Absolute Calibration of Pressure and Particle Velocity Microphones

A. Absolute Calibration of Pressure Microphones

The Sound Pressure Level (SPL): 
\[ SPL = L_p = 10 \log_{10}(p_{rms}^2 / p_0^2) = 20 \log_{10}(p_{rms} / p_0) \text{ dB} \]

where \( p_0 \) = reference sound over-pressure amplitude = \(2.0 \times 10^{-5}\) \text{ rms Pascals} = \(20 \text{ rms } \mu \text{Pa} \)

= the \{average\} threshold of human hearing (at \( f = 1 \text{ KHz} \)).

Useful conversion factor(s): 
\[ p = 1.0 \text{ rms Pascal} \leftrightarrow SPL = 94.0 \text{ dB} \text{ (in a free-air sound field)} \]

\[ 1.0 \text{ rms Pa} = 1.0 \text{ rms N/m}^2 \]

We can determine (i.e. measure) the \textit{absolute} sensitivity of a pressure microphone \( e.g. \) in a free-air sound field – such as the great wide-open or in an anechoic room\{at an industry-standard reference frequency \( f = 1 \text{ KHz} \)\} side-by-side with a \{NIST-absolutely calibrated\} SPL Meter a distance of a few meters away from a sound source – \( e.g. \) a loudspeaker. We turn up the volume of the sound source until the SPL Meter \( e.g. \) reads a steady SPL of 94.0 dB (quite loud!) which corresponds to a \( p = 1.0 \text{ rms Pascal} \) acoustic over-pressure amplitude. We then measure the \textit{rms AC} voltage amplitude of the output of the pressure microphone \( V_{p-mic} \) (in \textit{rms Volts}) \( e.g. \) using a Fluke 77 DMM on \textit{RMS AC} volts. Thus, the \textit{absolute} sensitivity of the pressure microphone is:

\[ S_{p-mic} \left( V / \text{Pa}, @ f = 1 \text{ KHz}, SPL = 94 \text{ dB} \right) = V_{p-mic} \left( \text{rms Volts} \right) / 1.0 \left( \text{rms Pa} \right) \]

\( n.b. \) The Knowles Acoustics 1/10” diameter FG-23329-C05 omni-directional electret condenser pressure microphone + op-amp preamp that we routinely use in the UIUC Physics of Music/Musical Instruments Lab has a \textit{flat} frequency response from 100 Hz to 10 KHz, with a nominal sensitivity of \( S_{mic} = -53 \text{ dB} \)

relative to 1.0 \textit{rms Volt}/0.1 \textit{rms Pascal}. The KA omni-directional pressure microphone op-amp preamp has a voltage gain of 10× \( (\text{flat in frequency over the audio frequency range } \{20 \text{ Hz} - 20 \text{ KHz} \} \text{ to within } \pm 0.1 \text{ dB}) \), thus the nominal sensitivity of the KA omni-directional pressure microphone + op-amp preamp is \( S_{p-mic+preamp} = -43 \text{ dB} \) over the audio frequency range.

The typical mean/average sensitivity of the Knowles Acoustics omni-directional electret-condenser pressure microphone + preamp that we routinely use in the UIUC Physics of Music/Musical Instruments Lab, determined at \( f = 1 \text{ KHz} \) and \( SPL = 94.0 \text{ dB} \) using an \textit{EXTEC} 407768 SPL meter (\textit{C}-weighted) and a Fluke 77 DMM on \textit{RMS AC} volts is:

\[ \langle S_{p-mic}^{KA} \rangle = 280 \text{ rms mV/rms Pa} \]
B. Absolute Calibration of Particle Velocity Microphones

The Sound Particle Velocity Level (SUL):

\[ SUL = L_u \equiv 10 \log_{10} \left( \frac{u_{\text{rms}}^2}{u_0^2} \right) = 20 \log_{10} \left( \frac{u_{\text{rms}}}{u_0} \right) \ (dB) \]

where \( u_0 \) = reference particle velocity amplitude = \( 4.84 \times 10^{-8} \ \text{rms} \ \text{m/s} \) = the \{average\} threshold of human hearing (at \( f = 1 \ \text{KHz} \)).

**Useful conversion factor:** \( u = 2.42 \ \text{rms} \ \text{mm/s} \leftrightarrow SUL = \text{SPL} = 94.0 \ \text{dB} \) (in a free-air sound field)

We can determine \((i.e. \ measure)\) the absolute sensitivity of a particle velocity microphone in a free-air sound field \{\(e.g. \) at \( f = 1 \ \text{KHz} \}\}, side-by-side with a \{NIST-absolutely calibrated\} SPL Meter. We again turn up the volume until the SPL Meter \(e.g.\) reads 94.0 dB, which corresponds to \( u = 2.42 \ \text{rms} \ \text{mm/s} \) particle velocity amplitude, also known as \(1.0 \ \text{rms} \ \text{Pa} \). We then measure the RMS AC voltage amplitude of the output of particle velocity microphone \( V_{\text{mic}} \) (in \( \text{rms Volts} \)) \(e.g.\) using a Fluke 77 DMM on RMS AC Volts. Thus, the absolute sensitivity of the particle velocity microphone (at \( f = 1 \ \text{KHz} \)) is:

\[ S_{\text{mic}} \ (@ \text{SPL} = 94dB) = V_{\text{mic}} / 1.0 \ (\text{rms} \ \text{V} / \text{rms} \ \text{Pa}) \]

where in a free-air sound field:

\[ 1.0 \ \text{rms} \ \text{Pa} = 1.0(\text{rms} \ \text{Pascal}) \rho_0^{\text{air}} \text{cm}^3 / \text{kg} = 2.42(\text{rms} \ \text{mm/s}) \]

In a free-air sound field (such as the great wide-open), \(e.g.\) for monochromatic traveling plane waves, the 1-D particle velocity amplitude \( u \) is related to the over-pressure amplitude \( p \) by:

\[ u = p / \rho_0^{\text{air}} c_0^{\text{air}} \]

where \( \rho_0^{\text{air}} = 1.204 \ \text{kg/m}^3 \) and speed of sound in air is \( c_0^{\text{air}} = 344 \ \text{m/s} \) \(@\) NTP.

How does one physically measure particle velocity – \(i.e.\) how can we design & build a particle velocity transducer?

The Euler equation for inviscid fluid flow \(i.e.\) neglecting any/all dissipation is:

\[ \frac{\partial \bar{u}(\bar{r},t)}{\partial t} = -\frac{1}{\rho_0} \bar{\nabla} p(\bar{r},t) \]

For propagation of 1-D longitudinal sound waves \(e.g.\) in the \( z \)-direction, the 1-D Euler equation is:

\[ \frac{\partial u_z(\bar{r},t)}{\partial t} = -\frac{1}{\rho_0} \frac{\partial p(\bar{r},t)}{\partial z} \]

This equation physically tells us that if we measure the differential pressure amplitude \( \Delta p(\bar{r}) \) across a small longitudinal distance \( \Delta z \ll \lambda \) at a point in space \( \bar{r} \), the differential over-pressure amplitude \( \Delta p(\bar{r}) \) is linearly proportional to the time derivative of the longitudinal particle velocity amplitude \( u_z(\bar{r}) \) at that point. Integrating the above 1-D Euler equation with respect to time:

\[ u_z(\bar{r},t) = \int_{t=0}^{t} \left( \frac{\partial u_z(\bar{r},t')}{\partial t'} \right) dt' = -\frac{1}{\rho_0 \Delta z} \int_{t=0}^{t} \Delta p(\bar{r},t') dt' \]

for \( \Delta z \ll \lambda \)

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This equation tells us that the time-dependent 1-D particle velocity $u_z(\vec{r},t)$ is proportional to the integral of the time-dependent differential pressure $\Delta p(\vec{r},t)$ at the point $\vec{r}$.

For a harmonic/periodic, single-frequency sound field, the instantaneous differential pressure at the point $\vec{r}$ at time $t$ is: $\Delta p(\vec{r},t) = \Delta p(\vec{r}) \cos(\omega t + \varphi)$. Integrating this expression over time:

$$\int_{t'=\infty}^{t''=\infty} \Delta p(\vec{r},t') dt' = \Delta p(\vec{r}) \int_{t'=\infty}^{t''=\infty} \cos(\omega t' + \varphi) dt' = \left(\Delta p(\vec{r})/\omega\right) \sin(\omega t + \varphi)$$

Thus, we see that the 1-D particle velocity amplitude is proportional to the differential pressure amplitude at the point $\vec{r}$ via the relation: $u_z(\vec{r}) = \Delta p(\vec{r})/\omega \rho \Delta z$.

Note also that a −90° phase relation exists between the instantaneous 1-D particle velocity $u_z(\vec{r},t) = u_z(\vec{r}) \sin(\omega t + \varphi)$ and the instantaneous differential pressure $\Delta p(\vec{r},t) = \Delta p(\vec{r}) \cos(\omega t + \varphi)$.

It is possible to convert an omni-directional pressure microphone to a differential pressure microphone by carefully removing the rear/back cover of the microphone.

In the UIUC Physics of Music/Musical Instruments Lab, we use a so-modified, rectangular-shaped Knowles Acoustics EK-23132 omni-directional electret condenser differential pressure microphone, which has $\Delta z \sim 2 \text{ mm}$. The voltage output from this so-modified differential pressure mic is input to a simple op-amp integrator preamp circuit to carry out the above time integral operation. Hence, we obtain a signal output from this differential pressure mic + op-amp integrator preamp which is proportional to the 1-D particle velocity, $u_z(\vec{r},t)$!

Note that such a differential pressure mic only measures the component of the vector particle velocity $\vec{u}$ normal to the plane of the microphone element, i.e. $\vec{u}\cdot\hat{n} = u \cos \theta$. Thus, three such 1-D particle velocity microphones are needed to measure vector $\vec{u}$ at a specific $(x,y,z)$ point in space.

Note also that for $\lambda \gg \Delta z$, the response of a differential pressure mic increases linearly with frequency, i.e. $\Delta p(\vec{r}) \propto f$. However, as discussed above, the time integral of the differential pressure is inversely proportional to frequency, i.e. $\int_{t'=\infty}^{t''=\infty} \Delta p(\vec{r},t') dt' \propto 1/f$. The output of the integrating op-amp preamp has an RC time constant designed such that the output of the integrating op-amp decreases linearly with frequency above $f = 20 \text{ Hz}$. Hence, we see that overall frequency response of the KA particle velocity mic, which is the product of the frequency response of the differential pressure mic × the frequency response of the integrating op-amp has a flat frequency response above $f = 20 \text{ Hz}$!

The typical mean/average sensitivity of the Knowles Acoustics EK-23132 particle velocity/differential pressure mic + integrating op-amp preamp that we routinely use in the UIUC Physics of Music/Musical Instruments Lab, determined at $f = 1 \text{ KHz}$ and SPL = 94.0 dB using an EXTEC 407768 SPL meter (C-weighted) and a Fluke 77 DMM on RMS AC volts is:

$$\left[\text{S}^{\text{KA}}_{\text{u-mic}}\right] = 90 \text{ rms mV} / \text{rms Pa}^* = 90 \text{ rms mV} / 2.42 \text{ rms mm/s} = 3.72 \times 10^4 \text{ rms mV} / \text{rms m/s}$$
C. The Longitudinal Specific Acoustic Impedance, $Z$

In one dimension, the 1-D/longitudinal specific acoustic impedance ($n.b. a \text{ property of } \text{medium} \text{ in which sound waves propagate}$) is defined as:

$$Z \equiv \frac{p}{u} \left( \frac{Pa \cdot s}{m} = \frac{N \cdot s}{m^3} \equiv \text{Acoustic Ohms, } \Omega_a \right) \quad (\tilde{Z} \equiv \frac{p}{\tilde{u}} \text{ in } 3-D)$$

In order to determine the 1-D/longitudinal specific acoustic impedance $Z$, we use the raw rms voltage amplitudes associated with the signals output from the pressure and particle velocity mics and their respective absolutely-calibrated microphone sensitivities:

$$Z \left( \frac{Pa \cdot s}{m} \right) = \frac{p \left( \text{rms } Pa \right)}{u \left( \text{rms } m/s \right)} = \frac{V_{p-mic}^{KA \text{ rms } mV}}{V_{u-mic}^{KA \text{ rms } mV}} \frac{S_{p-mic}^{KA \text{ rms } mV/rms \ Pa}}{S_{u-mic}^{KA \text{ rms } mV/rms \ m/s}} = K_z \frac{V_{p-mic}^{KA \text{ rms } mV}}{V_{u-mic}^{KA \text{ rms } mV}}$$

Thus, for 1-D/longitudinal specific acoustic impedance measurements using the Knowles Acoustics $p$- and 1-D $u$-mics, the typical mean/average overall $Z$-conversion factor $K_z$ is:

$$K_z = \left( \frac{S_{u-mic}^{KA \text{ rms } mV/rms \ m/s}}{S_{p-mic}^{KA \text{ rms } mV/rms \ Pa}} \right) = \frac{3.72 \times 10^4 \text{ rms } mV/rms \ m/s}{280 \text{ rms } mV/rms \ Pa} = 132.8 \frac{Pa \cdot s}{m} \left( = 132.8 \Omega_a \right)$$

D. Time-Averaged Sound Intensity

For a harmonic/periodic single-frequency sound field, the time-averaged $\{\text{vector/3-D}\}$ sound intensity at the point $\vec{r}$ is:

$$\langle I \left( \vec{r} \right) \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T \tilde{u} \left( \vec{r}, t \right) p \left( \vec{r}, t \right) dt \left( \text{rms } W/m^2 \right)$$

The Sound Intensity Level (SIL), aka Loudness: $\text{SIL} = L_I \equiv 10 \log_{10} \left( I/I_0 \right) \ (dB)$

where $I_0 = \text{reference sound intensity level} = 10^{-12}$ Watts ($@ f = 1 \text{ KHz}$) and $I = \langle |\tilde{I}| \rangle$.

The following table gives some useful correspondences of $p$, $u$ and $I$ for various SPL’s:

<table>
<thead>
<tr>
<th>Sound Type</th>
<th>SPL (dB)</th>
<th>$p$ (rms Pa)</th>
<th>$u$ (rms m/s)</th>
<th>$I$ (rms W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artillery Fire</td>
<td>140</td>
<td>200</td>
<td>0.48</td>
<td>100</td>
</tr>
<tr>
<td>Rock Concert</td>
<td>120</td>
<td>20</td>
<td>0.048</td>
<td>1</td>
</tr>
<tr>
<td>Motorcycle</td>
<td>100</td>
<td>2.0</td>
<td>0.0048</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td><strong>Reference Level</strong></td>
<td><strong>94</strong></td>
<td><strong>1.0</strong></td>
<td>$2.4 \times 10^{-3}$</td>
<td><strong>2.8 \times 10^{-3}</strong></td>
</tr>
<tr>
<td>Vacuum Cleaner</td>
<td>80</td>
<td>0.2</td>
<td>$4.8 \times 10^{-4}$</td>
<td>$10^{-4}$</td>
</tr>
<tr>
<td>Normal Conversation</td>
<td>60</td>
<td>0.02</td>
<td>$4.8 \times 10^{-5}$</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>Whispering</td>
<td>40</td>
<td>0.002</td>
<td>$4.8 \times 10^{-6}$</td>
<td>$10^{-8}$</td>
</tr>
<tr>
<td>Empty Theater</td>
<td>20</td>
<td>$2 \times 10^{-4}$</td>
<td>$4.8 \times 10^{-7}$</td>
<td>$10^{-10}$</td>
</tr>
<tr>
<td><strong>Threshold of Hearing</strong></td>
<td><strong>0</strong></td>
<td><strong>2 \times 10^{-5}</strong></td>
<td><strong>4.8 \times 10^{-8}</strong></td>
<td><strong>10^{-12}</strong></td>
</tr>
</tbody>
</table>