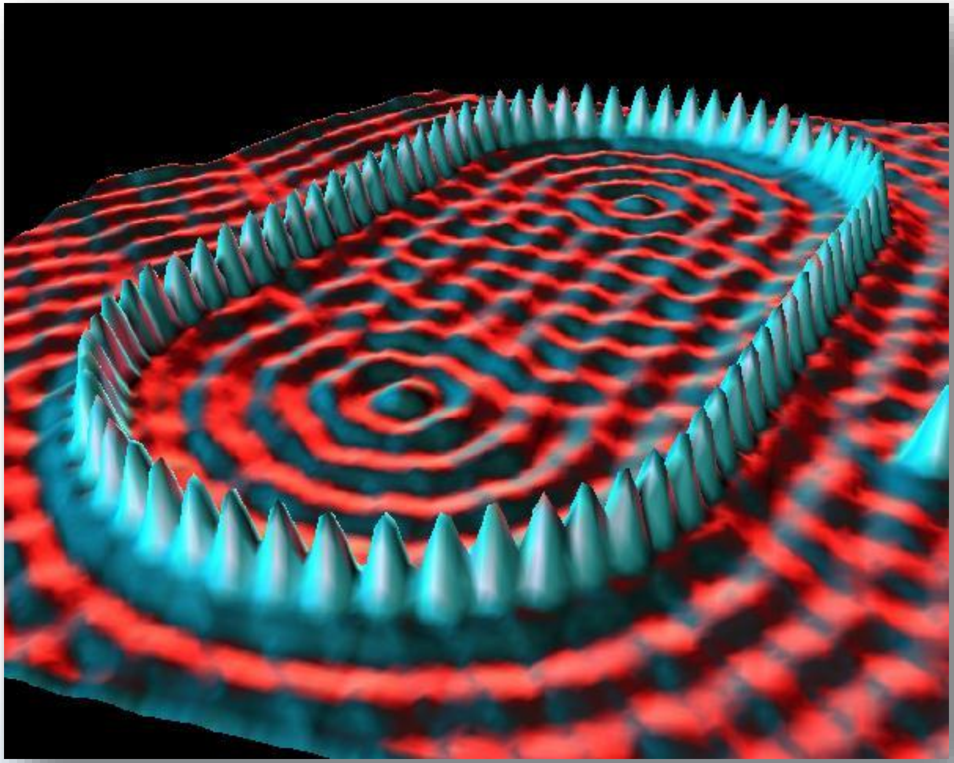


# Phys 102 – Lecture 24

**The classical and Bohr atom**

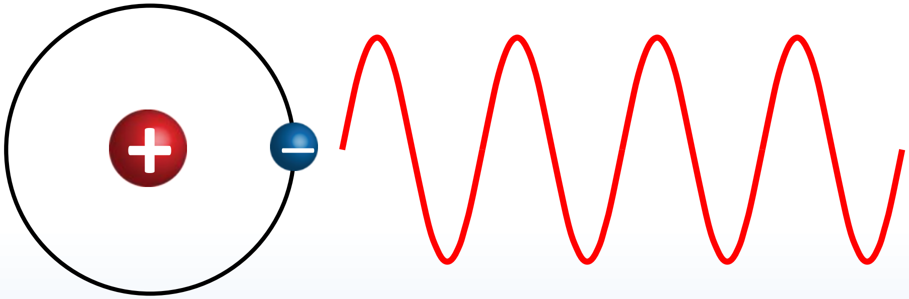
# *State of late 19th Century Physics*

- Two great theories *"Classical physics"*
  - Newton's laws of mechanics & gravity
  - Maxwell's theory of electricity & magnetism, including EM waves
- But... some unsettling problems
  - Stability of atom & atomic spectra
  - Photoelectric effect
  - ...and others
- New theory required *Quantum mechanics*

# *Stability of classical atom*

*Prediction* – orbiting $e^-$ is an oscillating charge & should emit EM waves in every direction

Recall Lect. 15 & 16



$$\bar{u}_{tot} = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0} B_{rms}^2$$

EM waves carry energy, so $e^-$ should lose energy & fall into nucleus!

Classical atom is NOT stable!

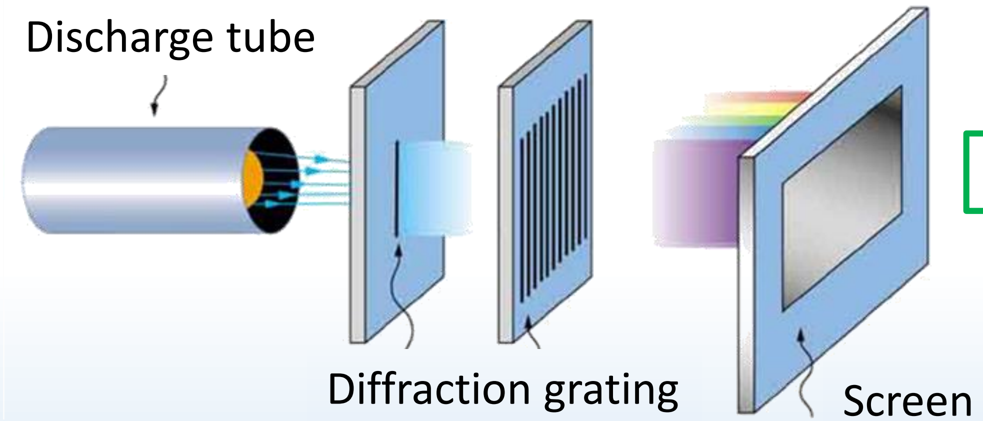
Lifetime of classical atom = $10^{-11}$ s

*Reality* – Atoms are stable

# Atomic spectrum

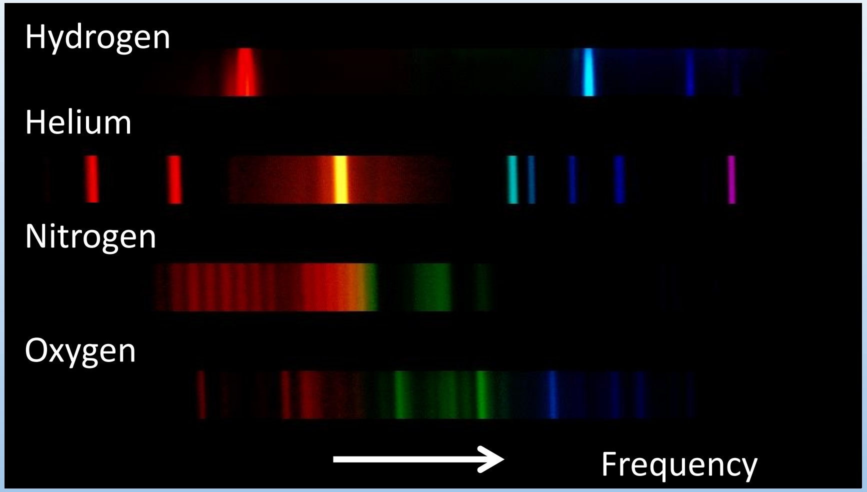
*Prediction* – $e^-$ should emit light at whatever frequency *f* it orbits nucleus

$$d \sin\theta = m\lambda$$




Recall Lecture 22

*Reality* – Only *certain* frequencies of light are emitted & are different for different elements



# *Quantum mechanics*

Quantum mechanics explains stability of atom & atomic spectra (and many other phenomena...)

QM is one of most successful and accurate scientific theories

Predicts measurements to $<10^{-8}$ (ten parts per billion!)

*Wave-particle* duality – matter behaves as a wave

Particles can be in many places at the same time
Processes are probabilistic not deterministic
Measurement changes behavior

Certain quantities (ex: energy) are *quantized*

# Matter waves

Matter behaves as a wave with *de Broglie* wavelength

Wavelength of matter wave

$$\boxed{\lambda = \frac{h}{p}}$$

Planck's constant

$$h = 6.626 \times 10^{-34}\,\mathrm{J \cdot s}$$

Momentum *mv* of particle

Ex: a fastball ($m$ = 0.5 kg, $v$ = 100 mph ≈ 45 m/s)

$$\lambda_{fastball} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{0.5 \cdot 45} = 3 \times 10^{-35}\,\mathrm{m}$$

20 orders of magnitude smaller than the proton!

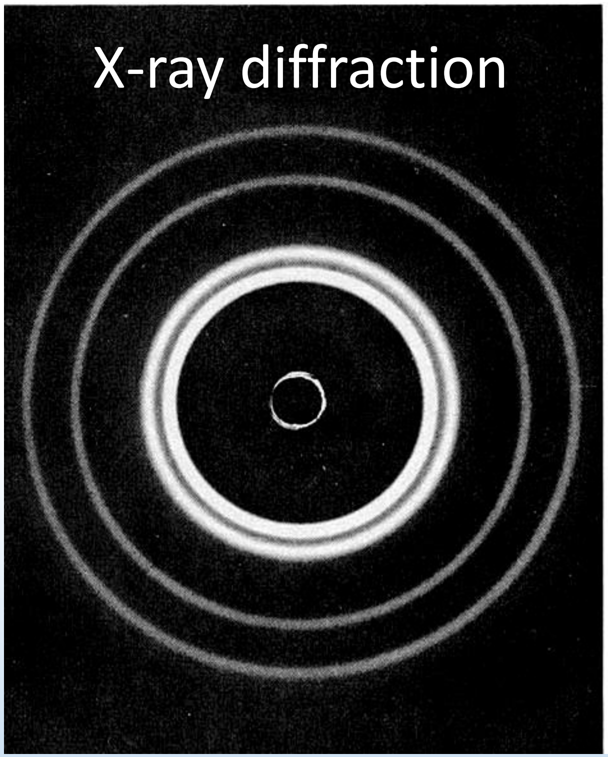
Ex: an electron ($m = 9.1\times10^{-31}$ kg, $v = 6\times10^{5}$ m/s)

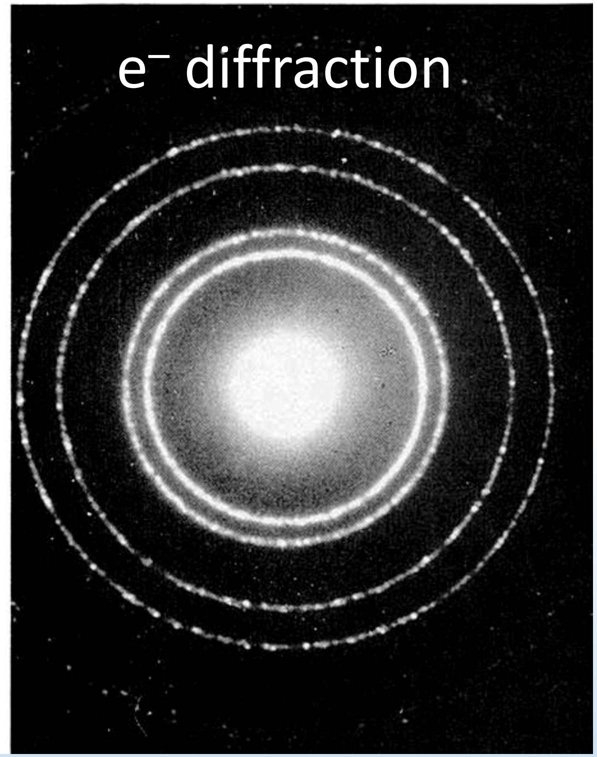
$$\lambda_{electron} = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{9.11 \times 10^{-31} \cdot 6 \times 10^{5}} = 1.2\,\mathrm{nm}$$

X-ray wavelength

How could we detect matter wave? Interference!

# *X-ray vs. electron diffraction*



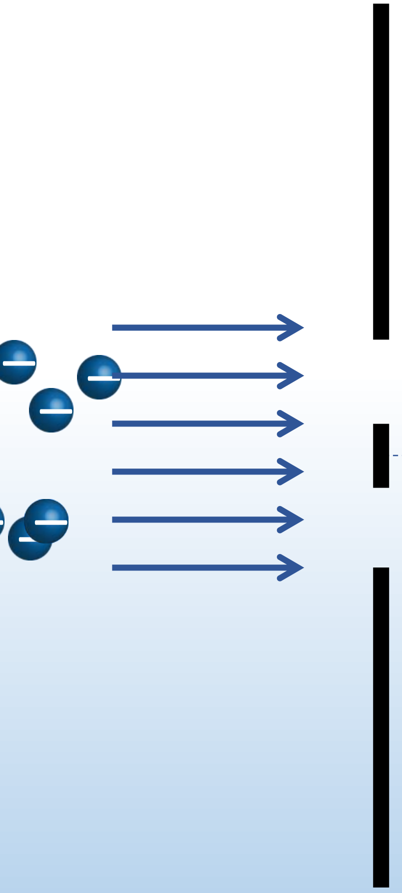




DEMO

Identical pattern emerges if de Broglie wavelength of $e^-$ equals the X-ray wavelength!

# *Electron diffraction*

Beam of mono-energetic $e^-$ passes through double slit

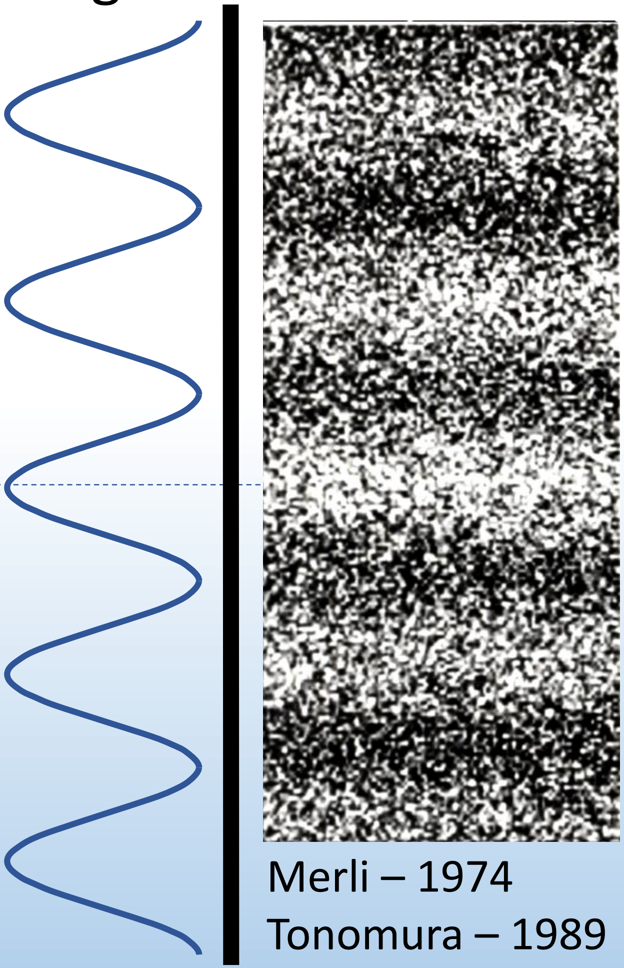


Interference pattern = probability

$$d \sin\theta = m\lambda \qquad \lambda = \frac{h}{p}$$

Wait! Does this mean $e^-$ passes through both slits?

What if we measure which slit the $e^-$ passes through?

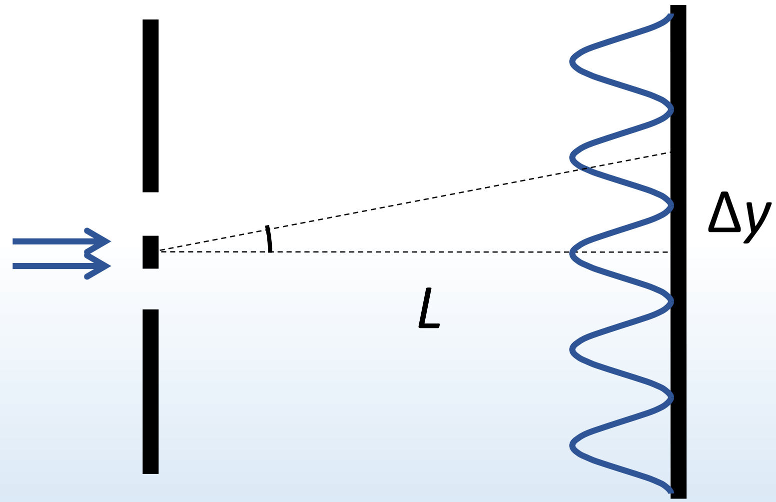


Merli – 1974
Tonomura – 1989

# ACT: Double slit interference

Consider the interference pattern from a beam of mono-energetic electrons *A* passing through a double slit.

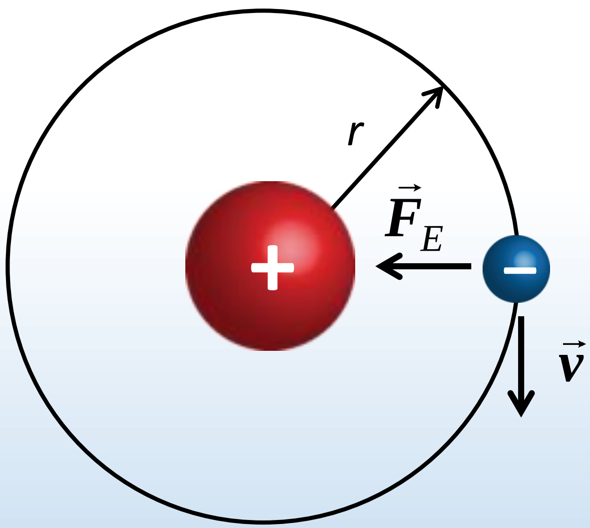
Now a beam of electrons *B* with 4x the energy of *A* enters the slits. What happens to the spacing $\Delta y$ between interference maxima?




A. $\Delta y_B = 4\ \Delta y_A$
B. $\Delta y_B = 2\ \Delta y_A$
C. $\Delta y_B = \Delta y_A$
D. $\Delta y_B = \Delta y_A / 2$
E. $\Delta y_B = \Delta y_A / 4$

# The "classical" atom

Negatively charged electron orbits around positively charged nucleus




Hydrogen atom

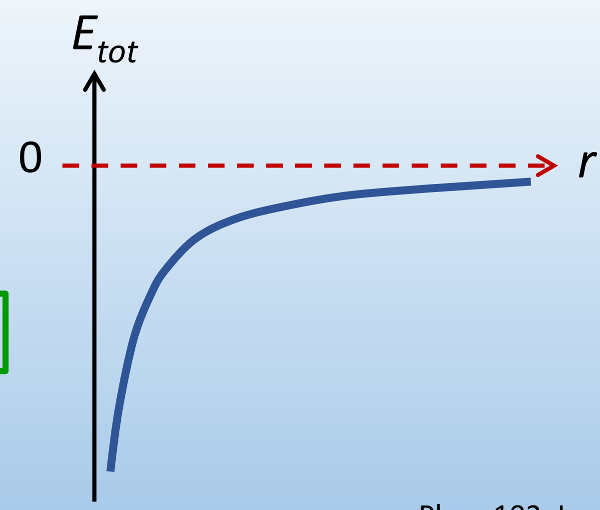
Orbiting e⁻ has centripetal acceleration:

$$F_E = k\frac{e^2}{r^2} = \frac{mv^2}{r} \qquad \text{so} \quad \frac{ke^2}{r} = mv^2$$

Total energy of electron:

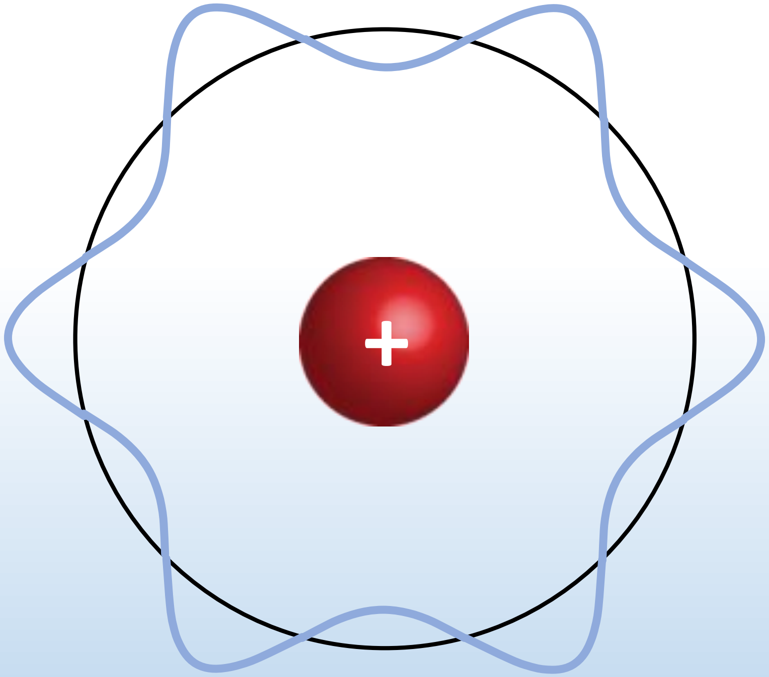
$$E_{tot} = K + U = \frac{1}{2}mv^2 - k\frac{e^2}{r} = -\frac{1}{2}\frac{ke^2}{r}$$

Recall Lect. 4



# The Bohr model

$e^-$ behave as waves & only orbits that fit an integer number of wavelengths are allowed




Orbit circumference

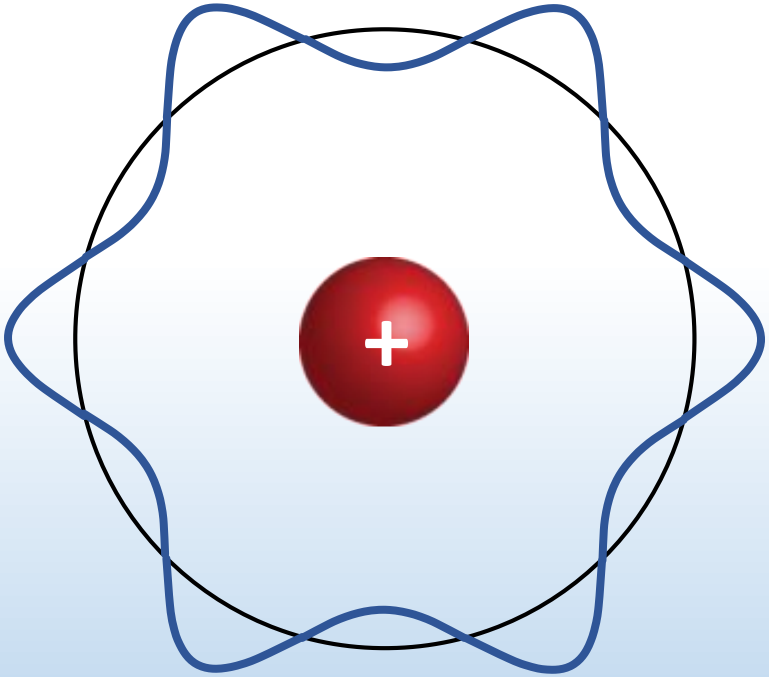
$$2\pi r = n\lambda \qquad n = 1, 2, 3\ldots$$

Angular momentum is *quantized*

$$L_n = n\hbar \qquad \hbar \equiv \frac{h}{2\pi}$$ "h bar"

# ACT: Bohr model

What is the quantum number *n* of this hydrogen atom?

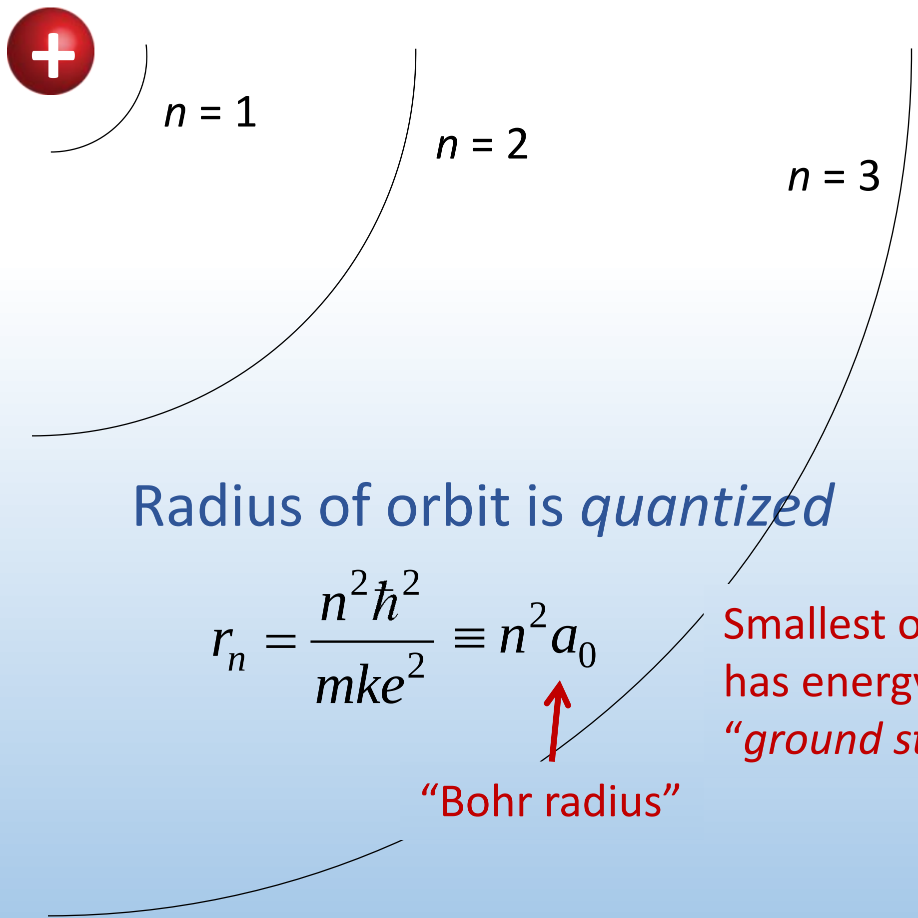



A. $n = 1$
B. $n = 3$
C. $n = 6$
D. $n = 12$

# *Energy and orbit quantization*

## Angular momentum is *quantized*

$$L_n \equiv pr = mvr = n\hbar \qquad n = 1, 2, 3\ldots$$

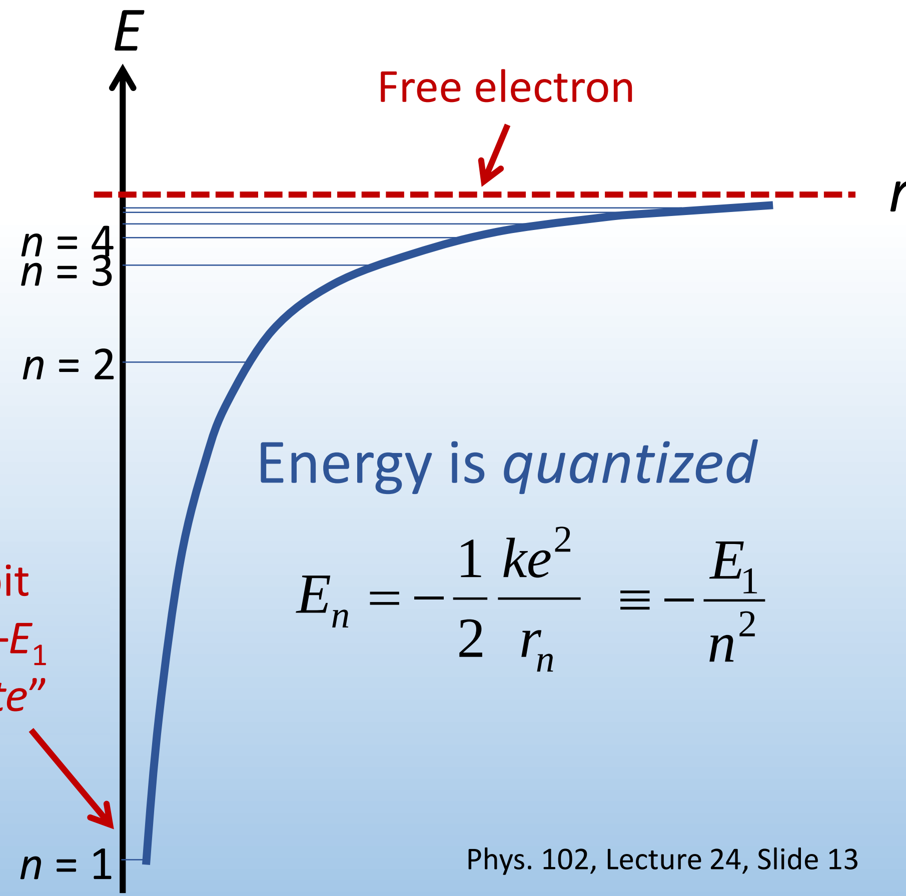



## Radius of orbit is *quantized*

$$r_n = \frac{n^2\hbar^2}{mke^2} \equiv n^2 a_0$$

"Bohr radius"

Smallest orbit has energy $-E_1$ *"ground state"*



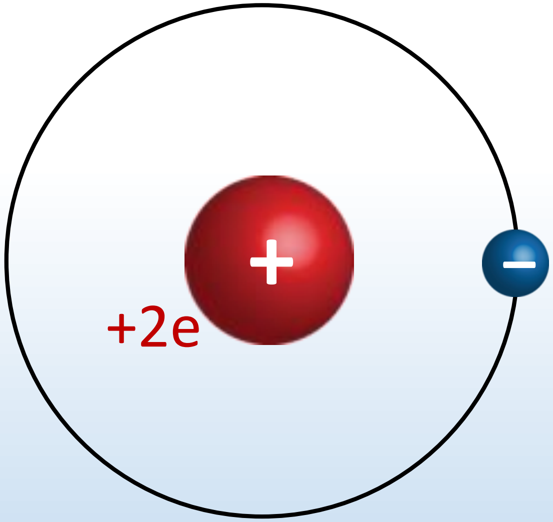

## Energy is *quantized*

$$E_n = -\frac{1}{2}\frac{ke^2}{r_n} \equiv -\frac{E_1}{n^2}$$

# ACT: CheckPoint 3.2

Suppose the charge of the nucleus is doubled (+2e), but the $e^-$ charge remains the same (–e). How does $r$ for the ground state ($n$ = 1) orbit compare to that in hydrogen?

For hydrogen: $r_n = \dfrac{n^2\hbar^2}{mke^2}$




A. 1/2 as large

B. 1/4 as large

C. the same

# ACT: CheckPoint 3.3

There is a particle in nature called a *muon*, which has the same charge as the electron but is 207 times heavier. A muon can form a hydrogen-like atom by binding to a proton.

How does the radius of the ground state ($n$ = 1) orbit for this hydrogen-like atom compare to that in hydrogen?

A. 207× larger

B. The same

C. 207× smaller

# Atomic units

At atomic scales, Joules, meters, kg, etc. are not convenient units

"Electron Volt" – energy gained by charge +1e when accelerated by 1 Volt: $U = qV$

1e = $1.6\times10^{-19}$ C, so 1 eV = $1.6\times10^{-19}$ J

Planck constant: $h = 6.626 \times 10^{-34}$ J·s
Speed of light: $c = 3 \times 10^{8}$ m/s

$$hc \approx 2\times10^{25}\,\mathrm{J\cdot m} = 1240\,\mathrm{eV\cdot nm}$$

Electron mass: $m = 9.1 \times 10^{-31}$ kg

$$mc^2 = 8.2\times10^{-13}\,\mathrm{J} = 511{,}000\,\mathrm{eV}$$

Since $U = \dfrac{ke^2}{r}$, $ke^2$ has units of eV·nm like $hc$

$$ke^2 \approx 1.44\,\mathrm{eV\cdot nm}$$

$$\frac{ke^2}{\hbar c} = 2\pi\frac{ke^2}{hc} \approx \frac{1}{137}$$

"Fine structure constant" (dimensionless)

# *Calculation: energy & Bohr radius*

Energy of electron in H-like atom (1 e$^-$, nuclear charge +Ze):

$$E_n = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -\frac{Z^2}{2n^2}mc^2\left(\frac{ke^2}{\hbar c}\right)^2 = -\frac{511{,}000\,\text{eV}}{2\cdot 137^2}\frac{Z^2}{n^2}$$

$$= -13.6\,\text{eV}\frac{Z^2}{n^2}$$

Radius of electron orbit:

$$r_n = \frac{n^2}{Z}a_0 = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z}\frac{\hbar c}{mc^2}\left(\frac{\hbar c}{ke^2}\right) = \frac{n^2}{Z}\frac{1240\,\text{eV}\cdot\text{nm}}{2\pi\cdot 511{,}000\,\text{eV}}137$$

$$= 0.0529\text{nm}\frac{n^2}{Z}$$

# ACT: Hydrogen-like atoms

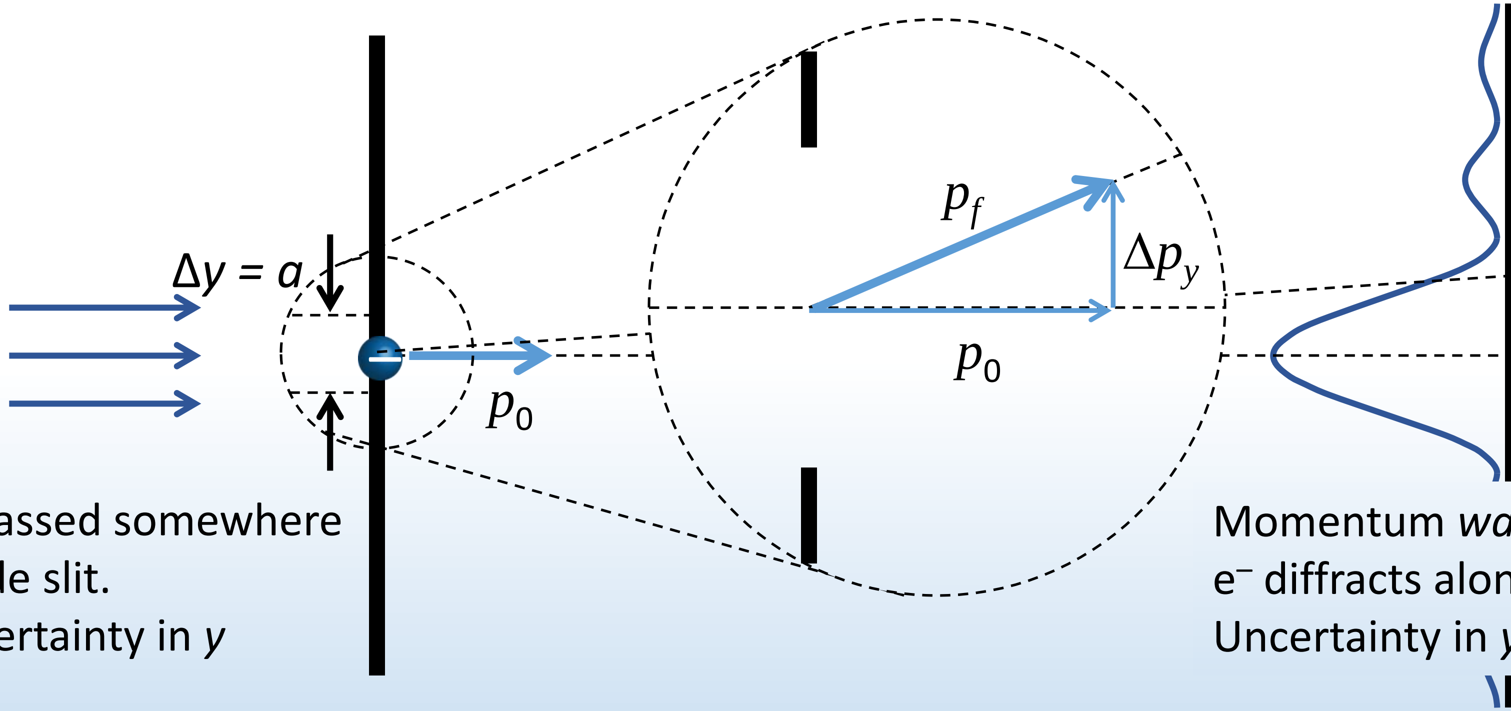
Consider an atom with a nuclear charge of +2e with a single electron orbiting, in its ground state ($n$ = 1), i.e. $He^{+}$.

How much energy is required to ionize the atom totally?

A. 13.6 eV
B. 2 × 13.6 eV
C. 4 × 13.6 eV

# Heisenberg Uncertainty Principle

$e^-$ beam with momentum $p_0$ passes through slit will diffract




$e^-$ passed somewhere inside slit.
Uncertainty in *y*

Momentum *was* along *x*, but $e^-$ diffracts along *y*.
Uncertainty in *y*-momentum

If slit narrows, diffraction pattern spreads out

Uncertainty in *y*

$$\Delta y \cdot \Delta p_y \geq \frac{\hbar}{2}$$

Uncertainty in *y*-momentum

# ACT: CheckPoint 4

The Bohr model cannot be correct! To be consistent with the Heisenberg Uncertainty Principle, which of the following properties *cannot* be quantized?

1. Energy is quantized $E_n = -\frac{E_1}{n^2}$
2. Angular momentum is quantized $L_n = p_n r_n = n\hbar$
3. Radius is quantized $r_n = n^2 a_0$
4. Linear momentum & velocity are quantized $p_n = \frac{\hbar}{n a_0}$

A. All of the above

B. #1 & 2

C. #3 & 4

# *Summary of today's lecture*

- Classical model of atom

  Predicts unstable atom & cannot explain atomic spectra

- Quantum mechanics

  Matter behaves as waves

  Heisenberg Uncertainty Principle

- Bohr model

  Only orbits that fit *n* electron wavelengths are allowed

    Explains the stability of the atom
    Energy quantization correct for single $e^-$ atoms ($H$, $He^+$, $Li^{++}$)
    However, it is *fundamentally* incorrect

Need complete quantum theory (Lect. 26)