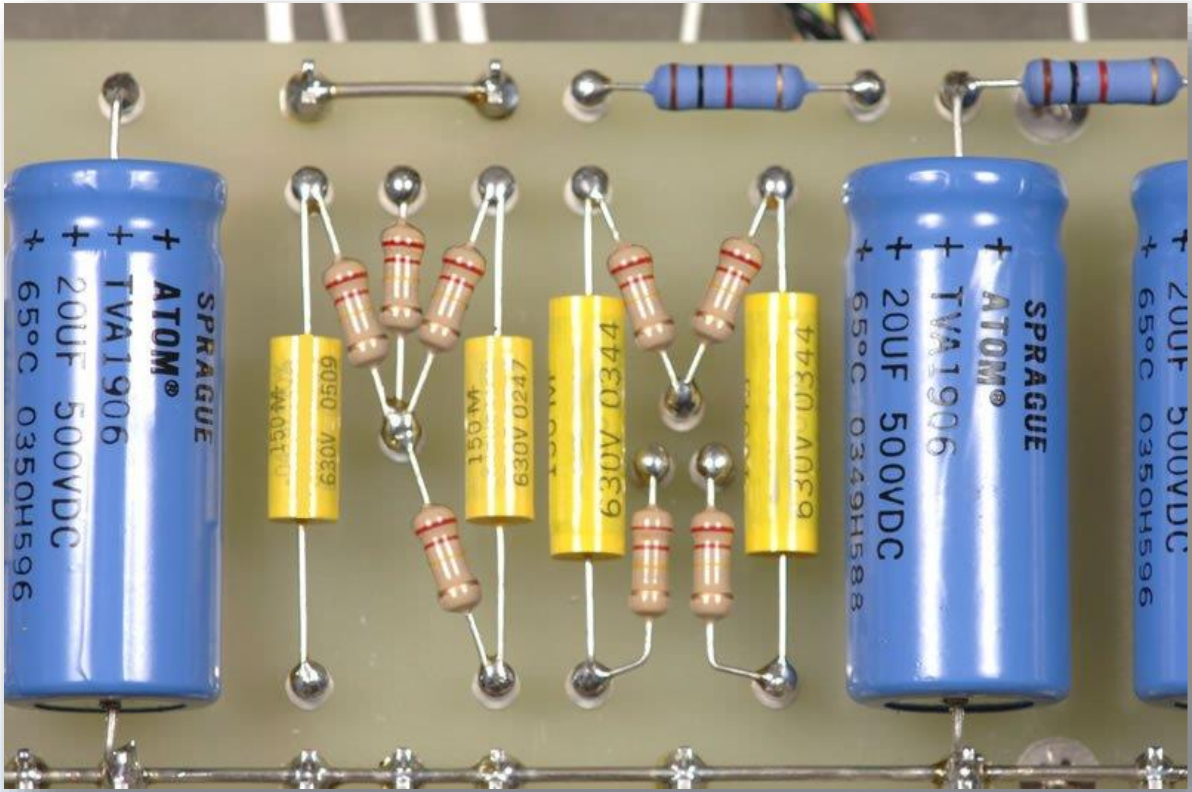


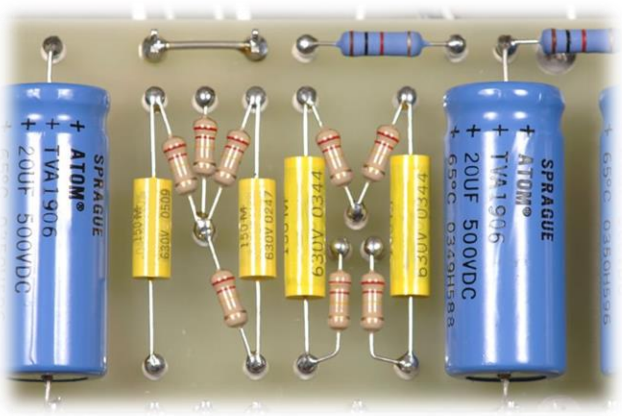
Phys 102 – Lecture 6

Circuit elements: resistors, capacitors, and batteries

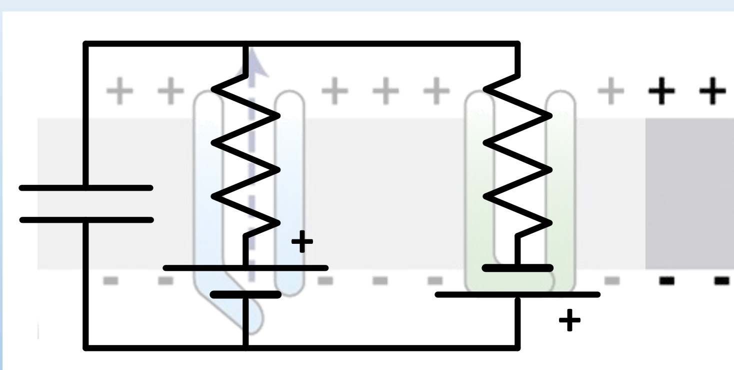
Today we will learn about...

Circuit elements that:

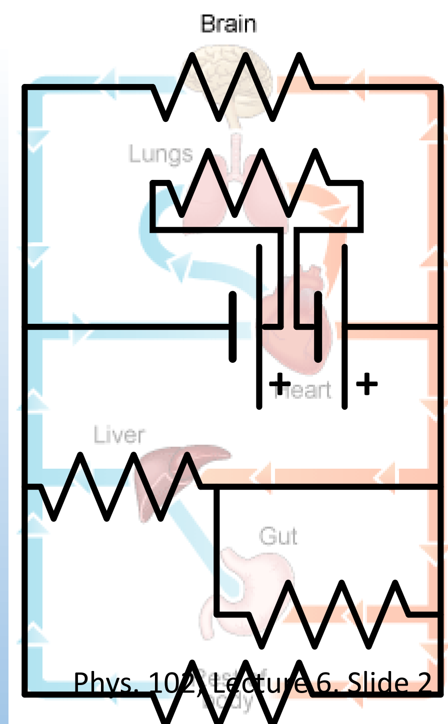
1) Serve as conduits for charge – *wires*
2) Pump charges around – *batteries*
3) Regulate flow of charge – *resistors*
4) Store and release charge – *capacitors*



These elements are idealizations of components in electronic circuits & in nature



Ex: neurons, circulatory system

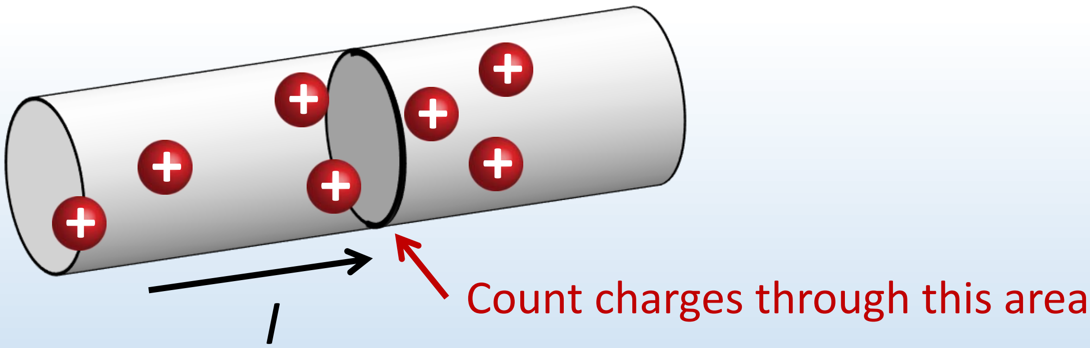


Electric current

Current – measure of flow of charge (+ charge, by convention)
Counts total charge ΔQ passing through area in a time interval Δt

$$I \equiv \frac{\Delta Q}{\Delta t}$$

Unit: A ("Amp" or "Ampere")
1A = 1C/s

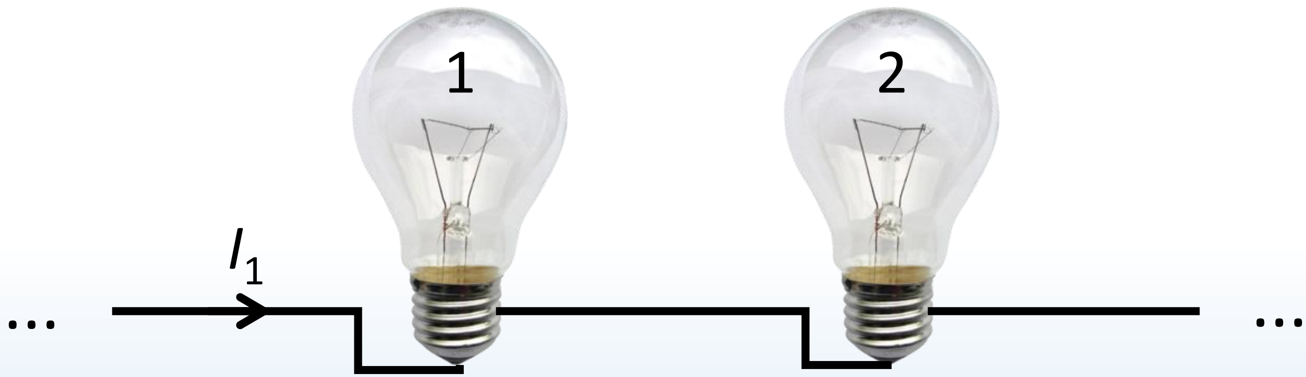


In electronic circuits, electrons (–e) carry current, flow *opposite* to current

In liquid or gas, both cations and anions can carry current

ACT: two light bulbs

Two light bulbs 1 and 2 are connected end-to-end by conducting wire. If a current I_1 flows through bulb 1, what is the current I_2 in bulb 2?



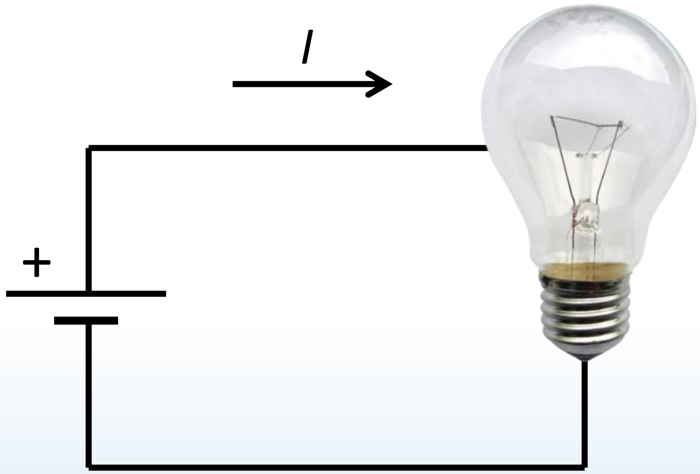
A. $I_2 < I_1$

B. $I_2 = I_1$

C. $I_2 > I_1$

Batteries & electromotive force

Battery – maintains a constant electric potential difference ("Electromotive force" – emf ε)

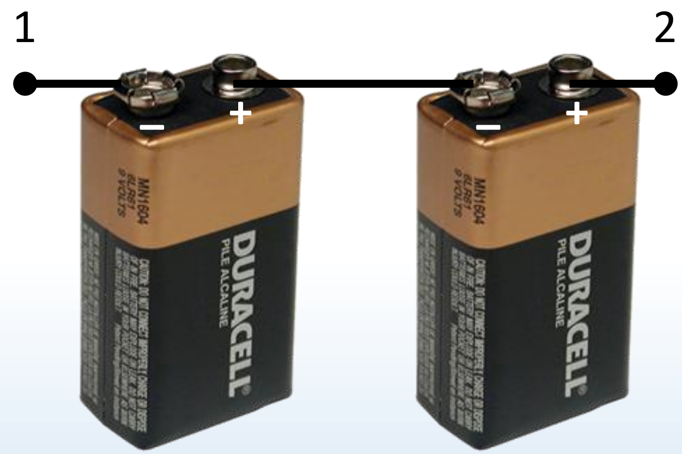


Electric potential is 9 V higher at + end relative to – end. Potential difference across a circuit element is its "voltage"

Electric potential difference drives current around circuit
Battery does NOT determine how much current flows
Battery does NOT generate new charges, it "pushes" charges, like a pump

ACT: Two batteries

Two 9 V batteries are connected end-to-end by conducting wire. What is the electric potential at point 2 relative to point 1?



A. +18 V

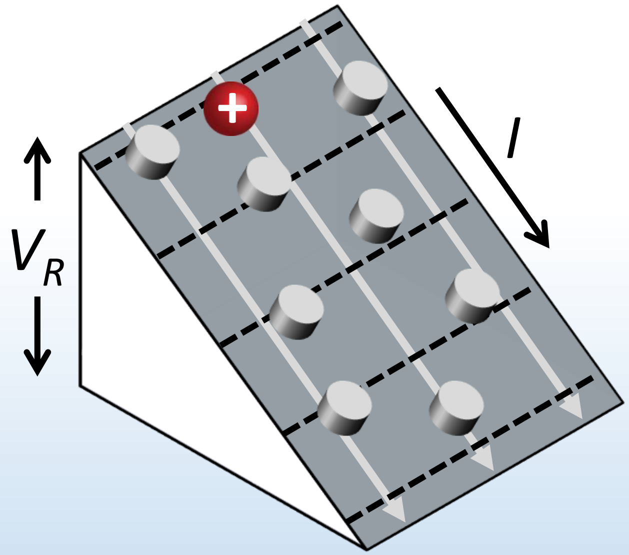
B. +9 V

C. –18 V

D. –9 V

Resistance and Ohm's law

Moving charges collide with each other, ions, defects inside material
Flow rate depends on electric potential difference



$I \propto V_R$ ← Double potential difference, double current

DEMO

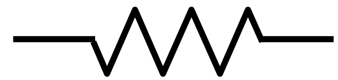
Resistance – proportionality constant between current and voltage

Ohm's law: $$R \equiv \frac{V_R}{I}$$ Units: Ω ("Ohms")

Potential difference causes current to flow ("downhill", by convention)
Resistance regulates the amount of flow

Physical resistance

__Resistor__ – circuit element designed to have resistance



Resistance depends on material parameters and geometry

Resistivity – density of scatterers

$$R = \rho \frac{L}{A}$$

Length – the longer the resistor, the more scattering

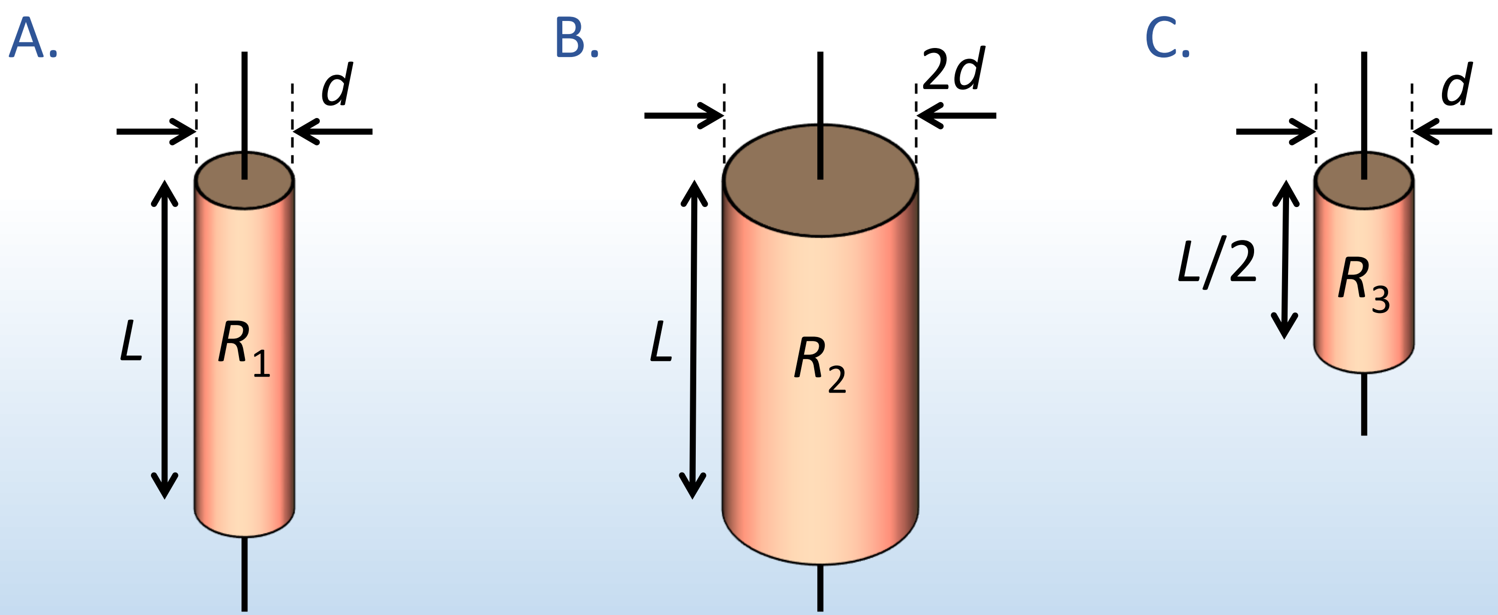
Cross sectional area – the wider the resistor, the more charges flow

DEMO

Material	ρ (Ω·m)
Copper	1.7×10^{-8}
Iron	9.7×10^{-8}
Sea water	0.22
Muscle	13
Fat	25
Pure water	2.4×10^{5}

ACT: CheckPoint 1.1

Which of the following three copper resistors has the *lowest* resistance?



Power generated and dissipated

Battery does work pumping charges through circuit

Ex: a 9 V battery does 9 J of work per 1 C of charge pumped

Power – rate of energy conversion

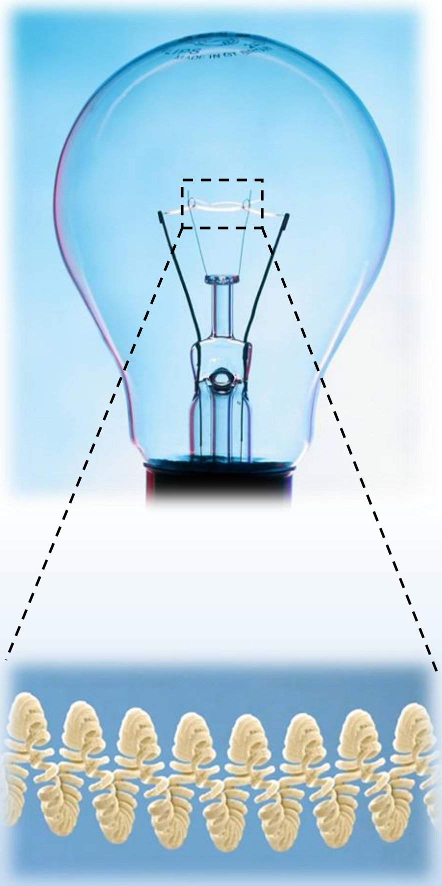
$$P_{batt} = \frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\varepsilon = I\varepsilon$$

Units: W ("Watts")
1 W = 1 J/s = 1 V A

Resistor *dissipates* electric potential energy
Charges lose electric potential energy in collisions inside resistor

$$P_{diss} = IV_R = I^2 R = \frac{V_R^2}{R}$$

Calculation: light bulb filament



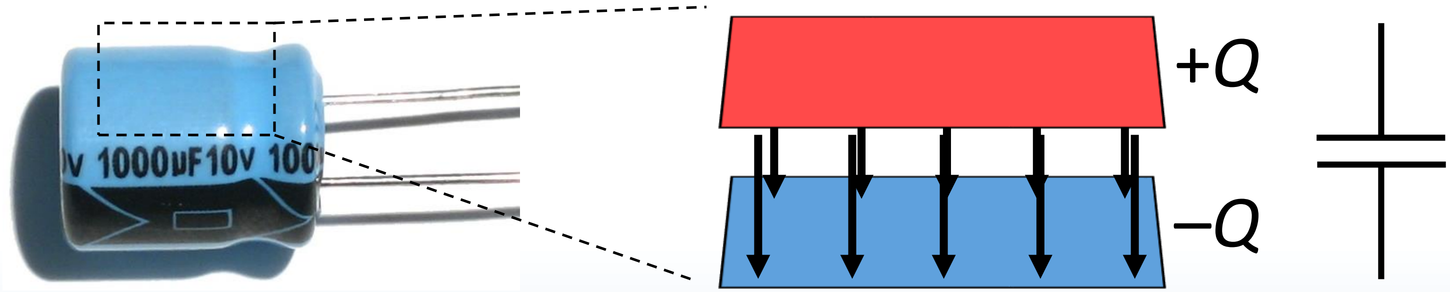
An incandescent light bulb is essentially a resistor that dissipates energy as heat and light. A typical light bulb dissipates 60 W with 120 V from an outlet.

The resistive element is a thin (40-μm diameter) *filament* of tungsten. How long must the filament be?

Capacitance

Capacitor – circuit element that stores separated charge

Consists of two conductors separated by a small gap



Capacitance – measures the ability to store charge Q given a voltage V_C applied between the conductors

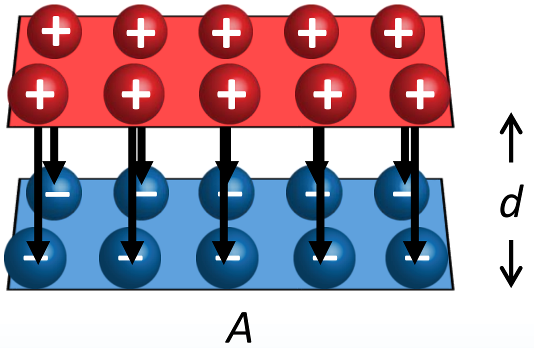
$$C \equiv \frac{Q}{V_C}$$

Units: F ("Farad")

1 F = 1 C/V

Physical capacitance

Parallel plate capacitor made up of two large conducting plates of area *A* separated by a small gap *d*



Electric field is uniform between plates (Recall Lect. 3)

$$E = \frac{Q}{\varepsilon_0 A}$$

Field strength ∝ density of field lines ∝ density of charges

Work to move +*q* from + to – plate in uniform *E* field (Recall Lect. 4)

$$W_E = +qEd = -\Delta U \qquad \Delta V = \frac{\Delta U}{q} = +Ed = V_C$$

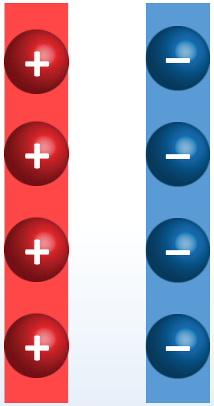
Capacitor voltage

For a parallel plate capacitor: $C = \frac{\varepsilon_0 A}{d}$

Capacitance depends on geometry

ACT: Parallel plates

A parallel plate capacitor carries a charge *Q*. The plates are then pulled a small distance further apart.



What happens to the charge *Q* on each plate?

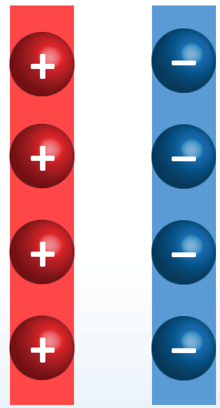
A. *Q* increases

B. *Q* stays constant

C. *Q* decreases

ACT: Parallel plates 2

A parallel plate capacitor carries a charge *Q*. The plates are then pulled a small distance further apart.



DEMO

The voltage V_C between the plates

A. Increases B. Stays the same C. Decreases

Dielectrics

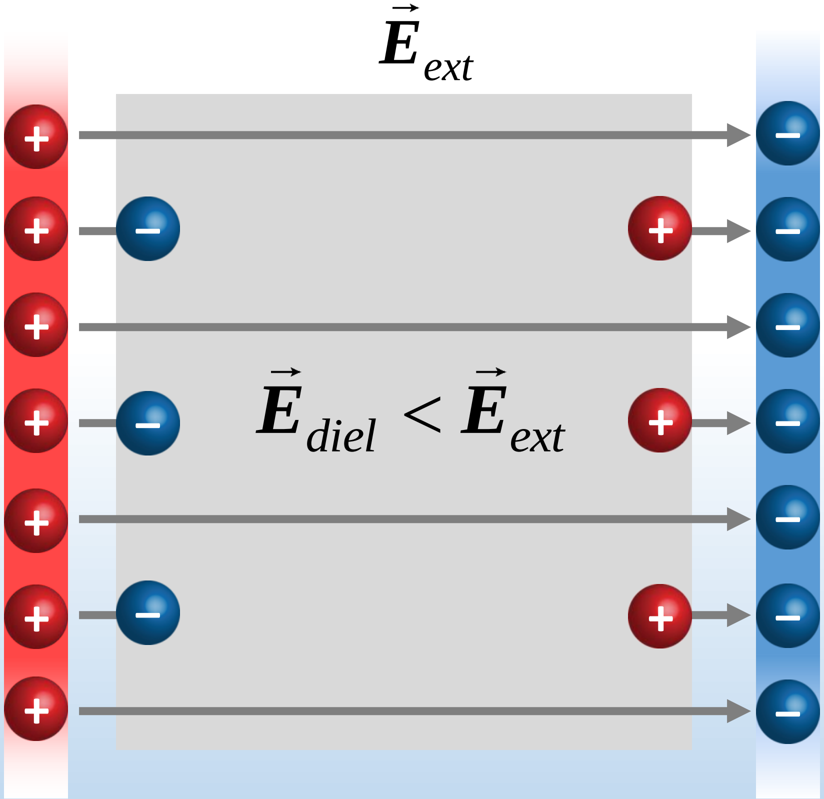
Imagine placing insulating material (dielectric) between plates

External field polarizes dielectric
Excess +*q* and –*q* charges build up on opposite planes

Parallel planes of +*q* and –*q* *create own E field*, cancel out part of external E field

$$\vec{E}_{diel} = \frac{\vec{E}_{ext}}{\kappa}$$

Dielectric constant κ > 1

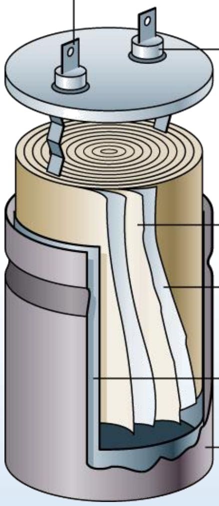


(Recall Lect. 3 – conductors)

Dielectric constant κ

Dielectric constant κ measures how much a material is polarized by electric field

Since $\vec{E} = \vec{E}_0 / \kappa$, need less E (or V) to store same Q, so $C = Q/V$ increases:



Dielectric constant

$$C = \kappa C_0$$

Capacitance with dielectric

Capacitance without dielectric

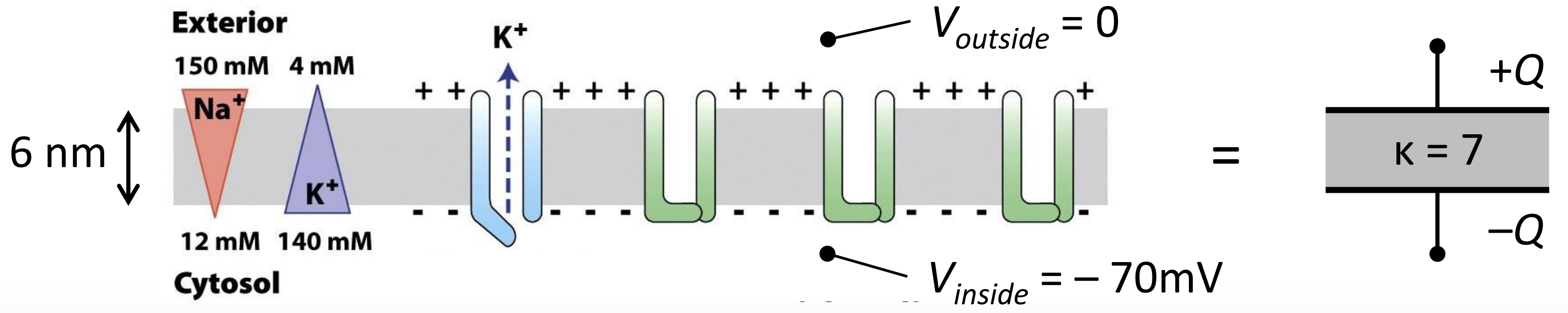
Material	κ (> 1)
Vacuum	1 (exactly)
Air	1.00054
Rubber	3-4
Glass	5
Cell membrane	7-9
Pure water	80

Capacitance depends on material parameters (dielectric) and geometry

Calculation: capacitance of a cell

Channels in a cell's membrane create a charge imbalance (recall Lect. 5), with + charge outside, – inside. The separated charge gives the cell *capacitance*, with the membrane acting as a dielectric ($\kappa = 7$).

Based on EXAM 1, FA09



What is the capacitance of a 1-μm^2 flat patch of cell?

At rest, a cell has a –70 mV voltage across it. How much charge is necessary to generate this voltage?

Capacitor energy

Separated charges have potential energy (Recall Lect. 4)

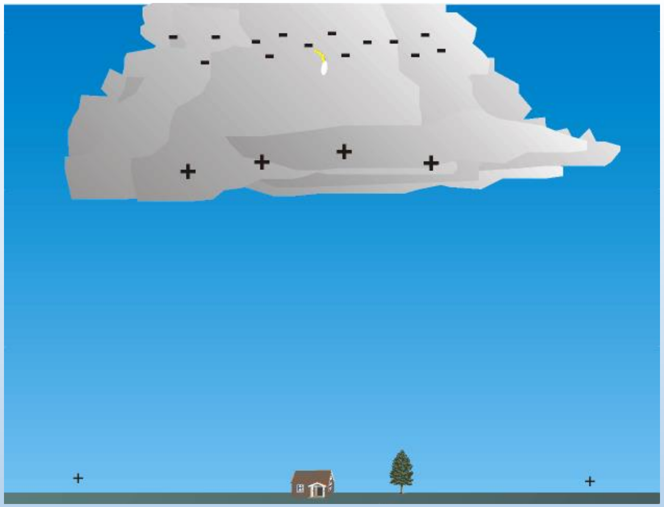
$$U_C = \frac{1}{2}QV_C = \frac{1}{2}CV_C^2 = \frac{1}{2}\frac{Q^2}{C}$$

Important factor of ½! Don't confuse this equation with $U = qV$ for individual charge q

Why separate charge?

Camera flash

Defibrillator

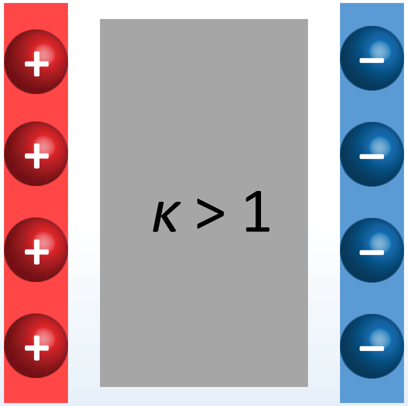


Lightning strike

A way to store *and* release energy

ACT: Capacitor dielectric

A parallel plate capacitor carries a charge Q. A dielectric with $\kappa > 1$ is inserted between the plates.



What happens to energy U_C stored in the capacitor?

A. U_C increases

B. U_C stays constant

C. U_C decreases

Summary of today's lecture

- Batteries generate emf ε, pump charges
- Resistors *dissipate* energy as power: $P = IV$

  Resistance: how difficult it is for charges to get through: $R = \rho L/A$

  Voltage determines *current*: $V = IR$

  Ideal wires have $R = 0$, $V = 0$
- Capacitors *store* energy as separated charge: $U = ½QV$

  Capacitance: ability to store separated charge: $C = \kappa\varepsilon_0 A/d$

  Voltage determines *charge*: $V = Q/C$
- Don't mix capacitor and resistor equations!