The classical and Bohr atom
State of late 19th Century Physics

• Two great theories “Classical physics”
  Newton’s laws of mechanics & gravity
  Maxwell’s theory of electricity & magnetism, including EM waves

• But... some unsettling problems
  Stability of atom & atomic spectra
  Photoelectric effect
  ...and others

• New theory required Quantum mechanics
Stability of classical atom

**Prediction** – orbiting $e^-$ is an oscillating charge & should emit EM waves in every direction

EM waves carry energy, so $e^-$ should lose energy & fall into nucleus!

Classical atom is NOT stable!

Lifetime of classical atom = $10^{-11}$ s

**Reality** – Atoms are stable
**Atomic spectrum**

*Prediction* – $e^{-}$ should emit light at whatever frequency $f$ it orbits nucleus

$$d \sin \theta = m\lambda$$

*Reality* – Only *certain* frequencies of light are emitted & are different for different elements
Quantum mechanics

Quantum mechanics explains stability of atom & atomic spectra (and many other phenomena...)

QM is one of most successful and accurate scientific theories

Predicts measurements to $<10^{-8}$ (ten parts per billion!)

Wave-particle duality – matter behaves as a wave

Particles can be in many places at the same time
Processes are probabilistic not deterministic
Measurement changes behavior

Certain quantities (ex: energy) are quantized
Matter waves

Matter behaves as a wave with de Broglie wavelength

\[ \lambda = \frac{h}{p} \]

Planck’s constant

\[ h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \]

Wavelength of matter wave

Momentum \( mv \) of particle

Ex: a fastball \((m = 0.5 \text{ kg}, v = 100 \text{ mph} \approx 45 \text{ m/s})\)

\[ \lambda_{\text{fastball}} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{0.5 \cdot 45} = 3 \times 10^{-35} \text{ m} \]

20 orders of magnitude smaller than the proton!

Ex: an electron \((m = 9.1 \times 10^{-31} \text{ kg}, v = 6 \times 10^{5} \text{ m/s})\)

\[ \lambda_{\text{electron}} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \cdot 6 \times 10^{5}} = 1.2 \text{ nm} \]

X-ray wavelength

How could we detect matter wave? Interference!
Identical pattern emerges if de Broglie wavelength of $e^-$ equals the X-ray wavelength!
Electron diffraction

Beam of mono-energetic $e^-$ passes through double slit

$$d \sin \theta = m\lambda \quad \lambda = \frac{h}{p}$$

Wait! Does this mean $e^-$ passes through both slits?

What if we measure which slit the $e^-$ passes through?

Merli – 1974
Tonomura – 1989
Consider the interference pattern from a beam of mono-energetic electrons $A$ passing through a double slit.

Now a beam of electrons $B$ with 4x the energy of $A$ enters the slits. What happens to the spacing $\Delta y$ between interference maxima?

A. $\Delta y_B = 4 \Delta y_A$
B. $\Delta y_B = 2 \Delta y_A$
C. $\Delta y_B = \Delta y_A$
D. $\Delta y_B = \Delta y_A / 2$
E. $\Delta y_B = \Delta y_A / 4$
The “classical” atom

Negatively charged electron orbits around positively charged nucleus

Orbiting $e^-$ has centripetal acceleration:

$$ F_E = k \frac{e^2}{r^2} = \frac{mv^2}{r} $$

so,

$$ \frac{ke^2}{r} = mv^2 $$

Total energy of electron:

$$ E_{tot} = K + U = \frac{1}{2} mv^2 - k \frac{e^2}{r} = -\frac{1}{2} \frac{ke^2}{r} $$

Recall Lect. 4

Phys. 102, Lecture 24, Slide 10
The Bohr model

e\textsuperscript{-} behave as waves & only orbits that fit an integer number of wavelengths are allowed

Orbit circumference

\[ 2\pi r = n\lambda \quad n = 1, 2, 3\ldots \]

Angular momentum is quantized

\[ L_n = n\hbar \quad \hbar \equiv \frac{\hbar}{2\pi} \quad \text{“h bar”} \]
What is the quantum number $n$ of this hydrogen atom?

A. $n = 1$
B. $n = 3$
C. $n = 6$
D. $n = 12$
Energy and orbit quantization

Angular momentum is quantized

\[ L_n \equiv pr = mvr = n\hbar \quad n = 1, 2, 3, \ldots \]

Radius of orbit is quantized

\[ r_n = \frac{n^2 \hbar^2}{mke^2} \equiv n^2 a_0 \]

Energy is quantized

\[ E_n = -\frac{1}{2} \frac{ke^2}{r_n} \equiv -\frac{E_1}{n^2} \]

Smallest orbit has energy \(-E_1\) "ground state"
Suppose the charge of the nucleus is doubled (+2e), but the $\text{e}^-$ charge remains the same (−e). How does $r$ for the ground state ($n = 1$) orbit compare to that in hydrogen?

For hydrogen:  
$$r_n = \frac{n^2 \hbar^2}{mke^2}$$

A. 1/2 as large  
B. 1/4 as large  
C. the same
There is a particle in nature called a muon, which has the same charge as the electron but is 207 times heavier. A muon can form a hydrogen-like atom by binding to a proton.

How does the radius of the ground state ($n = 1$) orbit for this hydrogen-like atom compare to that in hydrogen?

A. 207$\times$ larger  
B. The same  
C. 207$\times$ smaller
**Atomic units**

At atomic scales, Joules, meters, kg, etc. are not convenient units

“Electron Volt” – energy gained by charge +1e when accelerated by 1 Volt:  \( U = qV \)  
\[ 1e = 1.6 \times 10^{-19} \text{ C}, \text{ so } 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J} \]

Planck constant: \( h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \)  
\[ hc \approx 2 \times 10^{25} \text{ J} \cdot \text{m} = 1240 \text{ eV} \cdot \text{nm} \]

Speed of light: \( c = 3 \times 10^8 \text{ m/s} \)

Electron mass: \( m = 9.1 \times 10^{-31} \text{ kg} \)  
\[ mc^2 = 8.2 \times 10^{-13} \text{ J} = 511,000 \text{ eV} \]

Since \( U = \frac{ke^2}{r} \), \( ke^2 \) has units of eV⋅nm like \( hc \)  
\[ ke^2 \approx 1.44 \text{ eV} \cdot \text{nm} \]

\[ \frac{ke^2}{hc} = 2\pi \left( \frac{ke^2}{hc} \right) \approx \frac{1}{137} \]

“Fine structure constant” (dimensionless)
Calculation: energy & Bohr radius

Energy of electron in H-like atom (1 e\(^-\), nuclear charge +Ze):

\[
E_n = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -\frac{Z^2}{2n^2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = -\frac{511,000\text{ eV}}{2 \cdot 137^2} \frac{Z^2}{n^2}
\]

\[
= -13.6\text{ eV} \frac{Z^2}{n^2}
\]

Radius of electron orbit:

\[
r_n = \frac{n^2}{Z}a_0 = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z} \frac{\hbar c}{mc^2}\left(\frac{\hbar c}{ke^2}\right) = \frac{n^2}{Z} \frac{1240\text{ eV} \cdot \text{nm}}{2\pi \cdot 511,000\text{ eV}} \cdot 137
\]

\[
= 0.0529\text{ nm} \frac{n^2}{Z}
\]
ACT: Hydrogen-like atoms

Consider an atom with a nuclear charge of +2e with a single electron orbiting, in its ground state ($n = 1$), i.e. He$^+$. How much energy is required to ionize the atom totally?

A. 13.6 eV
B. $2 \times 13.6$ eV
C. $4 \times 13.6$ eV
Heisenberg Uncertainty Principle

e\textsuperscript{−} beam with momentum $p_0$ passes through slit will diffract

- $\Delta y = \alpha$
- $p_0$
- $p_f$
- $\Delta p_y$
- $\Delta y \cdot \Delta p_y \geq \frac{\hbar}{2}$

$e\textsuperscript{−}$ passed somewhere inside slit. Uncertainty in $y$

Momentum was along $x$, but $e\textsuperscript{−}$ diffracts along $y$. Uncertainty in $y$-momentum

If slit narrows, diffraction pattern spreads out
The Bohr model cannot be correct! To be consistent with the Heisenberg Uncertainty Principle, which of the following properties cannot be quantized?

1. Energy is quantized  \[ E_n = -\frac{E_1}{n^2} \]
2. Angular momentum is quantized  \[ L_n = p_n r_n = n\hbar \]
3. Radius is quantized  \[ r_n = n^2 a_0 \]
4. Linear momentum & velocity are quantized  \[ p_n = \frac{\hbar}{n a_0} \]

A. All of the above  
B. #1 & 2  
C. #3 & 4
Summary of today’s lecture

• Classical model of atom
  Predicts unstable atom & cannot explain atomic spectra

• Quantum mechanics
  Matter behaves as waves
  Heisenberg Uncertainty Principle

• Bohr model
  Only orbits that fit $n$ electron wavelengths are allowed
  Explains the stability of the atom
  Energy quantization correct for single e$^-$ atoms (H, He$^+$, Li$^{++}$)
  However, it is fundamentally incorrect

Need complete quantum theory (Lect. 26)