Phys 102 – Lecture 8

Circuit analysis and Kirchhoff’s rules
Recall from last time...

We solved circuits like... by combining series & parallel components

What about a circuit like...

Phys. 102, Lecture 8, Slide 2
Kirchhoff’s loop rule

Voltages around a loop sum to zero

\[ \sum \Delta V = 0 \]

Is voltage positive or negative?

- **Batteries**: + end is always at higher potential
- **Resistors**: higher/lower potential depends on current direction
- **Capacitors**: higher/lower potential depends on which plate has \(+Q/–Q\)

Label +/- for higher/lower electric potential

Go around loop and write \(+V_{\text{element}}\) if electric potential increases \(–V_{\text{element}}\) if it decreases

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**Calculation: single loop practice**

Calculate the current $I$ in the circuit

What if we go around the loop the “wrong” way?

What if we’re not given the current direction?

What if we pick the “wrong” direction?
Calculation: single loop practice

How can the current be driven opposite battery 2?

\[ \begin{align*}
V_1 &= 50 \text{ V} \\
R_1 &= 5 \Omega \\
R_2 &= 15 \Omega \\
V_2 &= 10 \text{ V}
\end{align*} \]
ACT: Checkpoint 1.1

Calculate the current through $R_1$.

A. $I_1 = 0.5$ A
B. $I_1 = 1.0$ A
C. $I_1 = 1.5$ A
ACT: Checkpoint 1.2

Calculate the current through $R_2$.

A. $I_2 = 0.5$ A  
B. $I_2 = 1.0$ A  
C. $I_2 = 1.5$ A
Nerve cell equivalent circuit

Neurons have different types of ion channels ($K^+$, $Na^+$, and $Cl^-$) that pump current into and out of cell – act like batteries!
Na⁺ channels have a “gate” (represented by the switch $S$) that allows or blocks ion flow. In its resting state, a Na⁺ channel is shut (i.e. switch $S$ is open). Which equation is correct?

A. $+\varepsilon_K - I_K R_K - I_K R_{Cl} - \varepsilon_{Cl} = 0$

B. $+\varepsilon_K - I_K R_K - I_{Na} R_{Na} - \varepsilon_{Na} = 0$

C. $+\varepsilon_K + I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$
Calculation: electric potential

Find the electric potential difference across the cell $V_{in} - V_{out}$ (Assume $V_{out} = 0$ for reference)

$\varepsilon_K = 80$ mV, $\varepsilon_{Na} = 60$ mV, $\varepsilon_{Cl} = 50$ mV

$R_K = 2$ M$\Omega$, $R_{Na} = 0.2$ M$\Omega$, $R_{Cl} = 5$ M$\Omega$

\[
\varepsilon_K - IR_K - IR_{Cl} - \varepsilon_{Cl} = 0
\]

or:

\[
V_{in} + \varepsilon_K - IR_K = V_{out}
\]

\[
V_{out} - IR_{Cl} - \varepsilon_{Cl} = V_{in}
\]
Kirchhoff’s junction rule

The sum of currents into a junction equals the sum of currents out of a junction

\[ \sum I_{in} = \sum I_{out} \]

Example:

\[ I_1 + I_2 + I_3 = I_4 + I_5 \]
ACT: Checkpoint 1.3

Calculate the current through the battery $I_B$.

A. $I_B = 0.5\, \text{A}$
B. $I_B = 1.0\, \text{A}$
C. $I_B = 1.5\, \text{A}$

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In the circuit, the current through $R_C$ is 0. What is the current through $R_3$ and the value of $R_x$?

No current flows through $R_C$ so $I_2 = I_3$ and $I_1 = I_x$

No current flows through $R_C$ so $V_C = 0$
During nerve impulse, Na⁺ channels open (i.e. switch S closes) and allow Na⁺ to enter the cell.

\[
V_{out} = 0
\]

\[
V_{in} = ?
\]

What happens to the currents through the channels and the potential in the cell?
Calculation: two loop circuit

Given the circuit to the right, find $I_K$, $I_{Na}$ and $I_{Cl}$ and $V_{in} - V_{out}$.

\[
\varepsilon_K = 80 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \varepsilon_{Cl} = 50 \text{ mV} \\
R_K = 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega
\]

1. Label all currents
2. Label +/− for all elements
3. Choose loop and direction
4. Write down voltage differences

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What is the correct expression for “Loop 3” in the circuit below?

A. $+\varepsilon_{Cl} - I_{Cl} R_{Cl} - I_{Na} R_{Na} + \varepsilon_{Na} = 0$

B. $+\varepsilon_{Cl} - I_{Cl} R_{Cl} + I_{Na} R_{Na} + \varepsilon_{Na} = 0$

C. $+\varepsilon_{Cl} + I_{Cl} R_{Cl} - I_{Na} R_{Na} + \varepsilon_{Na} = 0$
Calculation: two loop circuit

Given the circuit to the right, find $I_K$, $I_{Na}$ and $I_{Cl}$ and $V_{in} - V_{out}$.

$\varepsilon_K = 80 \text{ mV}$, $\varepsilon_{Na} = 60 \text{ mV}$, $\varepsilon_{Cl} = 50 \text{ mV}$

$R_K = 2 \text{ M}\Omega$, $R_{Na} = 0.2 \text{ M}\Omega$, $R_{Cl} = 5 \text{ M}\Omega$

We have 3 unknowns, need 3 equations

Loop 1:

Loop 2:

Loop 3:

5. Write down junction rule
What is the correct expression for junction in the circuit?

A. \( I_K + I_{Na} = I_{Cl} \)
B. \( I_{Na} + I_{Cl} = I_K \)
C. \( I_{Cl} + I_K = I_{Na} \)
Calculation: two loop circuit

Given the circuit to the right, find $I_K$, $I_{Na}$ and $I_{Cl}$ and $V_{in} - V_{out}$.

$\varepsilon_K = 80 \text{ mV}, \varepsilon_{Na} = 60 \text{ mV}, \varepsilon_{Cl} = 50 \text{ mV}$

$R_K = 2 \text{ M}\Omega, R_{Na} = 0.2 \text{ M}\Omega, R_{Cl} = 5 \text{ M}\Omega$

3 equations, 3 unknowns, the rest is algebra!

(1) $\varepsilon_K - I_K R_K - I_{Na} R_{Na} + \varepsilon_{Na} = 0$

(2) $\varepsilon_K - I_K R_K - I_{Cl} R_{Cl} - \varepsilon_{Cl} = 0$

(3) $I_{Na} + I_{Cl} = I_K$

Substitute Eq. (3) into Eq. (2) and rearrange

(2') $-30 - 7I_K + 5I_{Na} = 0$
Calculation: two loop circuits

Now 2 equations (1 and 2’), 2 unknowns ($I_K$ and $I_{Na}$)

\[(1) \quad +70 - I_K - 0.1I_{Na} = 0 \quad I_K = 70 - 0.1I_{Na} \]
\[(2') \quad +30 - 7I_K + 5I_{Na} = 0 \quad +30 - 7(70 - 0.1I_{Na}) + 5I_{Na} = 0 \]

Substitute $I_K$ in Eq. (1) into Eq. (2’) and rearrange

\[-460 + 5.7I_{Na} = 0 \quad I_{Na} = \frac{460 \text{ mV}}{5.7 \Omega} = 81 \text{nA} \]

Plug solution into Eq. (2’) to get $I_K$

\[+30 - 7I_K + 5 \cdot 81 = 0 \quad I_K = \frac{435 \text{ mV}}{7 \Omega} = 62 \text{nA} \]

Use junction Eq. (3) to get $I_{Cl}$

\[I_{Cl} = 62 - 81 = -19 \text{nA} \]
We found that $I_K = 62$ nA, $I_{Na} = 81$ nA and $I_{Cl} = -19$ nA. Which of the following statements is FALSE?

A. $I_K$ is out of the cell
B. $I_{Na}$ is into the cell
C. $I_{Cl}$ is into the cell
Calculation: two loop circuit

Find the new $V_{in} - V_{out}$:

$V_{out} = 0$

$V_{in} = -70 \text{mV}$

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Summary of today’s lecture

• Two basic principles:
  • Kirchhoff loop rule
    Voltages around circuit loop sum to zero (based on conservation of energy)
    \[ \sum \Delta V = 0 \]
  • Kirchhoff junction rule
    Currents into a circuit branch equal currents out (based on conservation of charge)
    \[ \sum I_{in} = \sum I_{out} \]
Summary of today’s lecture

• Basic approach to solving complex circuits:
  1. Label all currents
  2. Label +/- for all elements
  3. Choose loop(s) and direction(s)
  4. Write down voltage differences
  5. Write down junction rule

The rest is algebra!