Physics 102-1
Conservative Forces

NAME: _______________________________

LAB PARTNER(S): _______________________________

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Check the box next to the name of the person to whose report your group's data will be attached.

LAB SECTION: _______________________________

TA: _______________________________

DATE: _______________________________

DISCUSSION SECTION: _______________________________

If needed by your TA, please record:

E-MAIL ADDRESS: _______________________________

ADDITIONAL INFORMATION: _______________________________

I will now make this electron vanish
Equipment List
Colored pencils
E-field mapping tank
Stainless steel beaker
Suction bulb
E-field probe with banana jack
Two brass bars (each about 7 inches long) with banana jacks
Wires for connections in white basin
Brass ring
Fluke digital multimeter
Variac and Transformer
Level
(Read this and the appropriate sections of your textbook. Answer the questions before coming to lab)

Electrical and gravitational force laws

\[ F_{12} = G \frac{m_1 m_2}{r^2} \]

Figure 1. Newton's Universal Law of Gravitation

You are about to embark on a study of the electrical force. You will soon see, though, that the electrical force behaves in a manner quite similar to a force you already studied in Physics 101, the gravitational force. The form and structure of Newton's Law of Gravitation is shown in Figure 1.
Consider Coulomb's law, which governs the force between charged objects held a distance apart. It turns out that Coulomb's Law is structurally identical to Newton's Universal Law of Gravitation. In Physics 101 you learned that the magnitude of the force experienced by a mass $m_1$ due to the presence of a mass $m_2$ a distance $r$ away, as in Figure 1, is given by:

$$F_{12} = G \frac{m_1 m_2}{r^2} \quad \text{(Eq. 1)}$$

where $G$ is equal to $6.67 \times 10^{-11}$ N·m$^2$/kg$^2$. Thus, the force can be measured in Newtons if the masses are measured in kilograms and the distance is measured in meters.

Now compare Newton's Universal Law of Gravitation to Coulomb's Law. For charged objects, we say that the magnitude of the force on a charge $q_1$ due to the presence of a charge $q_2$, as in Figure 2, is given by

$$F_{12} = \frac{k |q_1 q_2|}{r^2} \quad \text{(Eq. 2)}$$

where $k$ is a constant that equals $9.0 \times 10^9$ N·m$^2$/C$^2$. The units of $k$ are chosen so that the force can be measured in Newtons if the charges are measured in Coulombs and the distance in meters. Note that one Coulomb of charge is equivalent to $6.3 \times 10^{18}$ single electron charges!

You might be wondering if you can have both electrical and gravitational forces operating on objects at the same time. The answer is indeed "yes!" However, in many situations one force is dominant over the other. Take for example the simplest hydrogen atom, which is composed of one electron and one proton. How do the electrical and gravitational forces between these subatomic particles compare?
Electron: \( m_e = 9.1 \times 10^{-31} \text{ kg} \)
\( q_e = -1.6 \times 10^{-19} \text{ C} \)

Proton: \( m_p = 1.7 \times 10^{-27} \text{ kg} \)
\( q_p = +1.6 \times 10^{-19} \text{ C} \)

Radius: \( r = 1.0 \times 10^{-10} \text{ m} \)

**Q3**
What is the magnitude of the electrical force on the electron from the proton? Is it attractive or repulsive?

\[ F_e = \quad \text{[N]} \]

**Q4**
What is the magnitude of the gravitational force on the electron from the proton? Is it attractive or repulsive?

\[ F_g = \quad \text{[N]} \]

**Q5**
Which force is the more dominant in the hydrogen atom, the gravitational or the electrical? By what factor? (Take the ratio.)

**Q6**
Which force are you more aware of on a daily basis? Does this answer match your answer above? Explain why this might be the case.
Electrical and gravitational fields

Gravitational or electrical fields are often used to understand the forces on masses or charges. For a point mass or a point charge we can draw so-called field lines to represent which way the force would point if we introduced a "test object" into the picture.

On the left side of Figure 4, a massive object creates and is surrounded by a gravitational field $G$. In the vicinity of this object is a "test mass." This situation is analogous to a satellite near the earth, where the satellite represents the test mass and the earth is the central massive object. While the gravitational field $G$ does not immediately tell us the force on the test mass or satellite, it does tell us the force per unit mass, or $F/m$, for any test mass we might choose. Additionally, the gravitational field has direction, as illustrated by the arrows in the figure. Thus, the concept of a gravitational field is both useful and powerful; it allows us to figure out how much force will be exerted on any test mass, and in what direction that force will be.

The same reasoning applies to electrical charges except that they can either attract or repel a "test charge." We can then conceive of an electric field $E$ which describes the force per unit charge, or $F/q$, for any test charge as shown on the right in Figure 4. Again, notice the direction of the field. The positive test charge shown in Figure 4 would naturally follow the path of a field line if set free. In instances where field lines become more complicated, this property can be quite helpful.

Q7

Draw an arrow on both the "test mass" and "positive test charge" in Figure 4, indicating the direction of the force on those objects. How did you determine the direction in each case?

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Energy and Work for a Conservative Force

By now you have encountered a variety of forces in your physics studies. The forces which hold atoms together and keep planets in orbits are both "1/r^2 forces." Accordingly, we might imagine that these forces are in some way special. The 1/r^2 forces are part of a class of forces known as "conservative" forces, and they have two important characteristics:

- work done by a conservative force acting on an object that is moving from one point to another depends only on the initial and final positions of the object, not on the path taken to travel there
- we can associate a potential energy function only for conservative forces

Work, Potential Energy, and Electric Potential

Work is done when a force acts on an object which is moving from one place to another. Mathematically, work done by a force acting on an object over a distance is defined as the projection of the force in the direction of the object's motion multiplied by the change in displacement of the object. The summation of these terms over a path from the object's initial position to its final position gives the total work done by the force on the object over the whole path length. The equation for work is

\[ W = \sum (F \cos \theta) \Delta s \]  

(Eq. 3)

where \( F \cos \theta \) represents the projection of the force vector \( F \) along the direction of motion, and \( \Delta s \) is the length of each small displacement along the path.

Consider either a uniform gravitational or electrical field as depicted in Figure 5. A massive or charged object starting somewhere at the top travels to the bottom following one of many different possible paths; we have illustrated two. Since these are conservative forces, the work done by the respective fields is the same no matter what path we take.

![Figure 5. Work done in gravitational and electrical fields](image-url)
Further, we can define a potential energy anywhere for either the massive or charged object in Figure 5. The potential energy of the object decreases when work is done by the field. The work $W_{cons}$ done by the conservative force field is related to the potential energy by

$$W_{cons} = PE_i - PE_f = -\Delta PE$$  \hspace{1cm} (Eq. 4)

Suppose we reverse the situation by having us do the work; that is, we input work against the force field by raising the object from the "final" location to the "initial" location as shown in Figure 5. As we raise the object, we will increase the potential energy of the object (we do "positive work") while, as shown in (Eq. 4), the force field does "negative work."

Given the mathematical similarity between the Coulomb force and the gravitational force, it should come as no surprise that experiments confirm that an electric field is also conservative. This means that the amount of work needed to move a charge from point A to point B is the same no matter what path is taken between the two points. A charge could be moved directly between two points or looped around in a set of elaborate circles and the work expended to take either path would be the same. In an electric field, the force is given by the expression $F = qE$ so the work done by an electric field on a charge $q$ traveling between points A and B is given by

$$W = \sum F \cos \theta \Delta s = \sum qE \cos \theta \Delta s$$  \hspace{1cm} (Eq. 5)

Recall from the previous example that as the gravitational field did work on an object, its potential energy decreased, which is the same as saying that $W = -\Delta PE$, or $\Delta PE = -W$. The same applies for electric fields, so

$$\Delta PE = -W = -\left(\sum qE \cos \theta \Delta s\right)$$  \hspace{1cm} (Eq. 6)

In dealing with work and potential energy in electric fields, it becomes rather handy to define an electric potential difference $\Delta V$ as the change in electrical potential energy $\Delta PE$ per unit charge between two points A and B, or

$$\Delta V = \frac{\Delta PE}{q} = \frac{PE_B}{q} - \frac{PE_A}{q} = V_B - V_A$$  \hspace{1cm} (Eq. 7)

You can also think of the electric potential difference as the work per unit charge that an external agent must perform to move a test charge from point A to B. The electric potential difference is also known as the voltage.

**Q8**

Are electrical potential energy and electric potential difference the same? Explain why or why not.

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Physics Lab 102-1

Conservative Forces

Investigation 1: Potentials and Equipotentials

To find out
• How to verify that the electric force is conservative
• How to map and examine the electric potential in two dimensions for a simple electrode configuration
• How to determine electric field lines from equipotential surfaces and vice versa

Preview
• Use a device to set up an electric field across a water tray and investigate the electric potential at various points in the tray
• Follow the values of the electrical potential energy per charge (voltage) along two paths in the field, and use this information to calculate the work needed to move a specific quantity of charge, measured in Coulombs, along either of these paths
• Discover a series of equipotential lines that occur in all electric fields
• Examine how the equipotentials are altered upon the introduction of a conducting ring

Figure 1. Diagram of a setup for mapping electric potential
Activity 1  Work in an Electric Field

1. Set up the tray as shown in Figure 1.
   - Use the level provided to level the water trays with at least 1 cm of water throughout.
   - Make sure that the electrodes are parallel and centered against the top and bottom walls of the tank.
   - Note that we have labeled one electrode the "positive electrode" and one the "negative electrode." They are connected to the red and black cables, respectively.

2. Set up the meter and turn on the electric field.
   - Turn the digital multimeter to read AC voltage, marked $\tilde{V}$ (we are actually using an alternating current field here for technical reasons.)
   - The multimeter has two probes and measures the electric potential difference (voltage) between the point where the red probe is touching and the point where the black probe is touching.
   - If you now place the test probe directly on top of the positive (red) electrode you will be in a position to measure the voltage difference from one electrode to the other.
   - Turn on the Variac power supply. Your meter should read 4.0 V. If it does not, you may need to adjust the Variac dial a bit or ask your TA to check your setup.

3. Try some measurements.
   - Poke the test probe into the water tank. Move it around a bit and get a feel for the voltage readings at different places in the tank.
   - Put the test probe half way between the two electrodes and record your reading here:
     Potential in the middle: __________ [V]

   **Prediction**  
   Suppose you put the test probe somewhere on the negative electrode. What do you expect the potential difference measured by Fluke digital multimeter the will be?

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4. Test your predictions.
   - Put the test probe on the negative electrode and record your reading here:
     Potential on the negative electrode : __________ [V]
Q1  Can you explain why you got this reading? What is the potential difference between two points on the same conductor? Is this true anywhere on the same conductor?

Q2  If you put 2.0 Coulombs of charge on the negative electrode (0 V) we would say it had 0 Joules of potential energy. If you put it midway between the two electrodes, how much potential energy would these same 2.0 Coulombs of charge have there? (Use the midway voltage you measured in Step 3 above in your calculation.)

Q3  How much work would you do if you moved the 2.0 Coulombs of (positive) charge from the midpoint to the positive electrode? (Again, use the midway voltage you measured in Step 3 in your calculation)
5. Calculate the work needed to move 3.0 Coulombs of charge along two different paths in our tank to a common ending point. **Remember:** if you move the charge and increase its potential energy, then you have done positive work (just like if you lift a book off of the floor). In this activity, you do positive work and increase the potential energy of the charge if you take action to oppose the electrodes (i.e., you move the charge opposite to the direction that the electrodes try to move the charge).

- Note in Figure 2 the points a, b and c.
- Measure the electrical potential values at the points a, b, and c.

Your data and formulas to calculate work along two different paths:

\[
V_a = \underline{\text{[V]}} \quad \quad \quad W_{ac} = q (V_c - V_a)
\]

\[
V_b = \underline{\text{[V]}} \quad \quad \quad W_{ab} = q (V_b - V_a)
\]

\[
V_c = \underline{\text{[V]}} \quad \quad \quad W_{bc} = q (V_c - V_b)
\]
Q4 How much work is required to move 3.0 Coulombs of charge along the direct path ac? (Use your data and show your calculations.)

Q5 How much work is required to move 3.0 Coulombs of charge along the compound and longer path ab + bc? Again, show your calculations.

Q6 How do your results compare? What does this tell you about the nature of the electric force field?

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Activity 2  Electric Field Lines and Equipotential Surfaces

It is possible that an electric charge can move along a path in an electric field without any work being done by the field? By definition, force is exerted along the direction of the field; no force is exerted along the direction perpendicular to the field, and no force means that no work is done. Thus, if we move a charge along a path which is everywhere perpendicular to $E$, no work will be done. We can envision a surface perpendicular to the field by considering a given point and looking at all of the paths which pass through this point and are perpendicular to the field. (In fact, many of the surfaces used to compute fields with Gauss's Law are examples of these surfaces.) How does the electric potential change along such a surface? By definition, $E$ points along the direction in which the potential changes the most. Perpendicular to $E$, the potential does not change at all. Consequently, all points on our perpendicular surface are at the same electrical potential, and the surface is called an equipotential surface.

You are already familiar with equipotential surfaces in a gravitational field; Figure 3 shows a common example – steps. For example, somebody on step 3 could walk anywhere along step 3 and gravity would do no work on that person. In other words, the gravitational potential energy is the same all along step 3 and therefore this step represents an equipotential surface. The other steps in the diagram are also equipotential surfaces.

![Figure 3. Equipotential steps](image-url)

The purpose of this part of the experiment is to explore the pattern of potential differences in the space between conducting equipotential surfaces. This pattern of potential differences can be related to the electric field created by charges on the conducting surfaces.
**Prediction**  
From your knowledge of electric fields and your previous measurements, predict and sketch several *electric field lines* in pencil on Figure 4.

1. Find several equipotential surfaces.
   - Use the test probe to discover a series of coordinates that all have the potential difference of 1.0 V with respect to the negative electrode.
   - Move your probe around and record coordinates directly on the sketch above. Be sure to use a different color pen or pencil from the one used for the prediction. Find and record the leftmost and the rightmost points that have readings of 1.0 V. Find at least 1 point per vertical grid line at this potential. Connect your points to form a continuous curve.
   - Repeat this procedure for 1.5 V and at least two other potential values.
Q7  Are the equipotential lines what you expected? Explain.

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2. Sketch new electric field lines.
   • Using the fact that electric field lines are always perpendicular to equipotential lines, sketch a set of electric field lines on your diagram in a different color than you have already used.
   • Put arrows on them to indicate the direction of the electric field.

3. Label all the lines.
   • Put legends on the diagrams to indicate which colors/lines are predictions and which are measurements.

Q8  Do these field lines agree with your prediction? If not, explain any differences.

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Activity 3  Rings and things

1. Add another conductor.
   • Place the conducting ring in the center of the tray between the two electrodes as in Figure 5.
2. Examine the electric potential inside the ring.
   - Move the probe around inside the ring. Record the meter's reading on the diagram for each of several points inside the ring.

Q9 Describe the electric potential inside the ring. What does this tell you about the electric field inside the ring?

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Q10 Suppose you placed a charged object in the middle of the ring. Which way, if any, would it move? Why?

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#3 ___________________________________________________________
**Prediction**  From your knowledge of electric fields, predict and sketch several electric field lines on Figure 5.

3. Check your predictions.  
   - Trace out at least five equipotential lines for a variety of potential differences in the range from 0 to 4 V. Make sure to draw a line at a potential difference of 2V, and also make sure to draw some equipotential lines close to the electrodes. Connect and sketch your equipotential lines on the diagram in a different color than what you have used previously.  
   - Label the values of your lines.  
   - Pay extra attention to what happens anywhere on the ring.  
   - Pay extra attention to what happens inside the ring.

4. Sketch the electric field lines.
   - Sketch a set of electric field lines on your diagram in a different color. Be sure to put arrows on them to indicate the direction of the electric field.

5. Label all the lines.
   - Put legends on the diagrams to indicate which colors/lines are predictions and which are measurements.

**Q11**  Explain how you knew how to draw your field lines. Do these field lines agree with your prediction? If not, explain any differences. What is different?

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**CLEAN UP CHECKLIST**

- [ ] Quit any computer programs.
- [ ] Put everything away.
- [ ] Collect the paper onto which data was taken (and annotated) and attach them to one of the lab reports.
- [ ] Make your setup look neat for the next group.
- [ ] Staple everything together, make sure you have answered all the questions and done all the activities, make sure the first page is completed with lab partners' names and check boxes, hand in your work.