equilibrium height of 15 cm. Note that the work done on the gas is $mg \Delta h$.

- a. $Q > 0$
- b. $Q < 0$
- c. $Q = 0$

8. If instead the container is perfectly insulated to ensure no heat flows in or out, what happens to the final temperature of the gas when the 2000-kg mass is added to the cap?

- a. The temperature increases.
- b. The temperature decreases.
- c. There is no change in the temperature.

9. Consider hollow spherical shells, as is shown on the figure. There is some amount of a hot He gas inside ($T=500$ K). The pressure inside equals the atmospheric pressure in both cases. The temperature of the environment outside the shell is $T=300$ K. The heat capacity of the walls is assumed negligible. Assume the wall thickness of each shell, with thermal conductivity κ , is much less than the shell's inner diameter.



What can we say about the cooling times for the two cases?

- The gas in the larger sphere takes four times longer to cool down.
- The gas in the larger sphere takes two times longer to cool down.
- The gas in the larger sphere cools down as fast as in the smaller sphere.
- The gas in the smaller sphere takes four times longer to cool down.
- The gas in the smaller sphere takes two times longer to cool down.

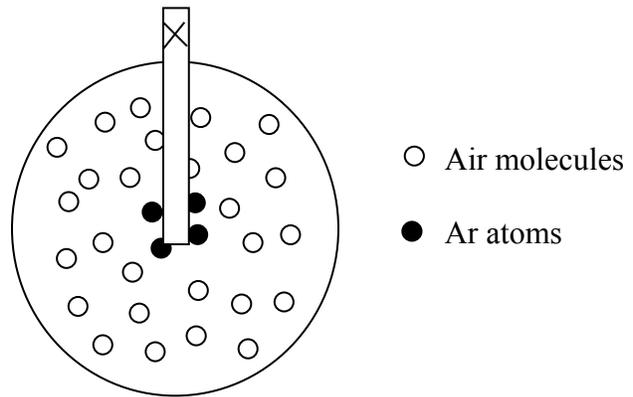
10. You have a cup of coffee, at initial temperature of 90°C . The mass of coffee is 0.2 kg. You would like to reduce the temperature to 50°C . How much cream needs to be added to the cup of coffee? Assume that the heat capacity of cream is the same as that of coffee and that the temperature of your cream is 20°C .

- 1.0 kg
- 0.27 kg
- 0.20 kg
- 0.15 kg
- 0.005 kg

11. On a cold winter day, the outside temperature is -18°C while the temperature inside the house is 20°C . Assume that the house is equipped with a double-paned window, with area 2 m^2 . Each glass pane is 5-mm thick, as is the layer of air between the two panes. The thermal conductivities of glass and air are $1\text{ W}/(\text{m}\cdot\text{K})$ and $0.03/(\text{m}\cdot\text{K})$, respectively; the thermal conductivity of the walls of the house can be neglected. How much heat has to be produced by the heating system in the room in order to compensate for the heat outflow through the window to keep the temperature in the room constant?

- 430 W
- 480 W
- 15.2 kW

12. Exactly 100 atoms of Ar gas (atomic weight 40 g/mole) are released from a capsule at the center of a spherical flask full of air. The radius of the flask is 0.5m. The average speed of the Ar atoms is 300 m/s and they have a mean free path of $0.1\ \mu\text{m}$. Assume that the process is dominated by diffusion only.



If the temperature is increased from 300K to 600K, what can we say about the average diffusion time, for the atoms to reach the walls of the flask?

- The diffusion time will be reduced by a factor of 1.414.
- The diffusion time will be reduced by a factor of 2.
- The diffusion time remains unchanged.
- The diffusion time increases by a factor of 1.414.
- The diffusion time increases by a factor of 2.

13. This problem is about a system consisting of 6 spin- $\frac{1}{2}$ atoms, so the total magnetic moment of the sample can range between $-6\mu_e$ and $+6\mu_e$. There is no magnetic field present, and all of the spin microstates are equally likely. What is the probability that the total magnetic moment of the system is $2\mu_e$?

- a. $1/64$
- b. $1/12$
- c. $7/64$
- d. $15/64$
- e. $30/64$

14. A sample of N identical monatomic spinless gas atoms at room temperature is in a container of volume V . A plate is removed from one end of the volume allowing the gas to expand into a larger volume such that the total volume containing the atoms is four times as large as the original one. Which of the following statements is true?

- a. The internal energy remains the same, but the dimensionless entropy increases by $N\ln 4$.
- b. The internal energy is reduced by a factor of 4, and the dimensionless entropy is unchanged.
- c. Both the entropy and the energy remain unchanged.
- d. The internal energy doubles, and the dimensionless entropy decreases by a factor of 4.
- e. The internal energy is reduced by a factor of 4, and the dimensionless entropy increases by $N\ln 4$.

The next two problems are related.

15. Consider a one-mole sample of a spin- $\frac{1}{2}$ paramagnet placed in a magnetic field large enough that all of the spins are aligned in the magnetic field direction. Call the spin entropy under these conditions $S(\infty)$. Now turn the magnetic field off.; the spins randomize and the spin entropy becomes $S(0)$. The sample is held at the same temperature in both cases. What is the change in entropy associated with the spin degrees of freedom of this sample, $S(0) - S(\infty)$?

- a. 16.633 J/K
- b. 11.529 J/K
- c. 8.316 J/K
- d. 5.764 J/K
- e. 33.266 J/K

16. As we discussed in class, the energy of the magnetic moment in the field will either be $-\mu_e B$ (if the moment is aligned with the field), or $+\mu_e B$ (if the moment is aligned against the field). For what value of applied magnetic field will one-third of the magnetic moments be aligned with the field? Hint: Think before you calculate.

- a. $kT \ln(2)/(2 \mu_e)$
- b. $kT \ln(3)/(2 \mu_e)$
- c. there is no value of the field for which 1/3 of the moments are aligned with the field.

17. You are cooking breakfast using a frying pan of mass 2 kg, heated to a temperature of 450 K. You can choose among frying pans made of Fe, Cu, or Al. Which one should you choose if your goal is to spend as little energy as possible to heat up the frying pan from room temperature to 450 K. The molecular weights of these metals are:

$m_{\text{Fe}} = 56 \text{ g/mole}$; $m_{\text{Cu}} = 64 \text{ g/mole}$; $m_{\text{Al}} = 27 \text{ g/mole}$.

- a. Cu
- b. Al
- c. Fe

18. A sample consists of three simple harmonic oscillators that are thermally coupled to each other. In the sample, 8 quanta of energy are shared by the three oscillators. By how much does the dimensionless entropy change if an additional oscillator is added, keeping the total number of quanta fixed at 8?

- a. $\Delta\sigma = 1.30$
- b. $\Delta\sigma = 1.38$
- c. $\Delta\sigma = 0.32$
- d. $\Delta\sigma = 1.13$
- e. $\Delta\sigma = 0.12$

The next two problems are related.

19. In a piece of copper at finite temperature, any Cu atom will have thermally driven oscillations about its equilibrium location. If the root-mean-square (rms) displacement at 20° C is 8.7×10^{-13} m, what is it at 30° C? Assume equipartition holds.

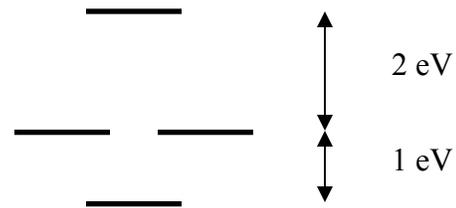
- a. 8.7×10^{-13} m
- b. 8.8×10^{-13} m
- c. 9.0×10^{-13} m
- d. 1.1×10^{-12} m
- e. 1.3×10^{-12} m

20. We said that we could model a crystalline solid as a collection of harmonic oscillators (i.e., each atom connected by effective springs in the x-, y-, and z-directions), giving a thermal energy of $3NkT$, in the limit $kT \gg \epsilon$, where ϵ is the spacing between adjacent harmonic oscillator energy levels. For this problem, consider what happens when we are NOT in that limit, and estimate the thermal energy in a 1-kg piece of copper when $\epsilon = kT = k(340\text{K}) = 4.7 \times 10^{-21}$ J.

- a. 77,000 J
- b. 44,000 J
- c. 28,000 J
- d. 26,000 J
- e. 133,000 J

The next three problems are related.

Consider a molecule that has the following energy-level structure, as shown in the diagram: A doubly-degenerate 1st excited state that is 1 eV above the singly-degenerate ground state, and 2 eV below a singly-degenerate highest level state.



21. What is the likelihood that the molecule is not in the ground state at $T = 3000\text{K}$?
- 0.01
 - 0.02
 - 0.04
 - 0.08
 - 0.16
22. If we compare this to a molecule with identical energy-level spacings as in the diagram, but with all the levels singly degenerate, how would the probability to be in the ground state be affected?
- The ground state would be more likely in the new molecule.
 - The ground state would be less likely in the new molecule.
 - The ground state would be equally likely in the new molecule.
23. What is the contribution of these internal energy levels to the entropy S of a collection of N of these molecules (as shown in the figure, i.e., with a doubly-degenerate first-excited state), in the limit of very high temperature? [i.e., ignore contributions from the motion and rotations of the molecules.]
- $Nk \ln(4)$
 - $Nk \ln(3)$
 - none of the above

The next two problems are related.

On a particular night in Champaign, a low-lying mist sits within $\sim 5\text{m}$ of the ground (*i.e.*, at 5 m, the mist density (number of mist droplets per m^3) is 10 times lower than it is at ground level).

24. Assuming $T = 20^\circ\text{C}$, and that the vertical mist density distribution is adequately modeled using a Boltzmann distribution, what is the approximate mass of a typical droplet? Assume $g = 9.8 \text{ m/s}^2$.

- a. $1.9 \times 10^{-22} \text{ kg}$
- b. $9.6 \times 10^{-22} \text{ kg}$
- c. $5.5 \times 10^{-23} \text{ kg}$
- d. $1.3 \times 10^{-23} \text{ kg}$
- e. $1.5 \times 10^{-24} \text{ kg}$

25. Compare the density of mist at a height of 15 m to the density at 5 meters.
Hint: Sketch the density as a function of altitude.

- a. The density at 15 m is smaller than it is at 5 m, by a factor of about 100.
- b. The density at 15 m is smaller than it is at 5 m, by a factor of about 3.
- c. The density at 15 m is larger than it is at 5 m, by a factor of about 3.

Physics 213 Formula Sheet

Spring 2015
(04/13)

Constants, Data, Definitions

$0 \text{ K} = -273.15 \text{ }^\circ\text{C} = -459.67 \text{ }^\circ\text{F}$
 $N_A = 6.022 \times 10^{23} / \text{mole}$
 $k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$
 $R = kN_A = 8.314 \text{ J/mol}\cdot\text{K} = 8.206 \times 10^{-2} \text{ l}\cdot\text{atm/mol}\cdot\text{K}$
 $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$ $1 \text{ liter} = 10^{-3} \text{ m}^3$
 $\text{STP} \rightarrow T = 0^\circ\text{C}; p = 1 \text{ atm}$
 $h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s} = 4.136 \times 10^{-15} \text{ eV}\cdot\text{s}$
 $\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J}\cdot\text{s}$
 $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
 $c = 2.998 \times 10^8 \text{ m/s}$
 $\mu_e = 9.2848 \times 10^{-24} \text{ J/T}$ $\mu_p = 1.4106 \times 10^{-26} \text{ J/T}$
 $m_e = 9.109 \times 10^{-31} \text{ kg}$ $m_p = 1836 m_e$
 $g = 9.8 \text{ m/s}^2$ $= 1.673 \times 10^{-27} \text{ kg}$

Fundamental Laws/Principles:

First law: $dU = dQ + dW$ Second Law: $d\sigma/dt \geq 0$

$P_i \propto \Omega_i \equiv e^{\sigma_i}$

Classical equipartition $\langle \text{energy} \rangle = \frac{1}{2} kT$ per quadratic term

Entropy & Temperature: $S = k\sigma = k \ln \Omega$; $\frac{1}{T} \equiv \left(\frac{\partial S}{\partial U} \right)_{V,N}$

Heat Capacities: $C_V \equiv (\partial U / \partial T)_V$; $C_p \equiv (\partial(U + pV) / \partial T)_p$

Special properties of α -ideal gases

$U = \alpha NkT = \alpha nRT$ $pV = NkT$ $p_{\text{tot}} = p_1 + p_2 + \dots$

$C_V = \alpha Nk = \alpha nR$ $C_p = C_V + Nk$ $n = \# \text{ moles} = N/N_A$

$c_p/c_v = (\alpha + 1)/\alpha = \gamma$ $W_{\text{by}} = NkT \ln(V_f/V_i)$

$VT^\alpha = \text{const.}$, or $pV^\gamma = \text{const.}$, $\gamma = (\alpha + 1)/\alpha$

$W_{\text{by}} = \alpha Nk (T_1 - T_2) = \alpha (p_1 V_1 - p_2 V_2)$

$\Delta S = C_V \ln(T_f/T_i) + Nk \ln(V_f/V_i)$

α	γ
● 3/2	5/3
●● 5/2	7/5

Processes, Heat Engines, etc

$\Delta U = Q - W_{\text{by}}$ $W_{\text{by}} = \int p dV$

Quasistatic: $dS = dQ/T$ so $\Delta S = \int (C/T) dT$

$dQ = dU + p dV$

$\epsilon_{\text{Carnot}} = 1 - T_C/T_H$

Diffusion and Heat Conduction

$D = (\ell^2/3\tau) = v\ell/3$ $\tau = \ell/v$

$\langle x^2 \rangle = 2Dt$ $\langle r^2 \rangle = 6Dt$

$J_x = \kappa \Delta T / \Delta x$, $\kappa = D_{\text{HC}}$ where $c = C_V/V$

$H_x = J A = \Delta T / R_{\text{th}}$ $R_{\text{th}} = d/\kappa A$ $\Delta L/L = \alpha \Delta T$

$T_A(t) = T_f + (T_{A0} - T_f) e^{-t/\tau}$, $\tau = R_{\text{th}} C_A$

Spins

$\Omega(N, N_{\text{up}}) = \frac{N!}{N_{\text{up}}! N_{\text{down}}!} = \frac{N!}{N_{\text{up}}! (N - N_{\text{up}})!}$; $\Omega(m) = 2^N \sqrt{\frac{2}{\pi N}} e^{-m^2/2N}$; $P(m) = \Omega(m) / 2^N$

$M = (N_{\text{up}} - N_{\text{down}}) \mu \equiv m\mu$, $M = N\mu \tanh(\mu B/kT)$

SHO

$P_n = (1 - e^{-\epsilon/kT}) e^{-n\epsilon/kT}$; $\langle E \rangle = \epsilon / (e^{\epsilon/kT} - 1)$ $\epsilon = hf$; $\Omega = \frac{(q + N - 1)!}{q!(N - 1)!}$

Counting, Bin Statistics, Entropy

Ω	Occupancy ($N \ll M$)		
	Unlimited	Single	Dilute
Distinct	M^N	$\frac{M!}{(M-N)!}$	M^N
Identical	$\frac{(N+M-1)!}{N!(M-1)!}$	$\frac{M!}{(M-N)!N!}$	$\frac{M^N}{N!}$

$\ln N! \approx N \ln N - N$

Equilibrium

Boltzmann: $P_n = \frac{d_n e^{-E_n/kT}}{Z}$; $Z \equiv \sum_i d_i e^{-E_i/kT}$

Free energies: $F \equiv U - TS$ $G \equiv U - TS + pV$

Chemical potential:

$\mu = \left(\frac{\partial F}{\partial N} \right)_{V,T} = \left(\frac{\partial G}{\partial N} \right)_{p,T}$; equilibrium $\sum_i (\Delta N_i) \mu_i = 0$

$\mu_i = kT \ln(n_i/n_{Ti}) - \Delta_i$ (ideal gas)

$n_Q = (2\pi mkT/h^2)^{3/2} = (10^{30} \text{ m}^{-3}) (m/m_p)^{3/2} (T/300\text{K})^{3/2}$

Semiconductors $n_e n_h = n_i^2$; $n_i = n_Q e^{-\Delta/2kT}$

Thermal Radiation

$J = \sigma_B T^4$, $\sigma_B = 5.670 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$ $\lambda_{\text{max}} T = 0.0029 \text{ m}\cdot\text{K}$

Particle	mass/mol
N ₂	28g
O ₂	32g
He	4g
Ar	40g
CO ₂	44g
H ₂	2g
Si	28g
Ge	73g
Cu	64g
Al	27g
1g	10^{-3} kg

