Phys 102 – Lecture 6

Circuit elements: resistors, capacitors, and batteries
Today we will learn about...

Circuit elements that:
1) Serve as conduits for charge – wires
2) Pump charges around – batteries
3) Regulate flow of charge – resistors
4) Store and release charge – capacitors

These elements are idealizations of components in electronic circuits & in nature

Ex: neurons, circulatory system
**Electric current**

*Current* – measure of flow of charge (+ charge, by convention)
Counts total charge $\Delta Q$ passing through area in a time interval $\Delta t$

\[
I \equiv \frac{\Delta Q}{\Delta t}
\]

Unit: A ("Amp" or "Ampere")

$1\text{A} = 1\text{C/s}$

In electronic circuits, electrons ($-e$) carry current, flow *opposite* to current.

In liquid or gas, both cations and anions can carry current.
Two light bulbs 1 and 2 are connected end-to-end by conducting wire. If a current $I_1$ flows through bulb 1, what is the current $I_2$ in bulb 2?

A. $I_2 < I_1$
B. $I_2 = I_1$
C. $I_2 > I_1$
Batteries & electromotive force

Battery – maintains a constant electric potential difference ("Electromotive force" – emf $\varepsilon$)

Electric potential is 9 V higher at + end relative to – end. Potential difference across a circuit element is its “voltage”

Electric potential difference drives current around circuit
Battery does NOT determine how much current flows
Battery does NOT generate new charges, it “pushes” charges, like a pump
Two 9 V batteries are connected end-to-end by conducting wire. What is the electric potential at point 2 relative to point 1?

A. +18 V
B. +9 V
C. −18 V
D. −9 V
Resistance and Ohm’s law

Moving charges collide with each other, ions, defects inside material. Flow rate depends on electric potential difference.

Potential difference causes current to flow ("downhill", by convention). Resistance regulates the amount of flow.

Ohm’s law: \[ R \equiv \frac{V_R}{I} \]

Units: \( \Omega \) ("Ohms")

Double potential difference, double current.
**Physical resistance**

*Resistor* – circuit element designed to have resistance

Resistance depends on material parameters and geometry

- **Resistivity** – density of scatterers
- **Length** – the longer the resistor, the more scattering
- **Cross sectional area** – the wider the resistor, the more charges flow

\[
R = \rho \frac{L}{A}
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho ) (( \Omega \cdot m ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>( 1.7 \times 10^{-8} )</td>
</tr>
<tr>
<td>Iron</td>
<td>( 9.7 \times 10^{-8} )</td>
</tr>
<tr>
<td>Sea water</td>
<td>0.22</td>
</tr>
<tr>
<td>Muscle</td>
<td>13</td>
</tr>
<tr>
<td>Fat</td>
<td>25</td>
</tr>
<tr>
<td>Pure water</td>
<td>( 2.4 \times 10^5 )</td>
</tr>
</tbody>
</table>

Phys. 102, Lecture 6, Slide 8
Which of the following three copper resistors has the *lowest* resistance?

A. $R_1$

B. $R_2$

C. $R_3$
**Power generated and dissipated**

Battery does work pumping charges through circuit

Ex: a 9 V battery does 9 J of work per 1 C of charge pumped

*Power* – rate of energy conversion

\[
P_{\text{batt}} = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t} \varepsilon = I\varepsilon
\]

Units: W (“Watts”)

1 W = 1 J/s = 1 V A

Resistor **dissipates** electric potential energy

Charges lose electric potential energy in collisions inside resistor

\[
P_{\text{diss}} = IV_R = I^2R = \frac{V_R^2}{R}
\]
Calculation: light bulb filament

An incandescent light bulb is essentially a resistor that dissipates energy as heat and light. A typical light bulb dissipates 60 W with 120 V from an outlet.

The resistive element is a thin (40-μm diameter) filament of tungsten. How long must the filament be?
Capacitance

*Capacitor* – circuit element that stores separated charge
Consists of two conductors separated by a small gap

*Capacitance* – measures the ability to store charge $Q$ given a voltage $V_C$ applied between the conductors

$$C \equiv \frac{Q}{V_C}$$

Units: F (“Farad”)
$1 \text{ F} = 1 \text{ C/V}$
Physical capacitance

Parallel plate capacitor made up of two large conducting plates of area $A$ separated by a small gap $d$

Electric field is uniform between plates (Recall Lect. 3)

$$E = \frac{Q}{\varepsilon_0 A}$$

Field strength $\propto$ density of field lines $\propto$ density of charges

Work to move $+q$ from $+$ to $-$ plate in uniform $E$ field (Recall Lect. 4)

$$W_E = +qEd = -\Delta U$$

$$\Delta V = \frac{\Delta U}{q} = +Ed = V_C$$

For a parallel plate capacitor:

$$C = \frac{\varepsilon_0 A}{d}$$

Capacitance depends on geometry
A parallel plate capacitor carries a charge $Q$. The plates are then pulled a small distance further apart.

What happens to the charge $Q$ on each plate?

A. $Q$ increases
B. $Q$ stays constant
C. $Q$ decreases
A parallel plate capacitor carries a charge $Q$. The plates are then pulled a small distance further apart.

The voltage $V_C$ between the plates

A. Increases  B. Stays the same  C. Decreases
Imagine placing insulating material (dielectric) between plates

External field polarizes dielectric
Excess $+q$ and $-q$ charges build up on opposite planes

Parallel planes of $+q$ and $-q$ create own $E$ field, cancel out part of external $E$ field

$$\vec{E}_{\text{diel}} = \frac{\vec{E}_{\text{ext}}}{\kappa}$$

Dielectric constant $\kappa > 1$
Dielectric constant $\kappa$

Dielectric constant $\kappa$ measures how much a material is polarized by electric field.

Since $E = \frac{E_0}{\kappa}$, need less $E$ (or $V$) to store same $Q$, so $C = \frac{Q}{V}$ increases:

$$C = \kappa C_0$$

### Capacitance with dielectric

<table>
<thead>
<tr>
<th>Material</th>
<th>$\kappa$ (&gt; 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1 (exactly)</td>
</tr>
<tr>
<td>Air</td>
<td>1.00054</td>
</tr>
<tr>
<td>Rubber</td>
<td>3-4</td>
</tr>
<tr>
<td>Glass</td>
<td>5</td>
</tr>
<tr>
<td>Cell membrane</td>
<td>7-9</td>
</tr>
<tr>
<td>Pure water</td>
<td>80</td>
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Capacitance depends on material parameters (dielectric) and geometry.
Calculation: capacitance of a cell

Channels in a cell’s membrane create a charge imbalance (recall Lect. 5), with + charge outside, – inside. The separated charge gives the cell capacitance, with the membrane acting as a dielectric ($\kappa = 7$).

Based on EXAM 1, FA09

What is the capacitance of a 1-μm² flat patch of cell?

At rest, a cell has a –70 mV voltage across it. How much charge is necessary to generate this voltage?
Capacitor energy

Separated charges have potential energy (Recall Lect. 4)

\[ U_C = \frac{1}{2} QV_C = \frac{1}{2} CV_C^2 = \frac{1}{2} \frac{Q^2}{C} \]

Important factor of \( \frac{1}{2} \)! Don’t confuse this equation with \( U = qV \) for individual charge \( q \)

Why separate charge?

- Camera flash
- Defibrillator
- Lightning strike

A way to store and release energy
A parallel plate capacitor carries a charge $Q$. A dielectric with $\kappa > 1$ is inserted between the plates.

What happens to energy $U_C$ stored in the capacitor?

A. $U_C$ increases
B. $U_C$ stays constant
C. $U_C$ decreases
Summary of today’s lecture

• **Batteries** generate emf $\varepsilon$, pump charges

• **Resistors** *dissipate* energy as power: $P = IV$
  
  Resistance: how difficult it is for charges to get through: $R = \rho L/A$

  Voltage determines current: $V = IR$

  *Ideal wires* have $R = 0$, $V = 0$

• **Capacitors** *store* energy as separated charge: $U = \frac{1}{2}QV$

  Capacitance: ability to store separated charge: $C = \kappa \varepsilon_0 A/d$

  Voltage determines charge: $V = Q/C$

• Don’t mix capacitor and resistor equations!