

Last Name: _____ First Name _____ Network-ID _____
Discussion Section: _____ Discussion TA Name: _____

Instructions—

Turn off your cell phone and put it away.

This is a closed book exam. You have ninety (90) minutes to complete it.

1. Use a #2 pencil; do **not** use a mechanical pencil or a pen. Fill in completely (until there is no white space visible) the circle for each intended input – both on the identification side of your answer sheet and on the side on which you mark your answers. If you decide to change an answer, erase vigorously; the scanner sometimes registers incompletely erased marks as intended answers; this can adversely affect your grade. Light marks or marks extending outside the circle may be read improperly by the scanner.
2. Print your last name in the **YOUR LAST NAME** boxes on your answer sheet and print the first letter of your first name in the **FIRST NAME INI** box. Mark (as described above) the corresponding circle below each of these letters.
3. Print your NetID in the **NETWORK ID** boxes, and then mark the corresponding circle below each of the letters or numerals. Note that there are different circles for the letter “I” and the numeral “1” and for the letter “O” and the numeral “0”. **Do not** mark the hyphen circle at the bottom of any of these columns.
4. **This Exam Booklet is Version A.** Mark the **A** circle in the **TEST FORM** box at the bottom of the front side of your answer sheet.
5. Stop **now** and double-check that you have bubbled-in all the information requested in 2 through 4 above and that your marks meet the criteria in 1 above. Check that you do not have more than one circle marked in any of the columns.
6. Do **not** write in or mark any of the circles in the STUDENT NUMBER or SECTION boxes.
7. On the **SECTION line**, print your **DISCUSSION SECTION**. (You need not fill in the COURSE or INSTRUCTOR lines.)
8. Sign (**DO NOT PRINT**) your name on the **STUDENT SIGNATURE line**.

Before starting work, check to make sure that your test booklet is complete. You should have 11 12 numbered pages plus two Formula Sheets.

Academic Integrity—Giving assistance to or receiving assistance from another student or using unauthorized materials during a University Examination can be grounds for disciplinary action, up to and including dismissal from the University.

Exam Grading Policy—

The exam is worth a total of 114 points, and is composed of three types of questions:

MC5: *multiple-choice-five-answer questions, each worth 6 points.*

Partial credit will be granted as follows.

- (a) If you mark only one answer and it is the correct answer, you earn **6** points.
- (b) If you mark *two* answers, one of which is the correct answer, you earn **3** points.
- (c) If you mark *three* answers, one of which is the correct answer, you earn **2** points.
- (d) If you mark no answers, or more than *three*, you earn **0** points.

MC3: *multiple-choice-three-answer questions, each worth 3 points.*

No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **3** points.
- (b) If you mark a wrong answer or no answers, you earn **0** points.

TF: *true-false questions, each worth 2 points.*

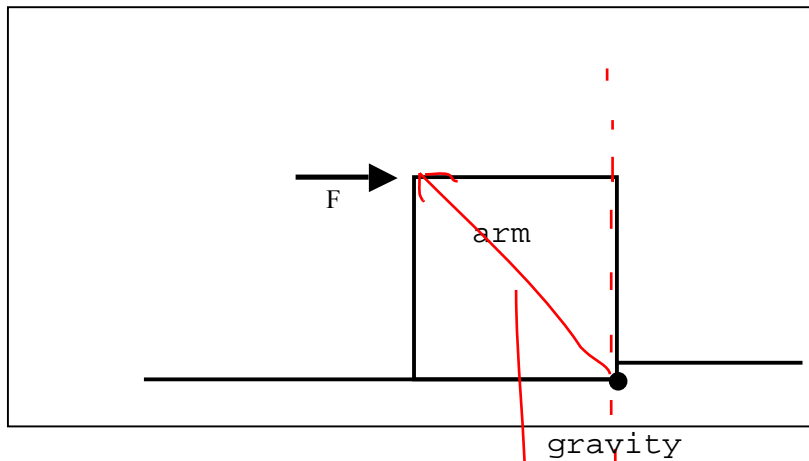
No partial credit.

- (a) If you mark only one answer and it is the correct answer, you earn **2** points.
- (b) If you mark the wrong answer or neither answer, you earn **0** points.

THE EXAM BOOKLET MUST BE TURNED IN AT THE END OF THE EXAM

Unless told otherwise, you should assume that the acceleration of gravity near the surface of the earth is 9.8 m/s^2 downward and ignore any effects due to air resistance.

The following 2 problems concern the same physical situation.



1. A solid, uniform cube 1 meter on a side is pushed against a narrow step on the floor (see figure); the block is then free to rotate about the axis marked with a solid black dot. A force $F = 13\text{N}$ is applied horizontally to the edge opposite the narrow step. What is the magnitude of the torque around the axis marked by the black dot due to the force F ?

- a. 7 Nm
- b. 9 Nm
- c. 11 Nm
- d. 13 Nm
- e. 15 Nm

|Torque| = (force) times (the length of the arm projected onto the perp direction to the force) (the direction of the dotted line)
 = 13 times 1 = 13 Nm.

2. Suppose the mass of the cube is 15 kg. What is the minimum torque required to tip the block up and cause it to rotate around the axis?

- a. 8 Nm
- b. 15 Nm
- c. 32 Nm
- d. 64 Nm
- e. 74 Nm

The largest torque is required at the moment we lift one end of the cube from the floor, because the torque due to gravity is maximum:

$$mg \text{ times } (1/2) = 15 \times 9.8 \times 0.5 = 73.5 \text{ Nm.}$$

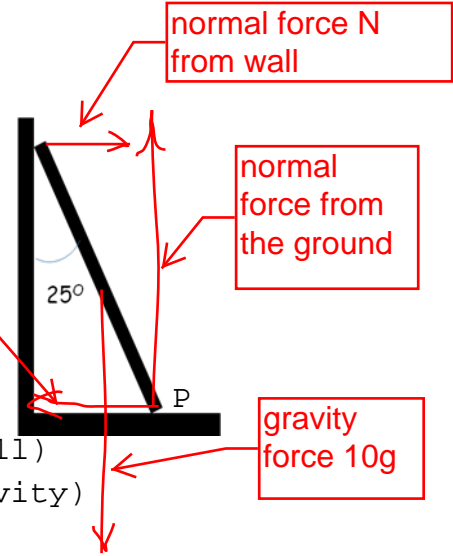
This is counterclockwise, so we need at least a clockwise torque of this magnitude.

1/2 m is the 'arm length' projected onto the direction perpendicular to the force.

In this type of problems, itemize all the forces acting on the rigid body first. Recall the 'free-body diagram.'

The following 2 problems concern the same physical situation.

3. Consider a ladder of length 5m leaning against a house at an angle of 25 deg from the vertical. The mass of the ladder is 10 kg and the interaction between the wall and the ladder is frictionless. Assume the mass of the ladder is distributed uniformly. What is the normal force from the wall on the ladder?



- a. 22.8 N
- b. 61.4 N
- c. 77.5 N
- d. 88.2 N
- e. 98.3 N

Let L be the length of the ladder.

friction force

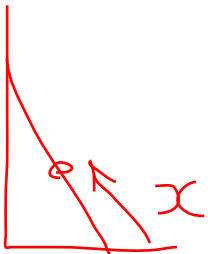
Torque around P must vanish.
 (-)N cos 25 x L (torque due to the wall)
 10g x sin 25 x (L/2) (that due to gravity)

The sum of these two torques must vanish:
 $N \cos 25 = 5g \sin 25$ or $N = 5g \tan 25 = 22.849...N.$

4. A person stands on the ladder and walks up. Does the magnitude of the normal force from the wall increase or decrease?

- a. Increases.
- b. Decreases.

This should be intuitively clear (imagine you are the wall!), but formally,
 $-NL \cos 25 + 5gL \sin 25 + MgX \sin 25 = 0,$
 where M is the mass of the person, and X its position along the ladder from P. Certainly, N increases with X.



5. The recent earthquake in Chile shortened the length of the day by 1.26 millionths of a second. Assume no external torques acting on the earth. Did the rotational energy of the earth increase or decrease?

- a. Increased.
- b. Decreased.

Two key points:
 (1) ω increased.
 (2) The total angular momentum should not change.
 Since $L = I\omega$, (1) and (2) imply I decreased.

The rotational energy is $K = L^2/2I$, so K must have increased.

The following 3 questions concern the same physical situation

6. A ball (solid uniform sphere) rolls up a plane that is inclined at an angle of 30 deg to the horizontal, without slipping. Would the angular acceleration be the same if the ball were rolling down the hill?

- a. Yes
- b. No

They must be the same.

If you do not believe this, let us calculate α .

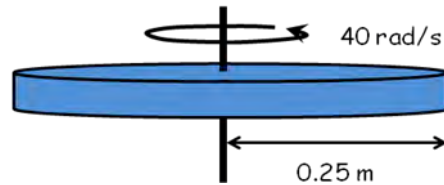
$I \alpha = R \times f$, $I = (2/5)MR^2$, so $(2/5)MR \alpha = f$.

$MR \alpha = Mg \sin 30 - f$. Therefore, $(7/5)MR \alpha = Mg \sin 30$ irrespective of f .

This is the equation for the translational motion. Here, $a = R \alpha$ is used.

The following 2 questions concern the same physical situation

7. A uniform solid disk of mass 3.2 kg and radius 0.25 m is spinning with an angular frequency of 40 rad/s around the axis at the center of mass, perpendicular to the disk. What is the magnitude of the disk's angular momentum?



- a. 1 kg m²/s
- b. 2 kg m²/s
- c. 4 kg m²/s
- d. 8 kg m²/s
- e. 16 kg m²/s

$L = I \times \omega$,

$I = (1/2) MR^2$, so

$L = (1/2) 3.2 \times (.25)^2 \times 40 = 4.0 \text{ kg m}^2/\text{s}$.

8. Suppose a frictional force of 2 N is applied at the rim of the disk. What is the magnitude of the angular acceleration of the disk?

- a. 2.5 rad/s²
- b. 5 rad/s²
- c. 7.5 rad/s²
- d. 10 rad/s²
- e. 20 rad/s²

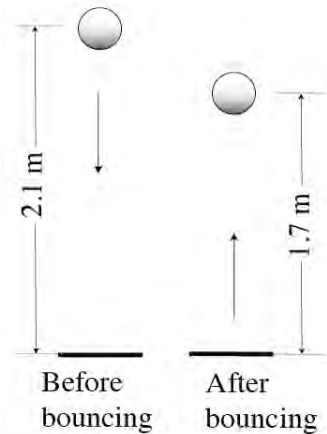
$I \alpha = \tau = R \times f$

Therefore,

$\alpha = Rf / ((1/2)MR^2) = 2f / MR = 2 \times 2 / 3.2 \times 0.25 = 5.0 \text{ rad/s}^2$

The following 3 questions concern related physical situations:

A ball falls vertically from the initial height of 2.1 m with no initial velocity to the floor and bounces vertically, reaching the highest point 1.7m from the floor as illustrated. Take the origin of the potential energy of the ball to be that on the floor. The mass of the ball



is 0.120 kg.

9. What is the loss of mechanical energy due to bouncing?

- a. 13 % of the initial potential energy.
- b. 19 % of the initial potential energy.
- c. 24 % of the initial potential energy.
- d. 28 % of the initial potential energy.
- e. 31 % of the initial potential energy.

The total mechanical energies can be seen from the maximum heights as mgh .

Before: mgh_B ,

After: mgh_A ,

so energy gain is $mgh_A - mgh_B$.

$(h_A - h_B)/h_B = -0.4/2.1 = -0.19$.

(Negative gain means loss.)

our sign convention: upward positive

10. What is the magnitude of the impulse on the ball from the floor when the ball bounces?

- a. 0.27 N·s
- b. 0.39 N·s
- c. 0.84 N·s
- d. 1.15 N·s
- e. 1.46 N·s

To compute the impulse, we must calculate the change in momentum: $\Delta p = \text{Impulse}$.

Speed just before bouncing is $\sqrt{2gh}$, so

Momentum before = $(-)(0.12) \sqrt{2 \times 9.8 \times 2.1} = (-)0.7699$,

Speed just after: $\sqrt{2 \times 1.7 \times 9.8} = 5.77$ m/s, so

Momentum after = 0.6927. Thus,

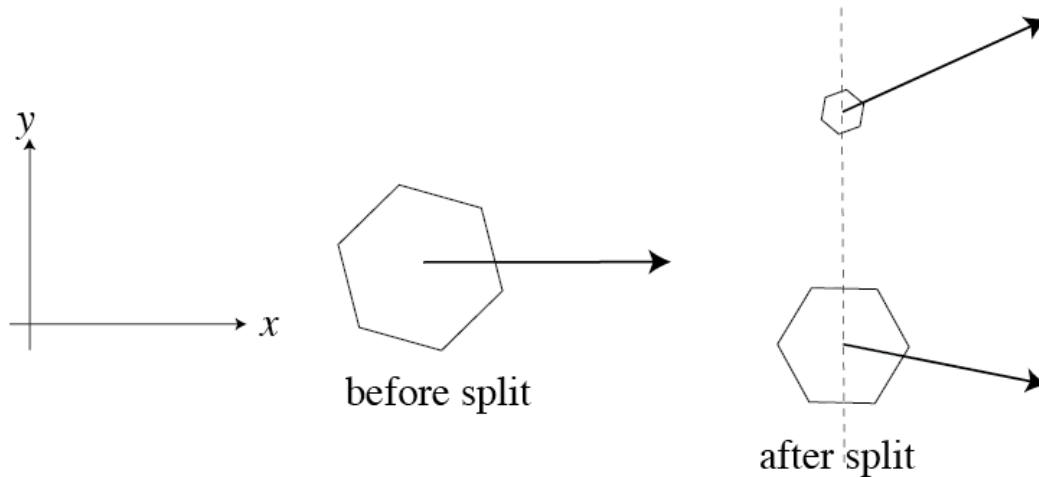
$\text{Imp} = 0.6927 - (-) 0.7699 = 1.463$ N·s.

11. Suppose instead that the mechanical energy is conserved during the bounce. Then the magnitude of the impulse on the ball from the floor would be

- a. larger than the case with energy loss.
- b. smaller than the case with energy loss.
- c. insufficient information is provided to give a definite answer.

This should be obvious, because the momentum after bouncing is larger with conservation of energy than without it.

12. A mass moving at a constant velocity in the x direction splits into two pieces without any external influence. After the split, the two pieces have velocity \mathbf{V}_1 and \mathbf{V}_2 , respectively. They have the same x -components (see the illustration; only their x -components are accurately given).



In this event

- a. Both energy and momentum are conserved.
- b. Energy is conserved, but not momentum.
- c. Momentum is conserved, but not energy.

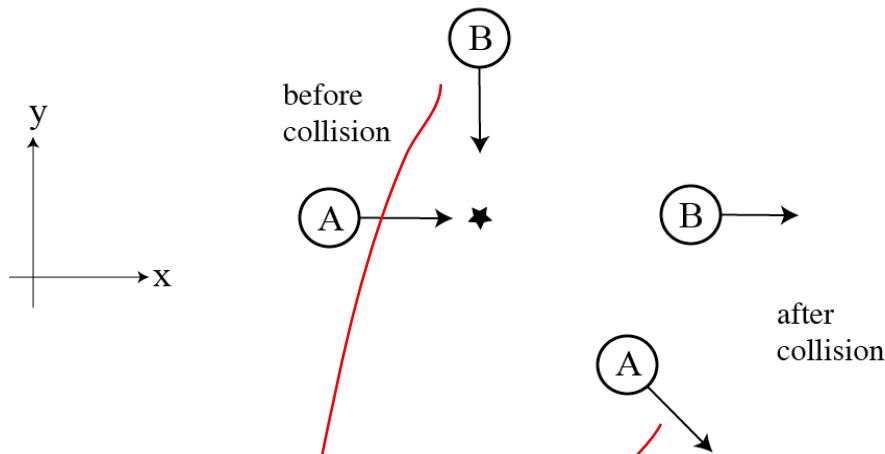
Since there is no external force, the momentum must be conserved. However, the energy need not be (perhaps due to chemical energy used to explode the mass). In this case, extra motion in the y direction requires more energy, so a does not hold.

13. There are two identical blocks (with the same mass) in a frictionless straight horizontal trough. Block B is initially stationary, and block A runs into block B. Can A reverse its direction of motion after collision?

- a. No, never.
- b. Yes.
- c. It depends on the details of the collision.

If it could, the conservation of momentum says B must be faster than A before collision. Therefore, the energy of the system must increase. The collision is a simple collision, so this is absolutely impossible.

The following 2 questions concern related physical situations:



Two dry ice pucks slide on a horizontal, frictionless surface. Puck A, which is 2 kg, moves at speed 7 m/s in the positive x direction and Puck B, which is 1 kg, moves at speed 8 m/s in the negative y direction as shown in the figure. The pucks collide near the star in the figure. After the collision Puck B is moving in the positive x direction, while Puck A is moving in the direction that makes 45 degrees with the positive x-axis as illustrated.

14. Find the speed of Puck A after collision.

- a. 1 m/s The y component of the total momentum must be conserved.
 b. 2 m/s $8 = 2 v \sin 45 = \sqrt{2} v$.
 c. $2\sqrt{2}$ m/s That is, $v = 8/\sqrt{2} = 4\sqrt{2}$ m/s.
 d. 4 m/s
 e. $4\sqrt{2}$ m/s

15. What was the magnitude of the impulse on Puck B due to Puck A when they collided?

- a. 2 N's The easiest way is to calculate the change of the B momentum vector. The speed v of B after collision may be obtained by
 b. 4 N's the conservation of the x-component of the momentum:
 c. 6 N's $2 \times 7 = 2 \times 4 + v$, or $v = 14 - 8 = 6$
 d. 8 N's The momentum of B before collision: (0, -8),
 e. 10 N's The momentum of B after collision: (6, 0).

Therefore, (6,8) is the momentum change = impulse.
 Its magnitude is 10 Ns.

A systematic solution of 14 and 15 on the next page

You must fully utilize the geometry (say, angles) supplied by the problem when you write down these vectors.

A systematic way to solve 14 and 15 is to write:

the velocity of B after collision = $(v, 0)$,

the velocity of A after collision = $(u, -u)$.

A before

B before

The conservation of momentum implies

$$2(7, 0) + (0, -8) = 2(u, -u) + (v, 0),$$

That is,

$$14 = 2u + v, \quad \leftarrow \text{x component}$$

$$-8 = -2u. \quad \leftarrow \text{y component}$$

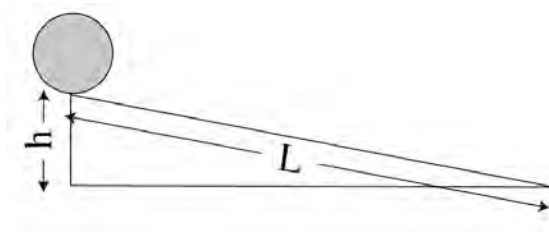
Therefore,

$$u = 4 \text{ m/s and } v = 6 \text{ m/s.}$$

The speed of A after collision is $\sqrt{2}u = 4\sqrt{2} \text{ m/s}$.

The following 2 questions concern related physical situations:

There is a slope with height h and the length of the slope $L = 20$ m as illustrated in the figure. **A** First, a solid uniform cylinder of radius $R = 0.2$ m is at the top of the slope starting from rest. It rolls down the slope without any slip.



16. When the disk reaches the end of the slope, its center of mass has a speed of 4 m/s. What is the height “ h ” of the slope?

- a. 0.73 m
 b. 0.92 m
 c. **1.22 m**
 d. 1.4 m
 e. 1.9 m
- The total kinetic energy: $K = (1/2)mv^2 + (1/2)I\omega^2$,
 and $v = R\omega$ (no slip), $I = (1/2)mR^2$.
 Thus, $K = (3/4)mv^2$. The conservation of energy implies
 $K = mgh$, so $h = 3v^2/4g = 12/9.8 = 1.2248$ m

angular acceleration

17. What is the torque on the disk around its center of mass due to the frictional force when it is rolling down the slope? You may assume that the moment of inertia of the disk is 0.03 kgm^2 .

- a. 0.03 N·m
 b. **0.06 N·m**
 c. 0.09 N·m
 d. 0.12 N·m
 e. 0.15 N·m
- The torque τ implies $I\alpha = \tau$, so we need α .
 One way is to use $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$.
 $\Delta\theta = L/R$ in radians; $\omega_0 = 0$.
 $\alpha = R\omega^2/2L$. Here, $\omega = 4/R = 20$.
 $\alpha = 0.2 \cdot 20^2/40 = 2$, so
 $\tau = I\alpha = 0.03 \times 2 = 0.06$ Nm.

This is a rather hard problem.

no energy loss

18. Consider two ways, A and B, for a ball to move from a height $z = h$ to the ground at $z = 0$: A) dropping in free fall, and B) rolling without slipping down a plane that has an angle α with the horizontal plane (the ground). Which of the following is true instantaneously before the ball hits the ground?

- a. The kinetic energy of case A is greater than case B.
 b. The kinetic energy of case B is greater than case A.
 c. **The kinetic energy in both cases is the same.**

K is equal to the initial potential energy in both cases.

19. Two students, George and Steve, run up a flight of stairs. Each student expends the same power to complete this task. George runs up in 5 seconds, and Steve runs up in 3 seconds. George weighs 200 pounds. How much does Steve weigh?

- a. 120 pounds
- b. 200 pounds
- c. 240 pounds
- d. 300 pounds
- e. 333 pounds

Let P be the power. Then, the energy spent are:

$$\text{George } 5P = M_G g h,$$

$$\text{Steve } 3P = M_S g h.$$

These are their final potential energies.

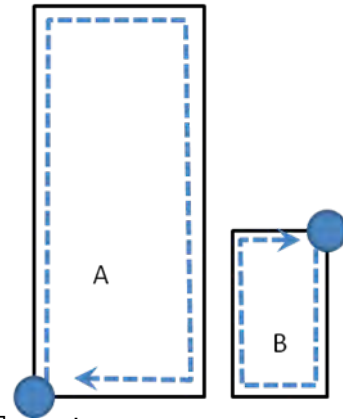
Therefore, $3P/M_S = 5P/M_G$ or

$$M_S = M_G (3/5) = 120 \text{ pounds.}$$

20. A mass (the solid ball in the figure) is circulated at a constant speed completely around two different paths (see figure). The area enclosed in path A is 4 x the area enclosed by path B. Gravity acts vertically.

- a. More work is done by gravity in path A than in path B.
- b. More work is done by gravity in path B than on path A.
- c. The work done by gravity in both cases is identical.

The potential energy difference is the work done by gravity. In both cases, there is no potential energy change. No work has been done!



21. Two brass balls with the same diameter roll down an inclined plane. They both start rolling down the ramp at the same time from the same place at the top of the ramp. Ball A is solid brass, and Ball B is a hollow brass ball with a shell thickness only 1/16th of the radius. Both balls start from rest. Which ball arrives at the bottom first?

- a. Ball A
- b. Ball B
- c. They both arrive at the bottom at the same time.

h is the initial height

The conservation of energy tells us

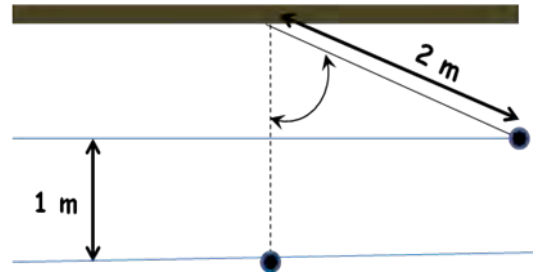
$$(1/2) Mv^2 + (1/2)I(v/R)^2 = Mgh.$$

From this we can obtain the translational speed v as

$$v^2 = 2gh/(1 + I/MR^2).$$

This equation tells us that the larger I/MR^2 , the slower the translational speed. This ratio is larger for B than for A, so B is slower.

22. A pendulum consists of a point mass of 5 kg and is hanging from a massless string of 2 meters. You pull the point mass to a height 1 meter from the resting position, and then let the ball go. What is the angular velocity of the pendulum when the point mass is exactly at the lowest point?



- a. 0.15 rad/s
- b. 2.2 rad/s
- c. 4.4 rad/s
- d. 8.9 rad/s
- e. 19.6 rad/s

The speed V is

$$(1/2) MV^2 = Mgh,$$

so

$$V = \sqrt{2gh} = \sqrt{2 \times 9.8} = 4.43 \text{ m/s},$$

which is equivalent to $\omega = V/R = 4.43/2 = 2.214 \text{ rad/s}$.

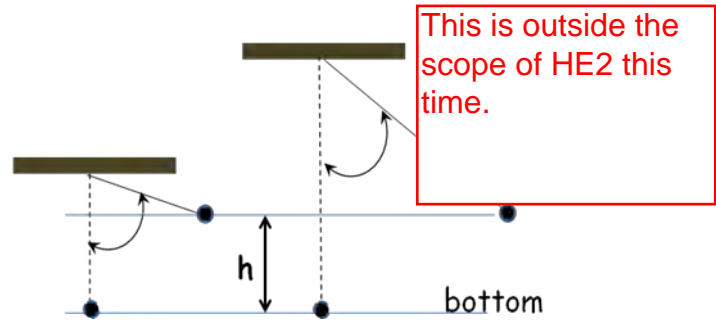
23. Consider two solid brass cylinders; cylinder A has a radius of 10 cm, and cylinder B has a radius of 20 cm. The cylinders have the same length. Both cylinders are released from the top of an inclined plane. When the cylinders reach the bottom of the plane, which cylinder has the greatest rotational kinetic energy?

- a. cylinder A
- b. cylinder B
- c. Both have the same rotational kinetic energy.

B is definitely heavier, so it must have larger kinetic energy than A. The ratio of the translational and rotational kinetic energies is determined by geometry or I/MR^2 , which is identical for A and B. That is, for both A and B K_T/K_R ratio is the same.

Therefore, B must have larger rotational kinetic energy than A.

24. Consider two pendulums with identical balls hanging from massless strings. The length of the smaller pendulum is $\frac{1}{2}$ that of the larger pendulum and both balls at the minimum have the same distance to the ground. If both balls are raised to the same height “h” (compared to their minimum height – that is, the bottom), which of the following statements is true?

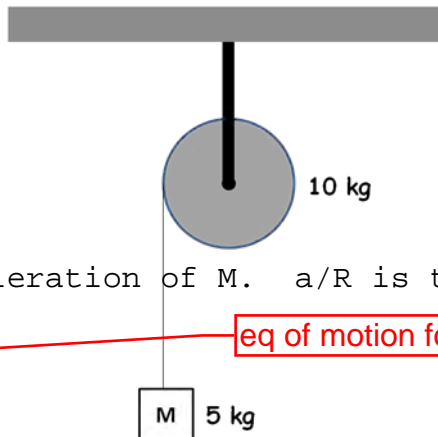


- a. The angular velocity of the smaller pendulum is greater than the angular velocity of the larger pendulum at the minimum position of the swinging pendulum.
- b. The translational velocity of the larger pendulum is larger than the translational velocity of the smaller pendulum ball at the minimum position of the swinging pendulum.
- c. The angular velocities of both pendulums are identical at the minimum position of the swinging pendulum.

The translational velocity is $v = \sqrt{2gh}$, identical for both.
 $\omega = v/L$; Shorter L means larger ω .

25. A freely hanging block of mass 5 kg is attached to a massless string, which is wrapped several times around a pulley (solid cylinder) of mass 10 kg and radius 0.1 m (see diagram). There is no friction in the pulley mechanism. What is the acceleration of the block?

- a. 2.9 m/s²
- b. 3.9 m/s²
- c. 4.9 m/s²
- d. 5.9 m/s²
- e. 6.9 m/s²



Let a be the DOWNWARD acceleration of M. a/R is the angular acceleration for the pulley.

no slip $\rightarrow 5a = 5g - T$, ← eq of motion for M

$I(a/R) = RT$, ← eq of motion for the pulley

where $I = (1/2)10R^2$.

Therefore, the second equation reads

$$(1/2)10 a = T.$$

Combining this and the first equation, we get $10a = 5g$. That is, $a = g/2 = 4.9 \text{ m/s}^2$.

**Check to make sure you bubbled in all your answers.
 Did you bubble in your name, network-ID, and exam version?**