Phys 102 – Lecture 9

RC circuits
Recall from last time...

We solved various circuits with resistors and batteries (also capacitors and batteries)

What about circuits that combine all three... ...RC circuits

Phys. 102, Lecture 8, Slide 2
RC circuits

Circuits that store and release energy controllably...

Camera flash

Defibrillator

Nerve cells
Today we will...

- Learn about RC circuits
  Charge on capacitors cannot change instantly, so behavior of RC circuit depends on time
- Analyze RC circuits under different situations
  Charging capacitors at short/long times
  Discharging capacitors at short/long times
  Time dependence
- Apply these concepts
  Nerve cells and nerve impulses (action potential)
Charging capacitor

Initially the capacitor is uncharged \((Q_0 = 0)\)
At \(t = 0\) we close switch \(S_1\).

Immediately after:
Current \(I_0\) flows around loop, through \(C\)
No charge on \(C\) \((Q_0 = 0)\)

After a long time \((t = \infty)\):
Charge on \(C\) builds until \(V_C = \varepsilon\). Current decreases to zero \((I_\infty = 0)\)
Both switches are initially open and the capacitor is uncharged. What is the current through light bulb 1 right after switch $S_1$ is closed?

A. $I_b = 0$
B. $I_b = \frac{\epsilon}{3R}$
C. $I_b = \frac{\epsilon}{2R}$
D. $I_b = \frac{\epsilon}{R}$
Both switches are initially open and the capacitor is uncharged. What is the voltage across the capacitor a long time after switch $S_1$ is closed?

A. $V_C = 0$
B. $V_C = \frac{\epsilon}{2}$
C. $V_C = \epsilon$
D. $V_C = 2\epsilon$
Initially the capacitor is fully charged ($Q_0 = C\varepsilon$) At $t = 0$ we close switch $S_2$.

*Immediately after:*
Current $I_0$ driven around loop, through $C$
Charge on $C$ from before ($Q_0 = C\varepsilon$)

*After a long time* ($t = \infty$):
Charge on $C$ dissipates until $V_C = 0$. Current decreases to zero ($I_\infty = 0$)
After $S_1$ has been closed for a long time, it is opened and $S_2$ is closed. What is the current through light bulb 2 right after $S_2$ is closed?

A. $I_b = 0$
B. $I_b = \frac{\varepsilon}{3R}$
C. $I_b = \frac{\varepsilon}{2R}$
D. $I_b = \frac{\varepsilon}{R}$

$V_C - I_b R = 0$
Now both $S_1$ and $S_2$ are closed. What is the current through light bulb 2 a long time after both switches are closed?

A. $I_b = 0$
B. $I_b = \frac{\varepsilon}{3R}$
C. $I_b = \frac{\varepsilon}{2R}$
D. $I_b = \frac{\varepsilon}{R}$

Phys. 102, Lecture 7, Slide 10
Summary: charging & discharging

- Charge (and therefore voltage, since $V_C = Q/C$) on capacitors cannot change instantly.

- **Short term** behavior of capacitor:
  - If the capacitor is charging, current $I$ drives charge onto it, and $Q$ increases (acts like a wire).
  - If the capacitor is discharging, current $I$ drives charge off of it, and $Q$ decreases (acts like a battery).

- **Long term** behavior of capacitor:
  - If the capacitor is fully charged, $I = 0$ and $Q$ is maximum (acts like an open circuit).
  - If the capacitor is fully discharged, $I = 0$ and $Q$ is minimum (acts like an open circuit).
Nerve cell equivalent circuit

Neurons have ion channels (K⁺, Na⁺, and Cl⁻) that pump current into and out of cell (it is polarized). Cell membrane also has capacitance.
Action potential

At rest, Na$^+$ channels in cell are closed. When stimulated, the cell’s voltage increases (depolarization). If a threshold is exceeded, the Na$^+$ channels open & trigger a nerve impulse (action potential)

\[
C_m \quad R_K \quad S \quad R_{Na} \\
I_K \quad I_{Na} \\
\varepsilon_K \quad \varepsilon_{Na}
\]

Phys. 102, Lecture 7, Slide 13
The neuron has been in resting state for a long time. What is the voltage across the membrane capacitance?

A. $V_C > \varepsilon_K$
B. $V_C = \varepsilon_K$
C. $V_C < \varepsilon_K$

$\varepsilon_K = 70 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV},$
$R_K = 2 \text{ M}\Omega, R_{Na} = 0.4 \text{ M}\Omega, C_m = 300 \text{ pF}$
Some time ago, the cell was stimulated and depolarized to $-60$ mV, less than threshold to open Na$^+$ channels. What happens next?

**Immediately after:**
- No current through Na$^+$ channel
- Current $I_K$ driven by K$^+$ channel
- Charge $Q_0$ on $C_m$ from $V_C = 60$ mV

**After a long time:**
- Current $I_K$ decays to 0
- Charge on $C_m$ returns to rest value

---

**Calculation: action potential I**

$V_{in} = -60$ mV

$V_{out}$

$V_{in} = -60$ mV

$C_m$

$R_K$

$R_{Na}$

$\varepsilon_K$

$\varepsilon_{Na}$

$\varepsilon_K = 70$ mV, $\varepsilon_{Na} = 60$ mV,

$R_K = 2$ M$\Omega$, $R_{Na} = 0.4$ M$\Omega$, $C_m = 300$ pF

Phys. 102, Lecture 7, Slide 15
**RC circuit time dependence**

### Charging:

\[ \varepsilon - I(t)R - \frac{Q(t)}{C} = 0 \]

**Charge builds up:**
\[ Q(t) = Q_{\infty} (1 - e^{-t/RC}) \]

**Current decays:**
\[ I(t) = I_0 e^{-t/RC} \]

### Discharging:

\[ \frac{Q(t)}{C} - I(t)R = 0 \]

**Charge decays:**
\[ Q(t) = Q_0 e^{-t/RC} \]

**Current decays:**
\[ I(t) = I_0 e^{-t/RC} \]

Note that \( RC \) has units of time!

\[ R \times C = \frac{[V]}{[I]} \times \frac{[Q]}{[V]} = \frac{[Q]}{[I]} = [t] \]

**Demo:**

**Phys. 102, Lecture 7, Slide 16**
Myelinated nerve cells

Action potentials propagate down nerve cell at rate determined by the cell’s $RC$ time constant.

With very few exceptions (ex: C fibres) human neuron fibres are *myelinated*. Myelin reduces $C$, decreasing time constant & increasing propagation speed.

Many neurodegenerative diseases (ex: MS) cause progressive de-myelination.
How long does the cell take to return to 90% of its resting voltage?

Cell voltage $V_{in} - V_{out} = -V_C = -Q/C$:

$$\Delta V(t) = 10e^{-t/R_KC_m}$$

$$\Delta V(t_{90}) = 10(1 - 0.9) = 10e^{-t_{90}/R_KC_m}$$

Take natural log of both sides:

$$t_{90} = -R_KC_m \ln(0.1)$$

$$= -(2 \times 10^9)(300 \times 10^{-12})(-2.3)$$

$$\approx 1.4 \text{ ms}$$
Calculation: action potential II

Now, the cell was stimulated and depolarized to $-50$ mV, over the threshold to open Na$^+$ channels. What happens next?

*Immediately after:*
- Current $I_{Na}$ through Na$^+$ channel
- Current $I_K$ driven by K$^+$ channel
- Charge $Q_0$ on $C_m$ from before

$V_{in} = -50$ mV

$\varepsilon_K = 70$ mV, $\varepsilon_{Na} = 60$ mV,
$R_K = 2 \, \text{M}\Omega$, $R_{Na} = 0.4 \, \text{M}\Omega$, $C_m = 300$ pF
A long time after stimulating the cell, which statement below holds TRUE?

A. All currents are 0
B. The currents $I_K = I_{Na} \neq 0$
C. Voltage across $C_m$ is 0

\[ \varepsilon_K = 70 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \]
\[ R_K = 2 \text{ M}\Omega, R_{Na} = 0.4 \text{ M}\Omega, C_m = 300 \text{ pF} \]
Action potential summary

If the stimulus exceeds –55 mV, the Na⁺ channels open, depolarize the cell & trigger an action potential.

Once a +40 mV potential is reached, the Na⁺ channels close again & the cell repolarizes to its resting potential.
Summary of today’s lecture

• RC circuits depend on time
  Charge on capacitors cannot change instantly

• Short/long times & charging/discharging
  \( t = 0: I \) flows \( Q \) onto/off of \( C \), \( Q \) increases/decreases (charging/discharging)
  \( t = \infty: I \) through \( C \) decays to 0, \( Q \) reaches maximum/minimum (charging/discharging)
  \( \tau = RC \): provides time to charge/discharge

Next week magnetism!

Phys. 102, Lecture 8, Slide 22