Chapter 5: Radiation Dosimetry

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Units for Exposure and Dose

Units for Absorbed Dose

- **Radiation damage** depends on the energy absorption from the radiation and is **approximately proportional to the concentration of absorbed energy in tissue**.
- The basic unit of radiation dose is expressed in terms of absorbed energy per unit mass of tissue, which is called **Gary (Gy)**

$$1Gy = 1J / Kg = 100rad$$

where Rad stands for Radiation Absorbed Dose, which is a non-SI unit.

- The Gary is universally applicable to all types of ionizing radiation dosimetry – irradiation due to external field of gamma rays, neutrons or charged particles as well as that due to internally deposited radioisotopes.

The SI Unit for Exposure

- For external radiation of any given energy flux, the absorbed dose to any point within an organism depends on the type and energy of radiation, the depth within the organism of the point at which the absorbed dose is required and the elemental composition of the absorbing medium at that point.
- The x-ray fields to which an organism may be exposed is normally specified in **exposure unit**.

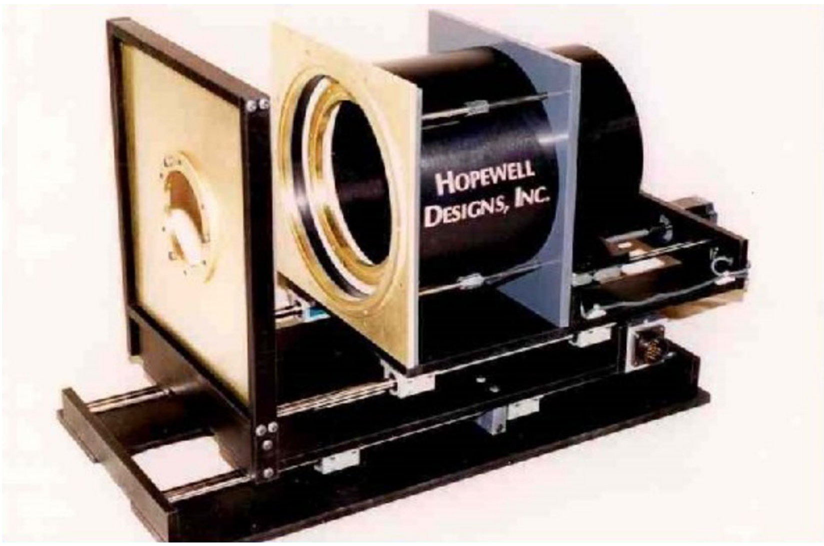
$$1\ X\ unit = 1C / Kg\ \text{air}$$

$$\begin{aligned} 1\ \text{X unit} &= 1\frac{\text{C}}{\text{kg air}} \times \frac{1\ \text{ion}}{1.6 \times 10^{-19}\ \text{C}} \times 34\frac{\text{eV}}{\text{ion}} \times 1.6 \times 10^{-19}\frac{\text{J}}{\text{eV}} \times 1\frac{\text{Gy}}{\text{J/kg}} \\ &= 34\ \text{Gy (in air)}. \end{aligned}$$

- Alternatively, exposure is also measured with the unit Roentgen (R),

$$1\ \text{R} = 2.58 \times 10^{-4}\ \text{C} \cdot \text{kg}^{-1}$$

Exposure Measurement: The Free Air Chamber



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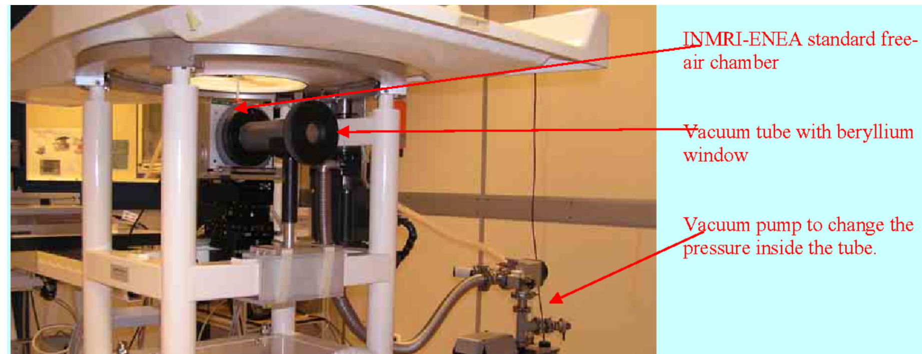


Figure 10. *Experimental set-up for the determination of the mass-air attenuation coefficients of SYRMA monochromatic x-ray beams*

ABSOLUTE AIR-KERMA MEASUREMENT IN A SYNCHROTRON RADIATION BEAM BY IONIZATION FREE-AIR CHAMBER

R. Laitano, M. Pimpinella, +7 authors A. Vascotto • Published 2007

A dosimetric procedure to assure the traceability to the air-kerma standard of the measurements performed in synchrotron radiation beams, was implemented at the "Istituto Nazionale di Metrologia delle Radiazioni Ionizzanti" of ENEA (INMRI-ENEA). To this purpose, absolute air-kerma measurements by the ENEA standard free-air chamber for low energy x-rays have been performed at the SYRMEP (SYnchrotron Radiation for Medical Physics) beamline of the ELETTRA light source in Trieste (Italy) where a... CONTINUE READING

Exposure Measurement: The Free Air Chamber

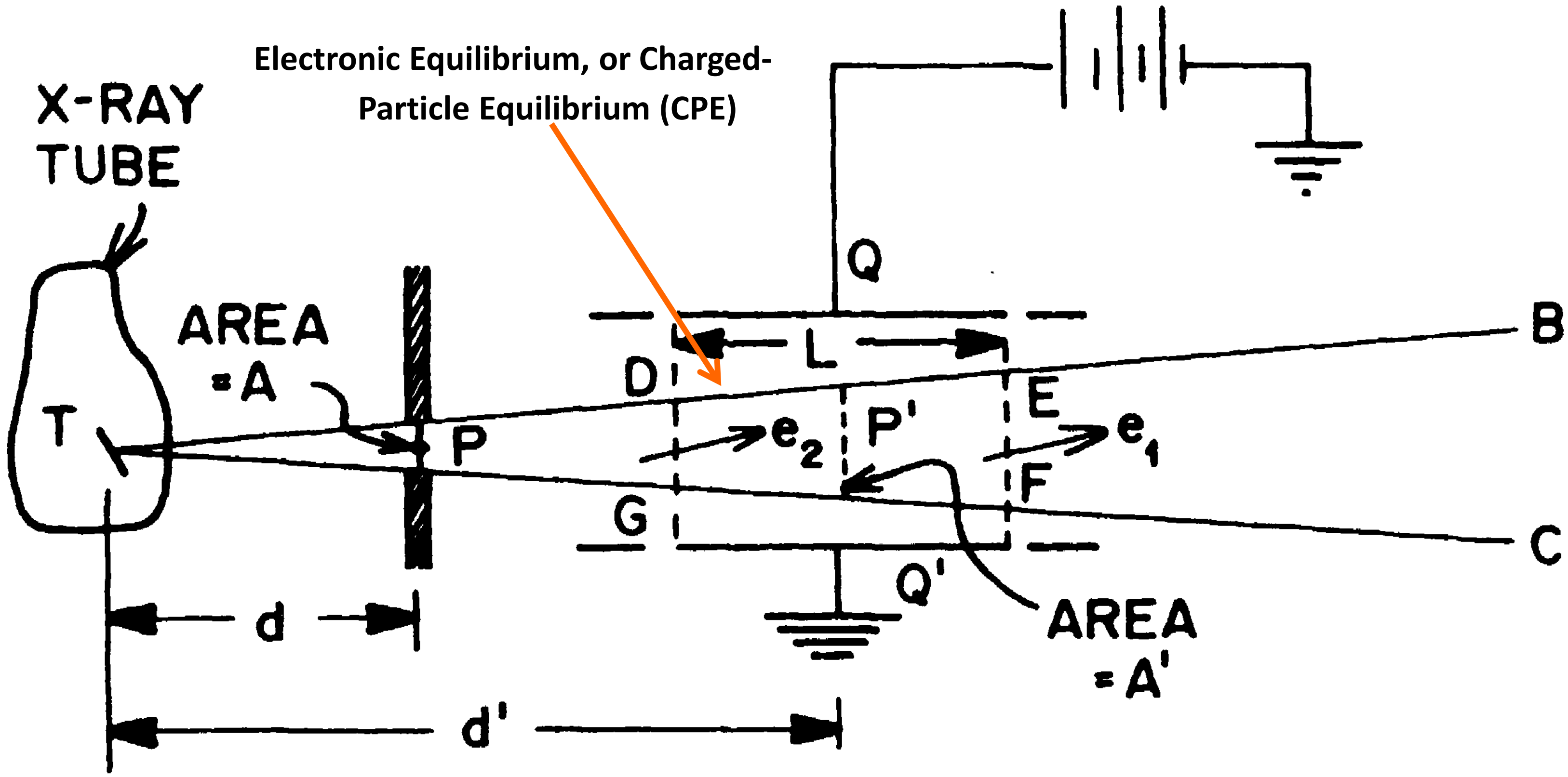


Fig. 12.1 Schematic diagram of the "free-air" or "standard" ionization chamber.

Turner, pp. 365.

Charged-Particle Equilibrium (CPE)

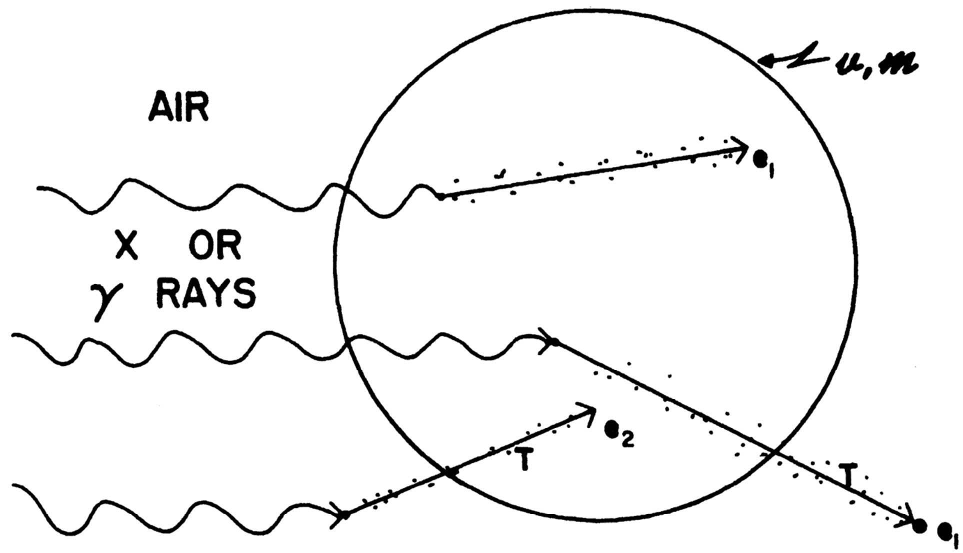


FIGURE 4.5. The role of CPE in the measurement of exposure *X*. The average exposure in the finite air volume *v* equals the total charge of either sign released in air by all electrons (e_1) that originate in *v*, divided by the air mass *m* in *v*. If CPE exists, each electron carrying an energy (say, *T*) out of *v* is compensated by another electron (e_2) carrying the same energy in. Thus the same ionization occurs in *v* as if all electrons e_1 remained there. The measurement of that charge divided by *m* is thus equivalent to a measurement of the average exposure in *v*. Radiative losses are assumed to escape from *v*, and any ionization they produce is not to be included in *X*.

From <<Introduction to Radiological Physics and Radiation Dosimetry>>

Exposure Measurement: The Free Air Chamber

The exposure in the volume $DEFG$ in roentgens would be determined directly if the total ionization produced only by those ions that originate from X-ray interactions in the truncated conical volume $DEFG$ could be collected and the resulting charge divided by the mass of air in $DEFG$. This mass is given by $M = \rho A' L$, where ρ is the density of air, A' is the cross-sectional area of the truncated cone at its midpoint P', and L, the thickness of the cone, is equal to the length of the collecting plates Q and Q'. Unfortunately, the plates collect all of the ions between them, not the particular set that is specified in the definition of the roentgen. Some electrons produced by X-ray interactions in $DEFG$ escape this volume and produce ions that are not collected by the plates Q, Q'. Also, some ions from electrons originally produced outside $DEFG$ are collected. Thus, only part of the ionization of an electron such as e_1 in Fig. 12.1 is collected, while ionization from an "outside" electron, such as e_2, is collected. When the distance from P to DG is sufficiently large (e.g., $\sim$ 10 cm for 300-keV X rays) electronic equilibrium will be realized; that is, there will be almost exact compensation between ionization lost from the volume $DEFG$ by electrons, such as e_1, that escape and ionization gained from electrons, such as e_2, that enter. The distance from P to DG, however, should not be so large as to attenuate the beam significantly between P and P'. Under these conditions, when a charge q is collected, the exposure at P' is given by

$$E_{P'} = \frac{q}{\rho A' L}. \tag{12.6}$$

Relationship between Exposure and Dose

The SI Unit for Exposure

- For external radiation of any given energy flux, the absorbed dose to any point within an organism depends on the type and energy of radiation, the depth within the organism of the point at which the absorbed dose is required and the elemental composition of the absorbing medium at that point.
- The x-ray fields to which an organism may be exposed is normally specified in **exposure unit**.

$$1\ X\ unit = 1C/Kg \text{ air}$$

$$1 \text{ X unit} = 1\frac{\text{C}}{\text{kg air}} \times \frac{1 \text{ ion}}{1.6 \times 10^{-19} \text{ C}} \times 34\frac{\text{eV}}{\text{ion}} \times 1.6 \times 10^{-19}\frac{\text{J}}{\text{eV}} \times 1\frac{\text{Gy}}{\text{J/kg}}$$
$$= 34 \text{ Gy (in air)}.$$

- Alternatively, exposure is also measured with the unit Roentgen (R),

$$1 \text{ R} = 2.58 \times 10^{-4} \text{ C} \cdot \text{kg}^{-1}$$

Calculation of Energy Transfer and Energy Absorption (Revisited)

Assuming $\mu_{en}x<<1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0(\mathrm{J}\cdot\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\Delta\dot{\Psi} = \dot{\Psi}_0 - \dot{\Psi} = \dot{\Psi}_0(1 - e^{-\mu_{en}x}) \approx \dot{\Psi}_0\mu_{en}x \ \ (J\cdot cm^{-2}\cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness x is

$$A\dot{\Psi}_0\mu_{en}x\ (J\cdot s^{-1})$$

Given the density of the material is ρ, the rate of energy absorption per unit mass (Dose Rate) in the slab is

$$\dot{D} = \frac{A(cm^2)\cdot\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\cdot\mu_{en}(cm^{-1})\cdot x\ (cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$\textit{Dose rate in the absorber:}\ \ \dot{\boldsymbol{D}}(J\cdot g^{-1}\cdot s^{-1}) = \dot{\boldsymbol{\Psi}}_{\mathbf{0}}(J\cdot cm^{-2}\cdot s^{-1})\frac{\boldsymbol{\mu}_{\boldsymbol{en}}(cm^{-1})}{\boldsymbol{\rho}(g/cm^3)}$$

Does-Exposure Relationship

- Another example (Cember, p178)

Consider a gamma-ray beam of quantum energy 0.3 MeV. If the photon flux 1000 quanta/cm^2/s, and the air temperature is 20°C, what is the exposure rate at a point in this beam and what is the absorbed dose rate for soft tissue at this point?

The exposure rate in C/kg/s is given by

$$\dot{X} = \frac{\phi \text{ photons/cm}^2\text{-s} \times E \text{ MeV/photon} \times 1.6 \times 10^{-13} \text{ J/MeV} \times \mu_a \text{ cm}^{-1}}{\rho_a \text{ kg/cm}^3 \times 34 \dfrac{\text{J/kg}}{\text{C/kg}}}$$

where

μ_a is the linear energy absorption coefficient for air for the photon energy and
ρ_a is the density of air.

ϕ is the photon flux (photons/cm^2/s).

Does-Exposure Relationship

The absorbed dose rate, in grays per second, is given by

$$\dot{D} = \frac{\frac{\phi \text{ photons/cm}^2}{\text{s}} \times E \text{ MeV/photon} \times 1.6 \times 10^{-13} \text{ J/MeV} \times \mu_m \text{ cm}^{-1}}{\rho_m \text{ kg/cm}^3 \times \frac{\text{J/kg}}{\text{Gy}}}$$

$$\dot{D} = \frac{\phi \cdot E \cdot \mu_m}{\rho_m}$$

where

μ_{m} is the linear energy absorption coefficient of the medium and
ρ_{m} is the density of the medium.

Does-Exposure Relationship

- The relationship between exposure and dose is obtained from the ratio of the absorbed dose rate and the exposure rate,

$$\frac{\dot{D}}{\dot{X}} = \frac{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_m)/\rho_m}{(\phi \times E \times 1.6 \times 10^{-13} \times \mu_a)/(\rho_a \times 34)}$$

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X} \text{ Gy/s}.$$

- For an X-ray flux that could induce 1 X-unit of exposure in air, it could lead to the following amount of dose in a given tissue

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a}$$

μ_m is the linear energy absorption coefficient of the medium and
ρ_m is the density of the medium.

μ_a is the linear energy absorption coefficient for air for the photon energy and
ρ_a is the density of air.

Does-Exposure Relationship

$$\dot{D} = 34 \times \frac{\mu_m/\rho_m}{\mu_a/\rho_a} \times \dot{X} \text{ Gy/s}.$$

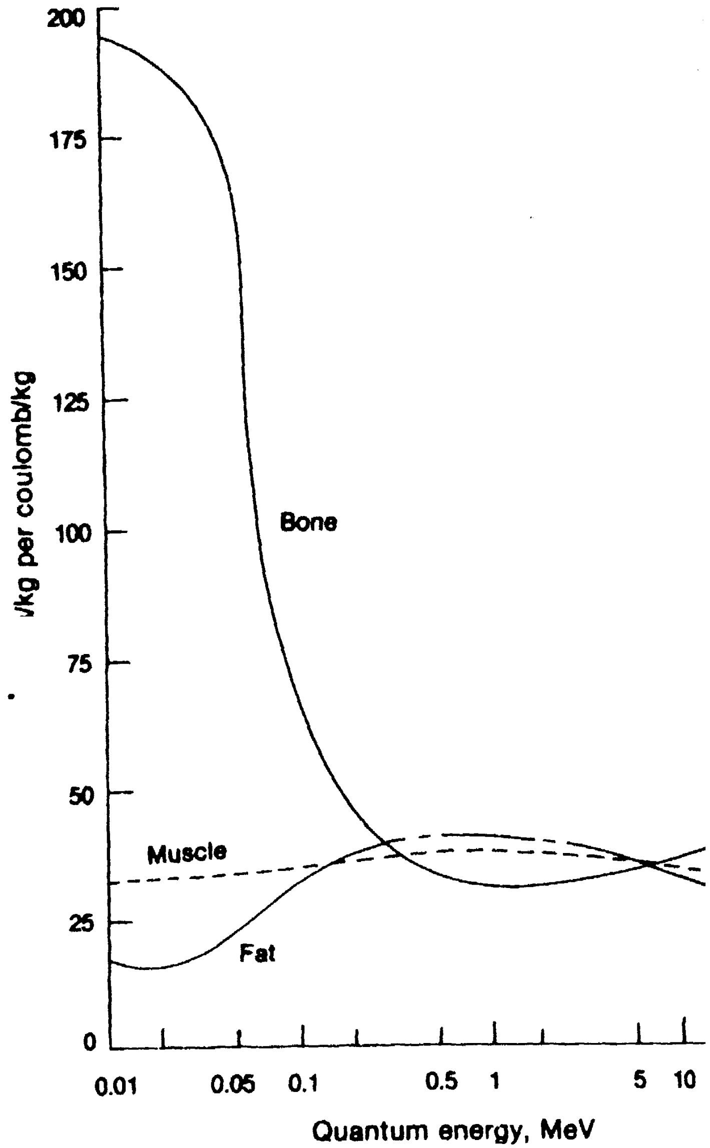


FIGURE 6.6. Energy absorption per X unit (coulomb per kilogram) exposure for several tissues. (Adapted from O. Glasser, *Medical Physics*, Vol. II. Yearbook Publishers, Chicago, IL, 1950. By permission.)

How is the human dose typically measured?

The Bragg-Gray Principle

Exposure Measurement: The Air Wall Chamber

Example 6.3

Chamber volume = 2 cm^3.

Chamber filled with air at STP.

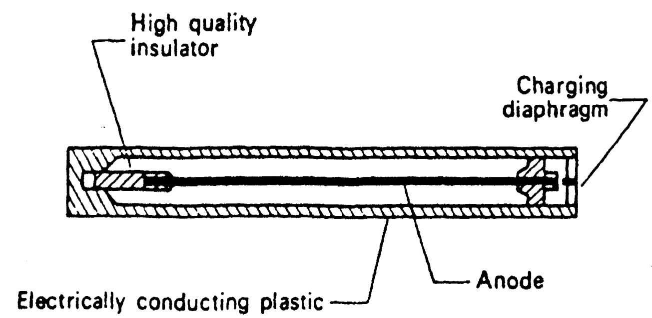
Electrical capacity = 5 $\mu\mu$F.

Voltage across chamber before exposure to radiation = 180 V.

Voltage across chamber after exposure to radiation = 160 V.

Exposure time $= \frac{1}{2}$ h.

Calculate the radiation exposure and the exposure rate.



Solution

The exposure is calculated as follows:

$$C \times \Delta V = \Delta Q \tag{6.8}$$

$$5 \times 10^{-12} \text{ farads} \times (180 - 160) \text{ volts} = 1 \times 10^{-10} \text{ coulombs.}$$

Since one exposure unit is equal to 1 C/kg, the exposure measured by this chamber is

$$\frac{1 \times 10^{-10}\ \text{C}}{2\ \text{cm}^3 \times 1.293 \times 10^{-6}\ \text{kg/cm}^3} = 3.867 \times 10^{-5}\ \text{C/kg},$$

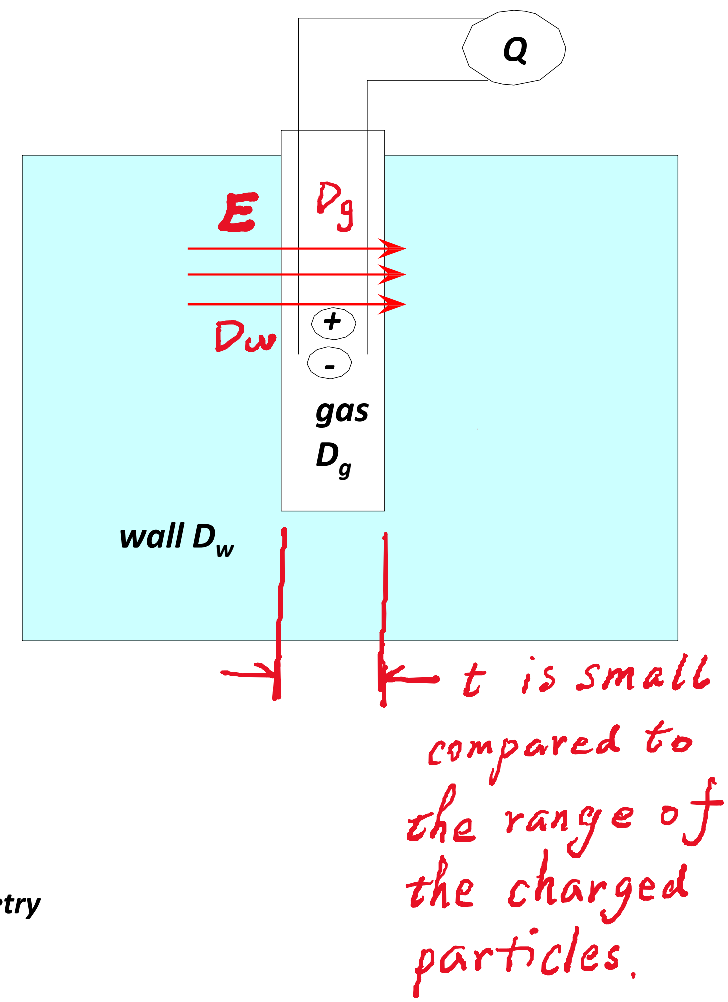
which corresponds to

$$3.867 \times 10^{-5}\ \text{C/kg} \times 3881 \frac{\text{R}}{\text{C/kg}} = 0.150\ \text{R},$$

Bragg-Gray Principle: Problem Statement

- Homogeneous medium, wall (w)
- Probe - cavity - thin layer of gas (g)
- Charged particles crossing w-g interface
- Objective: find a relation between the dose

  in a probe to that in the medium
- Basis for dosimetry

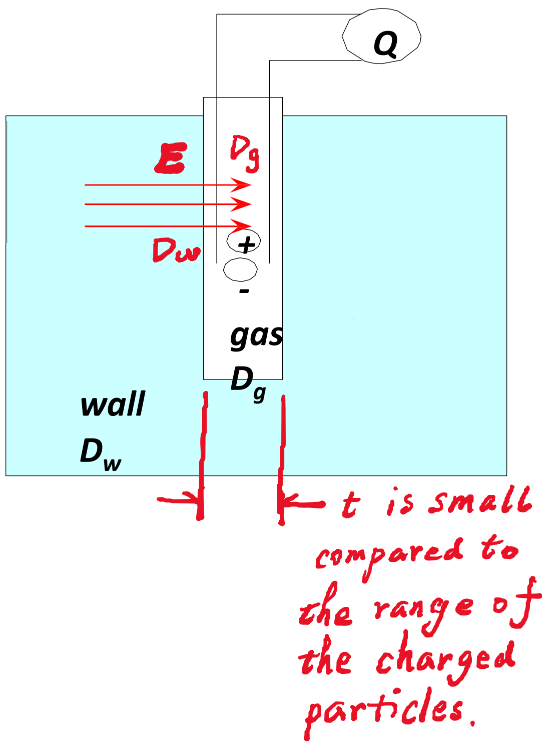


F.A. Attix, Introduction to Radiological Physics and Radiation Dosimetry

Bragg-Gray Principle

If we look at the very thin layers of wall media immediately adjacent to the interface, then the flux of the charged particles is almost unchanged across the boundary. The dose rate to the wall is given by

$$\dot{D}_w\left(\frac{J}{g\cdot s}\right)=\phi\left(\frac{particles}{s\cdot cm^2}\right)\cdot E\left(\frac{J}{particle}\right)\cdot \mu_w(cm^{-1})/\rho_w\left(\frac{g}{cm^3}\right),$$



where μ_w is the linear energy absorption coefficient.

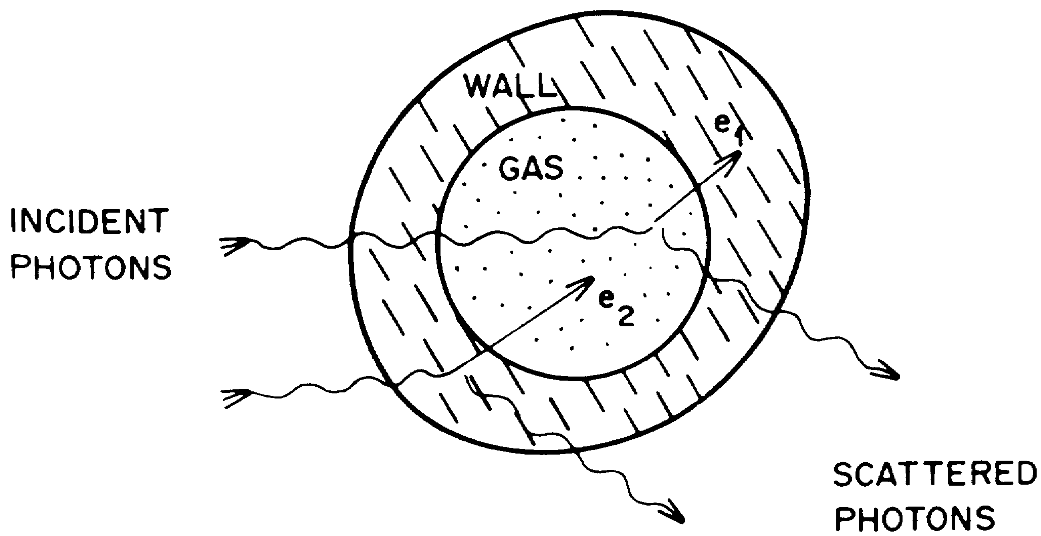
Then the ratio of dose (rate) in the wall and in the gas is

$$\frac{\dot{D}_w}{\dot{D}_g}=\frac{D_w}{D_g}=\frac{\mu_w/\rho_w}{\mu_g/\rho_g}$$

For charged particles, the <u>linear energy absorption coefficient μ</u> is roughly the same as the <u>linear stopping power</u>, $s=\frac{\mathrm{d}E}{\mathrm{d}x}$, therefore

$$\frac{\dot{D}_w}{\dot{D}_g}=\frac{D_w}{D_g}=\frac{\mu_w/\rho_w}{\mu_g/\rho_g}=\frac{s_w/\rho_w}{s_g/\rho_g}\ .$$

Absorbed Dose Measurement: Bragg-Gray Principle



Conditions for Reaching the Electronic Equilibrium

- ☞ Dimension of the gas volume is small compared to the range of the secondary charged particles.
- ☞ Wall thickness > maximum range of secondary charged particles.
- ☞ Wall thickness is not great enough to significantly attenuate the incident radiation.
- ☞ Wall and gas have similar atomic compositions.

Absorbed Dose Measurement: Bragg-Gray Principle

- The Bragg-Gray principle provides a means of relating ionization measurements in a gas volume to the absorbed dose in some convenient materials (such tissue equivalent materials) from which a dosimeter can be fabricated.
- If the gas cavity is surrounded by a wall medium of proper thickness to establish electronic equilibrium, then the energy absorbed per unit mass of the wall, dE_m/dM_m, is related to the energy absorbed per unit mass of gas, dE_g/dM_g, by

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

where S_m is the main mass stopping power of the wall medium and S_g is the mass stopping power of the gas to the secondary electrons.

Measurement of X- and Gamma Ray Dose

☞ For gamma rays with different energies

$$\frac{dE_m}{dM_m} = \frac{S_m}{S_g} \times \frac{dE_g}{dM_g}$$

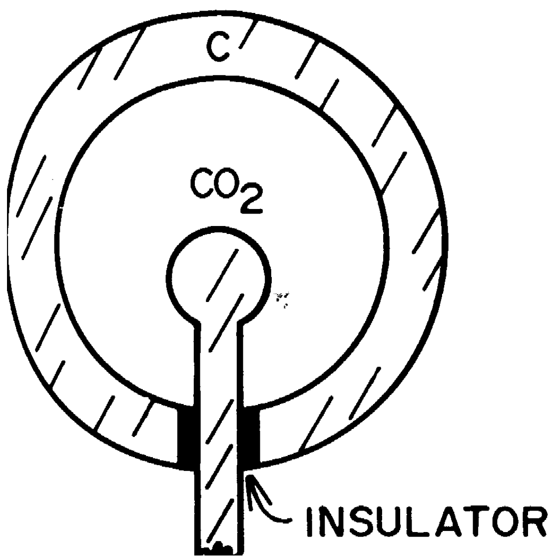


FIGURE 12.4. Cross section of graphite-walled CO_2 chamber for measuring photon dose.

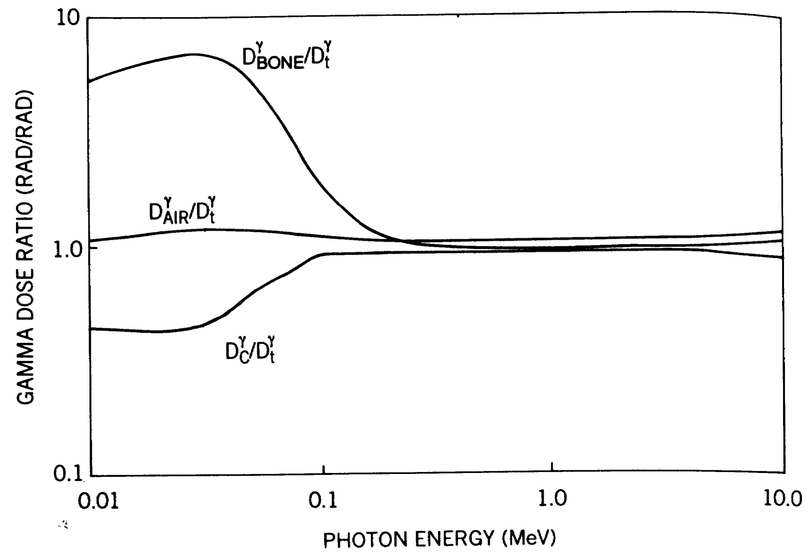


FIGURE 12.5. Ratio of absorbed doses in bone, air, and carbon to that in soft tissue, D_t.

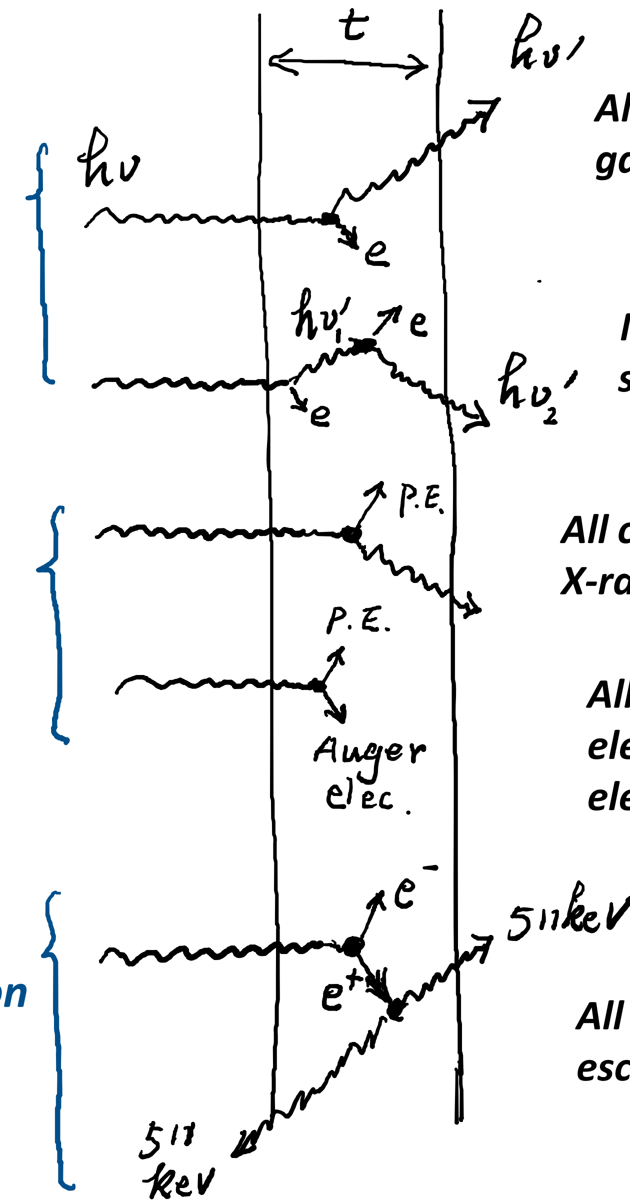
Kinetic Energy Released per Unit Mass (Kerma)

Energy Transfer by a Gamma Ray Beam

Compton scattering

Photoelectric effect

Pair production



All Compton scattered gamma rays escaped

Multiple Compton scattering ignored

All characteristic X-rays escaped

All photoelectrons, auger electrons and Compton recoil electrons are absorbed

All annihilation gamma rays escaped

Kinetic Energy Released per Unit Mass (Kerma)

- **Kerma**: Initial kinetic energy of the "primary" ionizing particles (including the photoelectrons, positron-electron pairs, recoil electrons and the scattered nuclei in case of fast neutrons) produced by the interaction of incident radiation per unit mass of the interacting medium.
- Measured in Gy (Joules per kilogram).
- An example (Cember, p183)

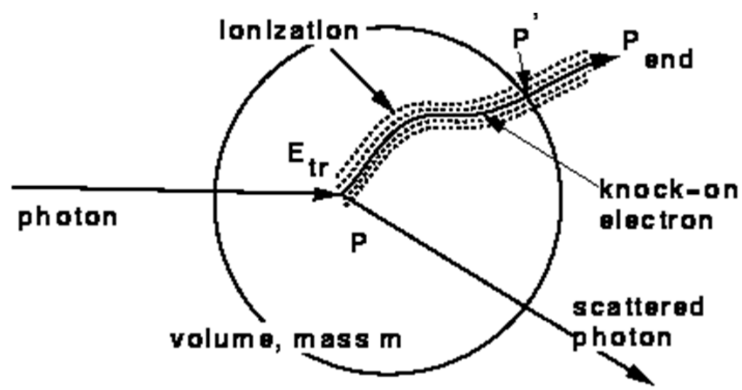


Figure 1: The exposure, air kerma and absorbed dose for a single photon which Compton scatters and transfers an energy $\mathbf{E_{tr}}$ to an electron at point **P**. The volume of interest is shown as a circle and the mass of this volume is m. The energetic electron set in motion at **P** slows down and stops at $\mathbf{P_{end}}$. As it slows down it loses energy which results in 30 ion pairs being created near the track, per keV of energy lost.

Kinetic Energy Released per Unit Mass (Kerma)

Example 6.6

A 10-MeV photon penetrates into a 100-g mass, and a pair-production interaction leads to a positron and an electron of 4.5 MeV each. Both charged particles dissipate all their kinetic energy within the mass through ionization and bremsstrahlung production. Three bremsstrahlung photons of 1.6, 1.4, and 2 MeV each are produced and escape from the mass before they interact. The positron, after expending all its kinetic energy, interacts with an ambient electron as they mutually annihilate one another to produce two photons of 0.51 MeV each. Calculate

(a) The kerma
(b) The absorbed dose.

Kinetic Energy Released per Unit Mass (Kerma)

(a) Kerma is defined as the *sum of the initial kinetic energies per unit mass of all charged particles produced by the radiation*. In this case, a positron-negatron pair of 4.5 MeV each (2 × 4.5 MeV) represents all the initial kinetic energy:

$$K = \frac{2 \times 4.5\ \text{MeV} \times 1.6 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}}}{0.1\ \text{kg} \times 1\ \frac{\text{J}}{\text{kg}} \Big/ \text{Gy}} = 1.44 \times 10^{-11}\ \text{Gy}.$$

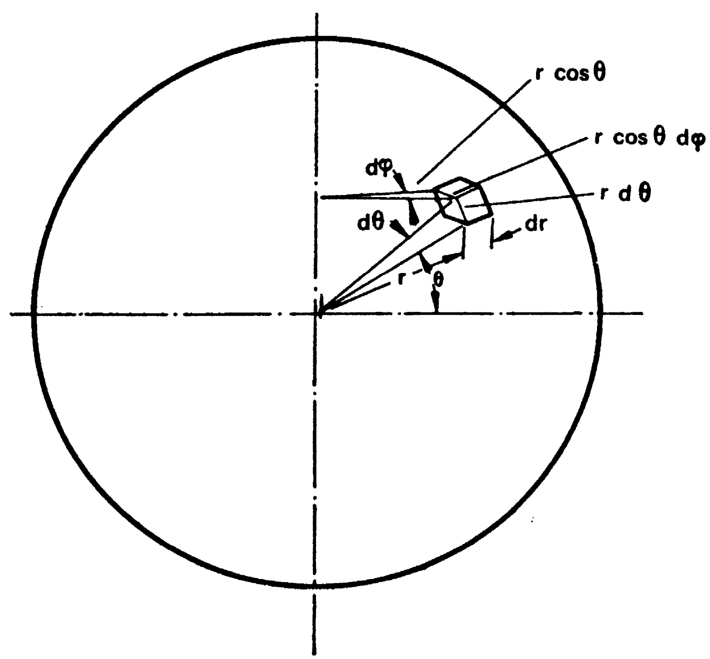
(b) Dose is defined as the *energy absorbed per unit mass*. Here we have th MeV of initial kinetic energy, of which (1.6 + 1.4 + 2) MeV was converted i bremsstrahlung and escaped from the mass. The absorbed dose, therefore is

$$D = \frac{(4.5 + 4.5)\ \text{MeV} - (1.6 + 1.4 + 2)\ \text{MeV}}{0.1\ \text{kg} \times 1\ \frac{\text{J}}{\text{kg}} \Big/ \text{Gy}} \times 1.6 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}} = 6.4 \times 10^{-12}\ \text{Gy}.$$

Radiation Dose Induced by Gamma Radiation

Specific Gamma-Ray Constant

- **Specific Gamma Ray Constant (Γ):** The gamma ray ***exposure rate*** from a point source of a unit activity at a unit distance. It is given in unit of coulombs per kilogram per hour at 1m from a 1 MBq point source.
- It is a useful numerical quantity for predicting the exposure resultant from a given gamma ray emitting radioisotope.



Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x \ll 1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0 (\mathrm{J \cdot cm^{-2} \cdot s^{-1}})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0 \mu_{en} x \ \ (J \cdot cm^{-2} \cdot s^{-1})$$

The rate of energy absorbed in the slab of area A ($\mathrm{cm^2}$) and thickness x is

$$A\dot{\Psi}_0 \mu_{en} x \ (J \cdot s^{-1})$$

Given the density of the material is ρ, the rate of energy absorption per unit mass (Dose Rate) in the slab is

$$\dot{D} = \frac{A(cm^2) \cdot \dot{\Psi}_0 (J \cdot cm^{-2} \cdot s^{-1}) \cdot \mu_{en}(cm^{-1}) \cdot x\ (cm)}{\rho(g \cdot cm^{-3}) \cdot A(cm^2) \cdot x(cm)},$$

$$Dose\ rate\ in\ the\ absorber:\ \ \dot{D}(J \cdot g^{-1} \cdot s^{-1}) = \dot{\Psi}_0 (J \cdot cm^{-2} \cdot s^{-1}) \frac{\mu_{en}(cm^{-1})}{\rho(g/cm^3)}$$

Specific Gamma-Ray Constant

TABLE 6.3. Specific Gamma-ray Constant of Some Radioisotopes

Isotope	Γ $\frac{R \cdot m^2}{Ci \cdot h}$ [a]	Γ $\frac{X \cdot m^2}{MBq \cdot h}$ [b]
Antimony 122	0.24	1.67E—09
Cesium 137	0.33	2.30E—09
Chromium 51	0.016	1.11E—10
Cobalt 60	1.32	9.19E—09
Gold 198	0.23	1.60E—09
Iodine 125	0.07	4.87E—10
Iodine 131	0.22	1.53E—09
Iridium 192	0.48	3.34E—09
Mercury 203	0.13	9.05E—10
Potassium 42	0.14	1.39E—09
Radium 226	0.825	5.75E—09
Sodium 22	1.20	8.36E—09
Sodium 24	1.84	12.80E—09
Zinc 65	0.27	1.88E—09

[a] From *Radiological Health Handbook*, rev. ed., U.S. Public Health Service, Bureau of Radiological Health, Rockville, MD, 1970.
[b] 1 X unit = 1 C/kg.

Cember, pp. 226.

Specific Gamma-Ray Constant

Given the specific gamma ray constant for an isotope, the exposure rate induced at a location at a distance, r, is simply

$$\dot{X} = \Gamma \frac{A}{r^2}$$

← *activity*

Example

(a) Estimate the specific gamma-ray constant for ^{137}Cs. (b) Estimate the exposure rate at a distance of 1.7 m from a 100-mCi point source of ^{137}Cs.

Solution

(a) The isotope emits only a 0.662-MeV gamma ray in 85% of its transformations (Appendix D). The average energy per disintegration released as gamma radiation is therefore $0.85 \times 0.662 = 0.563$ MeV. The estimated specific gamma-ray constant for ^{137}Cs is therefore $\Gamma =$ $0.28\ \mathrm{R\,m^2\,Ci^{-1}\,h^{-1}}$.

(b) From Eq. (12.28), the exposure rate at a distance $r = 1.7$ m from a point source of activity $C = 100$ mCi $= 0.1$ Ci is

$$\dot{X} = 0.28 \frac{\mathrm{R\,m^2}}{\mathrm{Ci\,h}} \times \frac{0.1\ \mathrm{Ci}}{(1.7\ \mathrm{m})^2} = 9.7 \times 10^{-3}\ \mathrm{R\,h^{-1}} = 9.7\ \mathrm{mR\,h^{-1}}. \qquad (12.29)$$

Turner, pp. 382.

Specific Gamma-Ray Constant

- **Specific Gamma Ray Constant (Γ):** The exposure rate from a gamma ray point source of unit activity and positioned at a unit distance. It is given in unit of coulombs per kilogram per hour at 1 m from a 1 MBq point source.
- It is a numerical quantity used to predict the exposure resultant from a given gamma ray emitting radioisotope – A measure of the ability of a gamma-ray source to deliver exposure and dose.

Internally Deposited Radioisotope (IV) Gamma Ray Emitters

- For gamma rays, one can no longer assume that the organ is infinitely large.
- For a uniformly distributed gamma ray emitting isotope, the dose rate from the isotope in an infinitesimal volume *dV to* Point ***p*** at a distance ***r*** away is

Mass energy absorption coefficient of the dose receiving media

$$d\dot{D} = C \cdot \Gamma \cdot \frac{e^{-\mu r}}{r^2} \cdot dV \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

where C is the concentration of the isotope, Γ is the specific gamma-ray emission, and μ is the linear energy absorption coefficient.

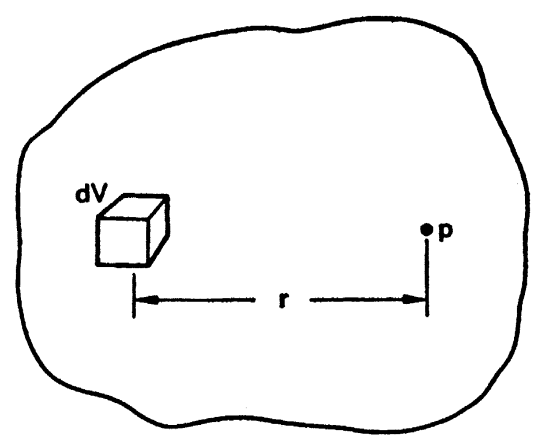


FIGURE 6.8. Diagram for calculating dose at point p from the gamma rays emitted from the volume element dV in a tissue mass containing a uniformly distributed isotope.

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

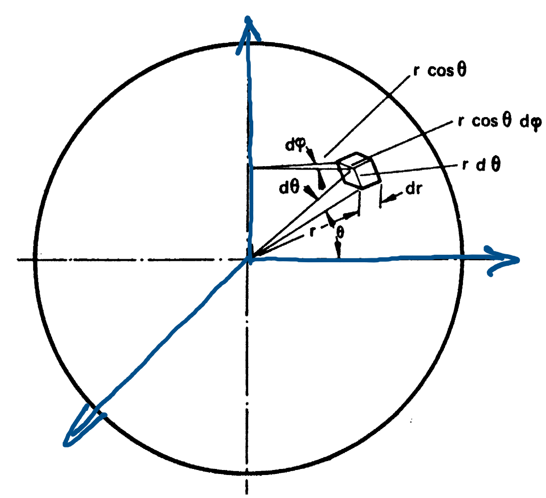
- As an example, for a uniform spherical source, the dose rate at the center is given by

$$\dot{D} = 4C\Gamma \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi/2} \int_{\varphi=0}^{\varphi=\pi} \frac{e^{-\mu r}}{r^2} \cdot r\, d\theta \cdot r\cos\theta\, d\varphi \cdot dr \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

$$= C\Gamma \cdot \frac{4\pi}{\mu}\left(1 - e^{-\mu R}\right) \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

- And the dose rate at the surface of the spherical source volume is given by

$$\dot{D}_{surface} = 0.5\, \dot{D}_{center}$$



Example 6.12

A spherical tank, capacity 1 m^3 and radius 0.62 m, is filled with aqueous ^{137}Cs waste containing a total activity of 37,000 MBq (1 Ci). What is the dose rate at the tank surface if we neglect absorption by the tank wall?

From Table 6.3 we find $\Gamma = 2.3 \times 10^{-9}$ X units/h/MBq at 1 m. Since water absorbs 38 Gy/X unit, the dose rate is 8.74×10^{-8} Gy/h/MBq at 1 m. The absorption coefficient of water for the 0.661 MeV gammas from ^{137}Cs is listed in Table 5.3 as 0.0327 cm^2/g. Since the density of water is 1 g/cm^3, the linear absorption coefficient is 0.0327/cm, or 3.27/m. The dose rate at the center of the sphere is found by substituting the respective values into Eq. (6.66):

$$\dot{D}_0 = C\Gamma \cdot \frac{4\pi}{\mu}(1 - e^{-\mu R}) \cdot \left(34 \frac{J/kg}{C/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_{air}/\rho_{air}}\right)$$

$$\dot{D}_0 = 3.7 \times 10^4 \text{ MBq/m}^3 \times 8.74 \times 10^{-8} \frac{\text{Gy} \cdot \text{m}^2}{\text{MBq} \cdot \text{h}} \times \frac{4\pi}{3.27 \text{ m}^{-1}}(1 - e^{-3.27 \times 0.62})$$

$$= 1.08 \times 10^{-2} \text{ Gy/h (1.08 rad/h)}.$$

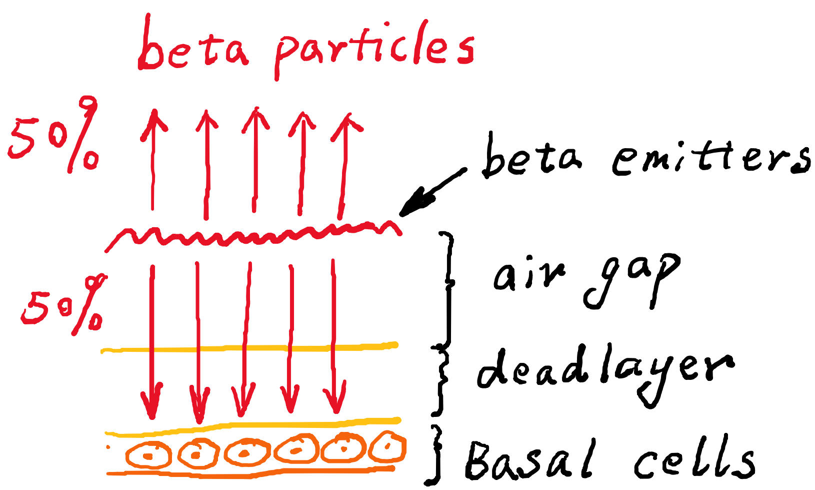
From Eq. (6.71), we see that the surface dose rate is $0.5 \times \dot{D}_0$. Therefore

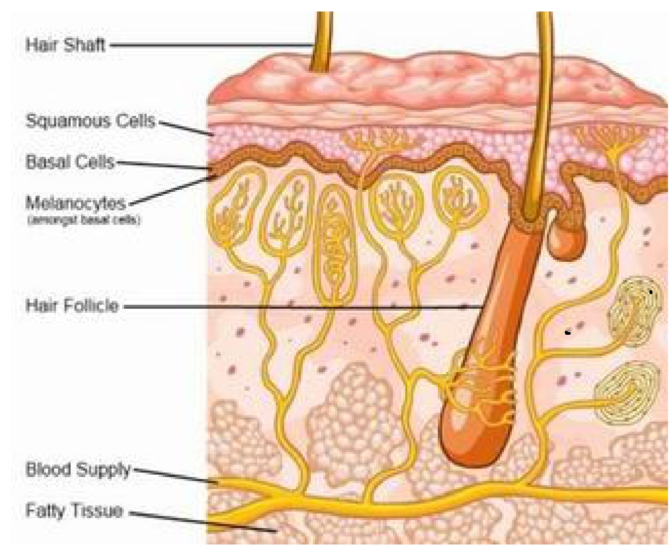
$$\dot{D}_{\text{surface}} = 0.5 \times 1.08 \times 10^{-2} = 0.54 \times 10^{-2} \text{ Gy/h (0.54 rad/h)}.$$

Radiation Dose Induced by Beta Radiation

Skin Dose from Surface Contamination

1. Beta particles are very easy to attenuate, for beta particles to contribute to skin dose, the source must be very close to the skin.
2. If we consider a thin layer of beta emitters placed parallel to the skin at a very small distance, we could assume that beta particles are traveling in parallel beams along two opposite directions only – 50% going up and 50% going down.
3. For skin dose, we only consider the dose deliver to the thin layer of Basel cells under the dead-layer (density thickness: 0.007g/cm^2) of the skin.





Calculation of Absorbed Dose (Revisited)

Assuming $\mu_{en}x<<1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident beam of particles is $\dot{\Psi}_0(\mathrm{J}\cdot\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1})$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0\mu_{en}x \ \ (J\cdot cm^{-2}\cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness x is

$$A\dot{\Psi}_0\mu_{en}x \ (J\cdot s^{-1})$$

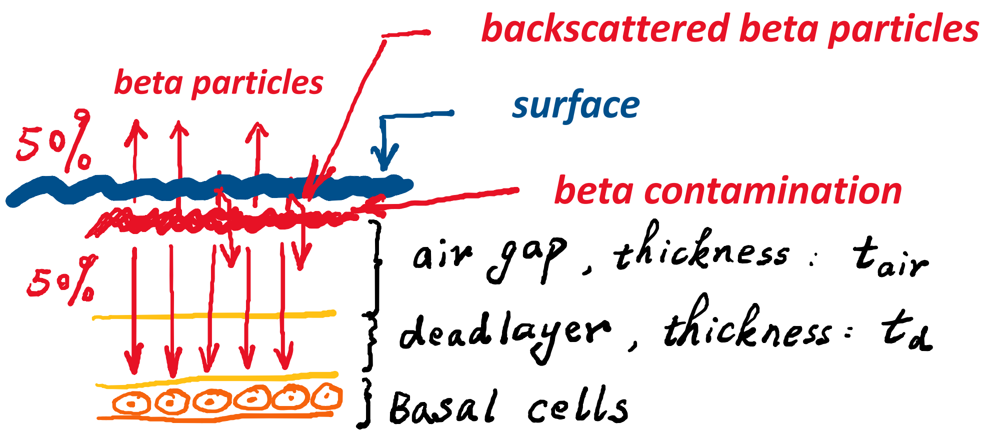
Given the density of the material is $\boldsymbol{\rho}(g/cm^3)$, and the linear energy absorption coefficient is $\boldsymbol{\mu_{en}}(cm^{-1})$, the **rate of energy absorption per unit mass (Dose Rate)** in the slab is

$$\dot{D} = \frac{A(cm^2)\cdot\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\cdot\mu_{en}(cm^{-1})\cdot x(cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$Dose\ rate\ in\ the\ absorber:\ \ \dot{D}(J\cdot g^{-1}\cdot s^{-1}) = \dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\frac{\mu_{en}(cm^{-1})}{\rho(g/cm^3)}$$

Skin Dose from Surface Contamination

Skin Dose from Surface Contamination



For a planar beta emitting surface, the surface dose rate may be easily calculated. Suppose the surface concentration is C_a Bq/cm^2, the dose rate to the basal cell region is

$$\dot{D} = C_a \cdot 0.5 \cdot f_b \cdot \bar{\bar{E}} \cdot e^{-\mu_{air} \cdot t_{air}} \cdot e^{-\mu_d \cdot t_d} \cdot \mu_\beta$$

- surface concentration is C_a (Bq/cm^2)
- Average energy of beta particles
- Mass energy absorption coefficient of tissue (cm^2/g)
- Backscattering correction, 1.25
- Attenuation by air
- Attenuation by dead skin layer

energy fluence rate, (J/cm^2/sec)

Beta Radiation – Dose from Surface Contamination An Example (Cember, p. 190)

Example 6.7

A solution of ^{32}P is spilled, and contaminates a large surface to an areal concentration of 37 Bq/cm^2. What is the estimated beta-ray-contact dose rate to the skin and the dose rate at a height of 1 m above the contaminated area?

For ^{32}P:

$E_m = 1.71$ MeV $\quad \overline{E} = 0.7$ MeV.

The beta absorption coefficients in air and in tissue are calculated by substituting 1.71 for the value of E_m in Eqs. (6.20) and (6.21):

$$\mu_{\beta,a} = 16(1.71 - 0.036)^{-1.4}\,\frac{cm^2}{g} = 7.78\,\frac{cm^2}{g}$$

$$\mu_{\beta,t} = 18.6(1.71 - 0.036)^{-1.37}\,\frac{cm^2}{g} = 9.18\,\frac{cm^2}{g},$$

Beta Radiation – Submersion Dose

Beta Radiation – Submersion Dose

- For **a small volume of air inside an infinite cloud of beta emitting radionuclide**, we have

$$\text{Rate of energy emission} = \text{rate of energy absorption}$$

- In an infinite cloud containing C Bq/m^3 of a beta emitter, the dose rate in a small volume inside the radioactive cloud is

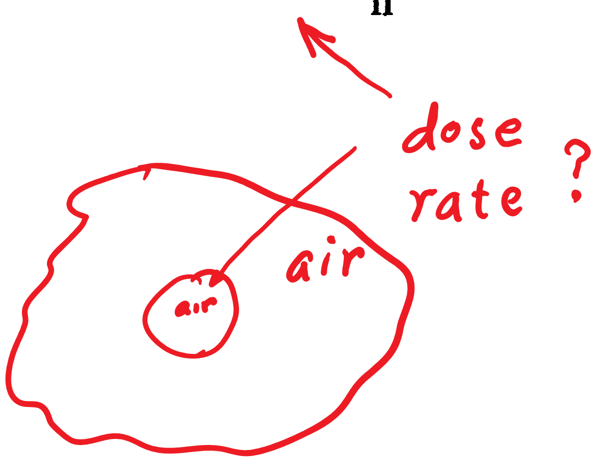
Beta Radiation – Submersion Dose

- For **a small volume of air inside an infinite cloud of beta emitting radionuclide**, we have

$$\text{Rate of energy emission} = \text{rate of energy absorption}$$

- In an infinite cloud containing C Bq/m^3 of a beta emitter, the dose rate in a small volume inside the radioactive cloud is

$$\dot{D}_{\text{inf}}, \frac{\text{mGy}}{\text{h}} = \frac{C\,\frac{\text{Bq}}{\text{m}^3} \times 1\,\frac{\text{tps}}{\text{Bq}} \times \overline{E}\,\frac{\text{MeV}}{\text{t}} \times 1.6 \times 10^{-13}\,\frac{\text{J}}{\text{MeV}} \times 3.6 \times 10^{3}\,\frac{\text{s}}{\text{h}}}{1.293\,\frac{\text{kg}}{\text{m}^3} \times 1\,\frac{\text{J}}{\text{kg}}\Big/\text{Gy} \times \frac{1\ \text{Gy}}{10^3\ \text{mGy}}}$$



$$\dot{D}_{\text{inf}} = 4.45 \times 10^{-7} \times C \times \overline{E}\,\frac{\text{mGy}}{\text{h}}$$

Skin Dose for Human Submersed in Radioactive Cloud Containing Beta Emitters

Understanding the Radiation from Cs-137

Decay scheme:

$$^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}.$$

What will happen to the excited Ba-137 nucleus?

http://faithandsurvival.com/wp-content/uploads/2023/04/fukushima-cesium-137-spread.jpg

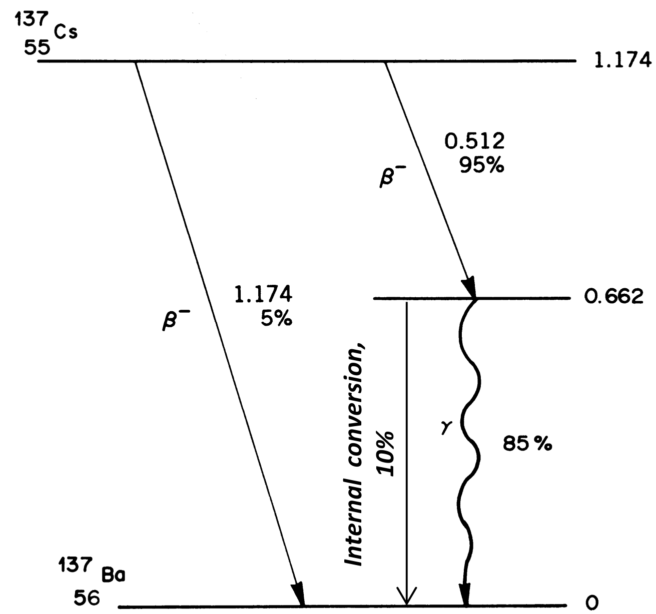
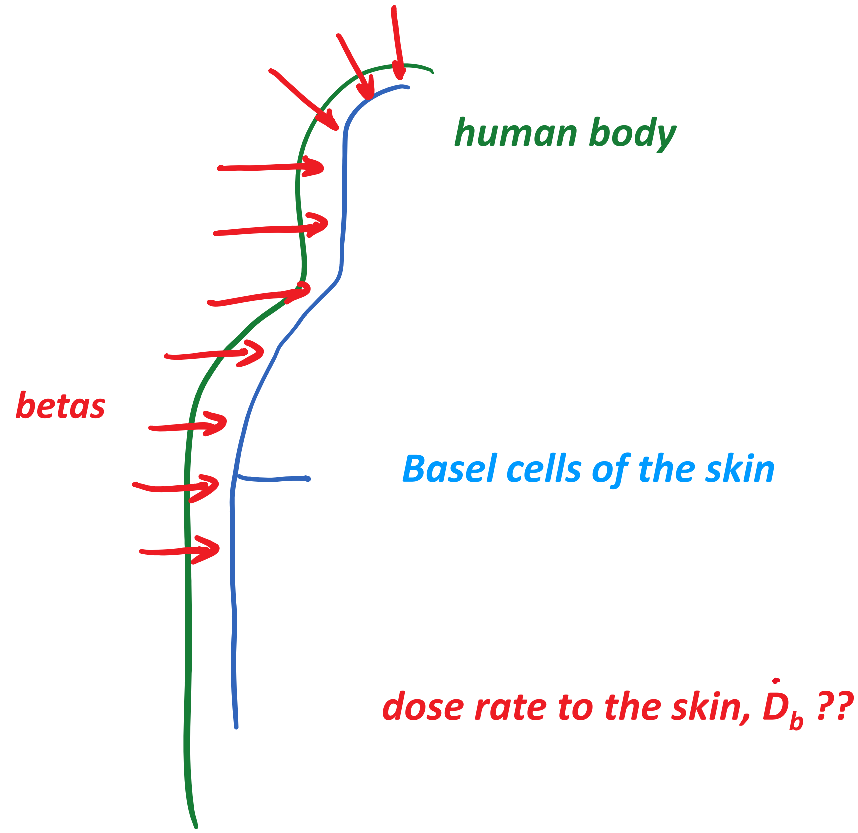


FIGURE 3.8. Decay scheme of $^{137}_{55}\mathrm{Cs}$.

Human Submersed in Radioactive Cloud Containing Beta Emitters

Rate of energy emission = rate of energy absorption



$$\dot{D}_b = 0.5 \times 1.1 \times \dot{D}_{\text{inf}}\,(\text{air}) \times e^{-\mu_{\beta,t} \times 0.007}$$

Human Submersed in Radioactive Cloud Containing Beta Emitters

- Because (a) the skin is irradiated from one side only, and (b) soft tissue absorbs about 10% more energy per kilogram than dose in air, the dose rate to the basal cells of the skin in a semi-infinite medium is

$$\dot{D}_{\text{inf}} = 4.45 \times 10^{-7} \times C \times \overline{E} \, \frac{\text{mGy}}{\text{h}}$$

$$\dot{D}_b = 0.5 \times 1.1 \times \dot{D}_{\text{inf}}\,(\text{air}) \times e^{-\mu_{\beta,t} \times 0.007}$$

- Therefore, the dose rate to the skin of a person immersed in a large cloud of concentration C Bq/m^3 is

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \overline{E} \times e^{-\mu_{\beta,t} \times 0.007} \, \frac{\text{mGy}}{\text{h}}$$

- Generally, if the cloud consists of several groups of beta particles with different maximum energies, the beta dose rate is

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \sum_i f_i \overline{E}_i e^{-\mu_{\beta i,t} \times 0.007} \, \frac{\text{mGy}}{\text{h}}$$

f_i : Fraction of the *i' th* group of beta particles

$\overline{E}_i$: Average energy of beta particles

$\mu_{\beta i,\ t}$: Linear energy absorption coefficient of tissue.

Beta Radiation – Submersion Dose

An example. Cember, pp. 231.

Calculate the dose rate to the skin of a person immersed in a large cloud of ^{85}Kr at a concentration of 37 kBq/m^3 (10^{-6} μCi/mL).

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \bar{E} \times e^{-\mu_{\beta,t} \times 0.007} \frac{\text{mGy}}{\text{h}}$$

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \sum_i f_i \bar{E}_i e^{-\mu_{\beta i,t} \times 0.007} \frac{\text{mGy}}{\text{h}}$$

Solution

Krypton-85 is a pure beta emitter that is transformed to ^{85}Rb by the emission of a beta particle whose maximum energy is 0.672 MeV and whose average energy is 0.246 MeV. The tissue absorption coefficient is calculated with Eq. (6.21):

$$\mu_{\beta,\text{t}} = 18.6(0.672 - 0.036)^{-1.37} = 34.6 \text{ cm}^2/\text{g},$$

and the skin dose is calculated with Eq. (6.38):

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \bar{E} \times e^{-(\mu_{\beta,\text{t}} \times 0.007)} \text{ mGy/h}$$

$$\dot{D}_b = 2.45 \times 10^{-7} \times 3.7 \times 10^4 \times 0.246 \times e^{-(34.6 \times 0.007)}$$

$$\dot{D}_b = 1.8 \times 10^{-3} \text{ mGy/h (0.18 mrads/h)}.$$

The Fukushima nuclear **disaster** is estimated to have released between 20-200 megacuries of **Krypton 85** from three melted down reactors

Organ Dose from Internally Deposited Beta Emitters

Internally Deposited Radioisotope (I) Corpuscular Beta Radiation

- For an infinitely large medium containing a uniformly distributed radioisotope, **the concentration of absorbed energy must be equal to the concentration of energy emitted by the isotope**.
- The energy absorbed per unit mass per transformation in a given organ is called the **specific effective energy** (SEE).
- For practical health physics purposes, "infinitely large" may be approximated by a tissue mass whose dimension exceed the range of the radiation.
- For alpha and beta radiation, this condition can be easily met, so that the SEE is simply the average energy of the radiation divided by the mass of the tissue in which it is distributed.

$$SEE\ (\alpha \text{ or } \beta) = \frac{\overline{E}(\alpha \text{ or } \beta)}{m} \frac{\text{MeV}}{t} \Big/ \text{kg}$$

Internally Deposited Beta Emitters – An Example Cember, pp. 234.

Calculate the daily dose rate to a testis that weighs 18 g and has 6660 Bq of ^{35}S uniformly distributed throughout the organ.

Sulfur is a pure beta emitter whose maximum-energy beta particle is 0.1674 MeV and whose average energy is 0.0488 MeV. The beta-ray dose rate from *q* Bq uniformly dispersed in *m* kg of tissue, if the specific effective energy is SEE MeV per transformation per kg, is

$$\dot{D}(\beta) = \frac{q \text{ Bq} \times 1 \text{ tps/Bq} \times \overline{E} \text{ MeV}/t \times 1.6 \times 10^{-13} \text{ J/MeV} \times 8.64 \times 10^{4} \text{ s/day}}{m \text{ kg} \times 1 \frac{\text{J}}{\text{kg}} / \text{Gy}}$$

$$= \frac{6.66 \times 10^{3} \times 1 \times 4.88 \times 10^{-2} \times 1.6 \times 10^{-13} \times 8.64 \times 10^{4}}{0.018 \times 1}$$

$$= 2.5 \times 10^{-4} \text{ Gy/day } (0.025 \text{ rad/day}).$$

Neutron Dose

Neutron dose to tissue:

- Fast neutron dose from elastic scattering (mostly from first-collision dose).
- Thermal neutron dose
  - neutron capture by H → gamma ray dose.
  - neutron capture by N → dose from the recoil nucleus.

Radiation Dose from Fast Neutrons

Example 6.16

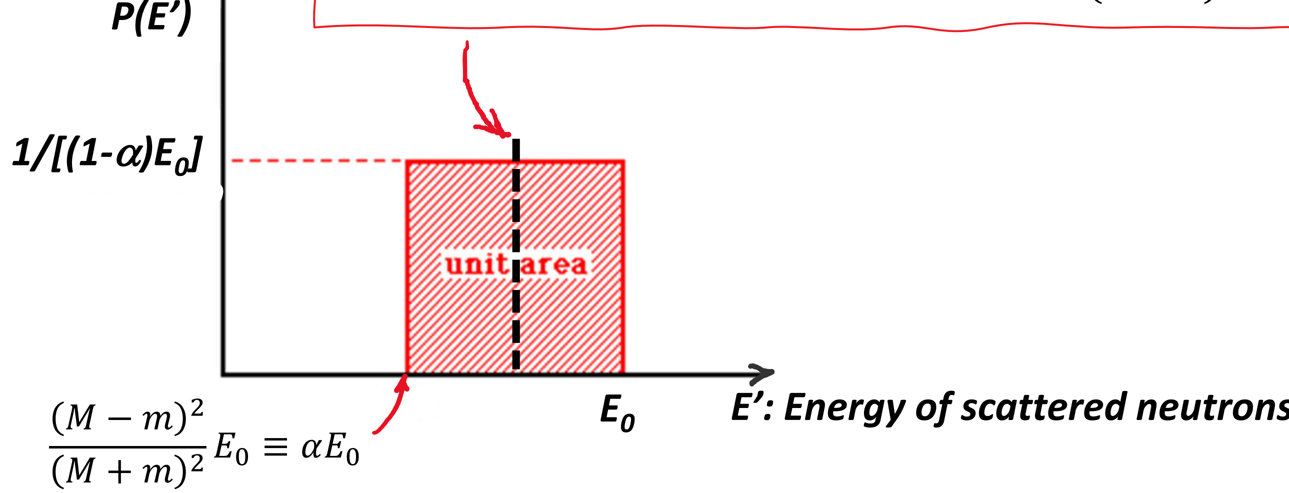
What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

How do Fast Neutrons Deposit Energy in Tissue?

Energy Spectrum of Scattered Neutrons (Revisited)

Average energy carried by the scattered neutron:

$$E'_{avg} = \frac{1+\alpha}{2} E_0 = \frac{1+\frac{(M-m)^2}{(M+m)^2}}{2} E_0 = \frac{M^2+m^2}{(M+m)^2} \cdot E_0$$



αE_0

Average energy transferred to the recoil nucleus:

$$E_{avg\_energy\_loss} = E_0 - E'_{avg} = \frac{2Mm}{(M+m)^2} \cdot E_0$$

Radiation Dose from Fast Neutrons (Revisited)

- For isotropic scattering, the average fraction of energy transferred in an elastic scattering with a nucleus of atomic mass number M is

$$f = \frac{2M}{(M+1)^2}$$

- The composition of soft tissue is shown below

TABLE 6.12. Synthetic Tissue Composition

Element	% Mass	*N*, atoms/kg	*f*
Oxygen	71.39	2.69×10^{25}	0.111
Carbon	14.89	6.41×10^{24}	0.142
Hydrogen	10.00	5.98×10^{25}	0.500
Nitrogen	3.47	1.49×10^{24}	0.124
Sodium	0.15	3.93×10^{22}	0.080
Chlorine	0.10	1.70×10^{22}	0.053

Source: Adapted from G. L. Brownell, W. H. Ellett, and A. R. Reddy, Absorbed Fractions for Photon Dosimetry. *J Nuclear Medicine*, Supplement No. 1, MIRD Pamphlet No. 3, February 1968. By permission.

Radiation Dose from Fast Neutrons

- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the **first collision dose**. The scattered neutron is **not** considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1\ \text{J/kg}\cdot\text{Gy}}, \tag{6.103}$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Radiation Dose from Fast Neutrons

Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Substituting the appropriate values into Eq. (6.103) yields

$$\dot{D}_n = \frac{2 \times 10^3\ n/\text{cm}^2 \cdot \text{s} \times 5\ \text{MeV}/n \times 1.6 \times 10^{-13}\ \text{J/MeV} \times 51.17\ \text{cm}^2/\text{kg}}{1\ \text{J/kg} \cdot \text{Gy}}$$

$$= 8.19 \times 10^{-8}\ \text{Gy/s}\left(8.19 \times 10^{-6}\ \text{rad/s}\right),$$

or

$$8.19 \times 10^{-8}\ \text{Gy/s} \times 10^6\ \mu\text{Gy/Gy} \times 3.6 \times 10^3\ \text{s/h}$$

$$= 295\ \mu\text{Gy/h}\ (29.5\ \text{mrad/h}).$$

The scattering cross sections of each of the tissue elements for 5-MeV neutrons are listed below:

ELEMENT	σ, x 10^{-24} cm^2	$N_i \sigma_i f_i$
O	1.55	4.628
C	1.65	1.502
H	1.50	4.485×10^1
N	1.00	1.848×10^{-1}
Na	2.3	7.231×10^{-3}
Cl	2.8	2.523×10^{-3}
		$\sum N_i \sigma_i f_i = 5.117 \times 10^1 \frac{\text{cm}^2}{\text{kg}}$

$$\dot{D}_n(E) = \frac{\phi(E) E \sum_i N_i \sigma_i f}{1 \text{ J/kg} \cdot \text{Gy}},$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,

E = neutron energy, in joules,

N_i = atoms per kilogram of the ith element,

σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,

f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

How do Thermal Neutrons Deposit Energy in Tissue?

Interaction of Slow and Thermal Neutrons (E<0.5eV) (Revisited)

The most important interactions between slow neutrons and absorbing materials are neutron-induced reactions, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

$$\text{neutron} + \text{target nucleus} \Rightarrow \begin{cases} \text{recoil nucleus} \\ \text{protons} \\ \text{alpha particles} \\ \text{fission fragments} \end{cases}$$

Neutron Induced Reactions

$$ {}_0^1n + {}_7^{14}N \rightarrow {}_6^{14}C + {}_1^1p $$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Thermal Neutron Dose from the $^{14}N(n,p)^{14}C$ Reaction

- Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

- For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg} \cdot \text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Neutron Induced Reactions

$${}_{0}^{1}n + {}_{1}^{1}H \rightarrow {}_{1}^{2}H + {}_{0}^{0}\gamma$$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Thermal Neutron Dose from the $^{1}H(n, \gamma)^{2}H$ Reaction

- For the $^{1}H(n, \gamma)^{2}H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

$$A = \phi N_H \sigma_H \text{ "Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

- The resulting gamma ray dose is illustrated with the following example.

Example 6.17

What is the absorbed dose rate to a 70-kg person from a whole body exposure to a mean thermal flux of 10,000 neutrons per cm^2 per second?

The dose rate due to the n, p reaction is calculated from Eq. (6.105)

$$\dot{D}_{np} = 1 \times 10^4 \times 1.49 \times 10^{24} \times 1.75 \times 10^{-24} \times 0.63 \times 1.6 \times 10^{-13}$$

$$= 2.628 \times 10^{-9}\ \text{Gy/s} \quad (2.628 \times 10^{-7}\ \text{rad/s}),$$

or

$$\dot{D}_{np} = 9.461\ \mu\text{Gy/h} \quad (0.95\ \text{mrad/h}).$$

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg} \cdot \text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

The autointegral gamma-ray dose rate is calculated with Eq. (6.82). The gamma-ray "activity," from Eq. (6.106) is

$$A = 10^4\ \text{cm}^2\ \text{s}^{-1} \times 5.98 \times 10^{25}\ \text{atoms/kg} \times 3.3 \times 10^{-25}\ \text{cm}^2/\text{atom}$$

$$= 1.973 \times 10^5\ \text{"Bq"/kg}.$$

$$A = \phi N_H \sigma_H\ \text{"Bq"/kg},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

The dose rate from this uniformly distributed gamma ray activity is calculated from Eq. (6.82):

$$\dot{D}_H = A \cdot E_r \cdot \varphi = 1.973 \times 10^5\ Bq/kg \cdot 2.23\text{MeV} \cdot 1.6 \times 10^{-16}\ J/MeV \cdot 0.278$$

$$= 1.19 \times 10^{-11}\ Gy/sec = 6.89 \times 10^{-8}\ Gy/\text{h}$$

The absorbed fraction, φ, for the 2.23-MeV gamma ray is found, by interpolating in Table 6.8 between the 2.000- and 4.000-MeV values, to be 0.278

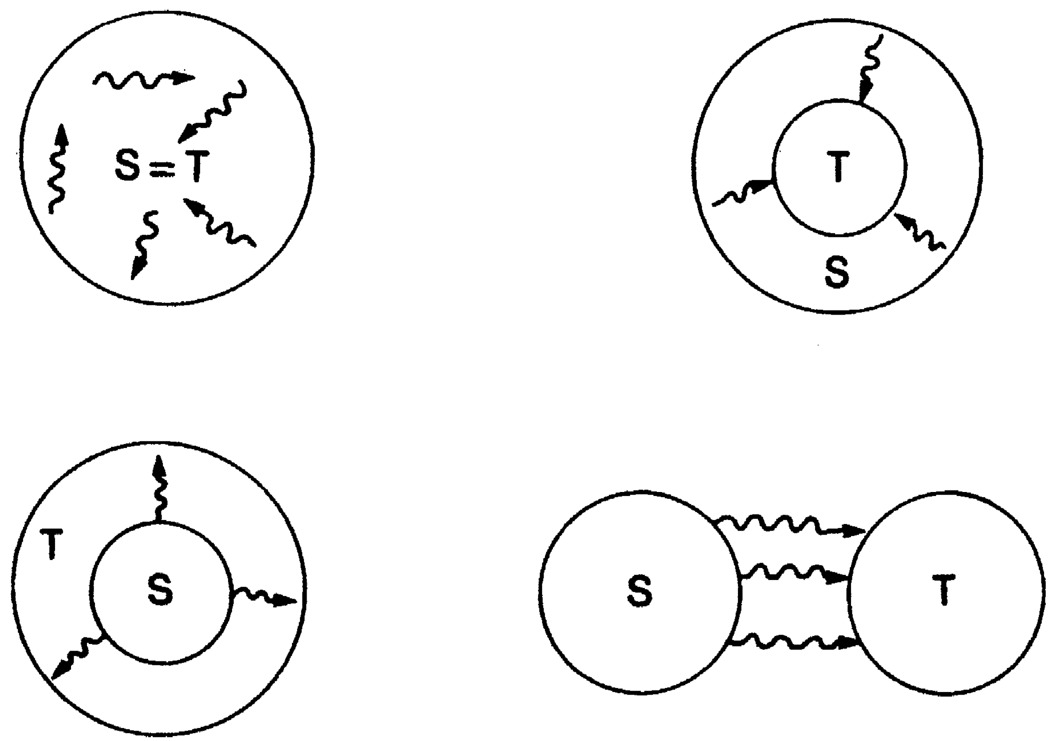
Accumulated Dose from Internally Deposited Radioactive Sources

Partial Absorption of Gamma-Ray Energy – MIRD Method

- To account for the partial absorption of gamma-ray energy in organs and tissues, the Medical Internal Radiation Dose (MIRD) Committee of the Society of Nuclear Medicine has developed a formal system for calculating the dose to a target organ or tissue from a source organ containing a uniformly distributed radioisotope.
- The **absorption fraction** – the fraction of the energy radiated by the source organ, which is absorbed by the target organ.

Partial Absorption of Gamma-Ray Energy – MIRD Method

- The **absorption fraction** – the fraction of the energy radiated by the source organ and absorbed by the target organ.



Partial Absorption of Gamma Ray Energy – MIRD Method

- The absorbed fraction are calculated by the application of **Monte Carlo** methods.

$$\text{Absorbed fraction} = \varphi = \frac{\text{energy absorbed by target}}{\text{energy emitted by source}}$$

- Standard data on the absorbed dose for photons of various energies for point isotropic sources and for uniformly distributed sources are published by MIRD in several Supplements to the Journal of Nuclear Medicine

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a] (Continued)

TARGET ORGAN	0.200 φ	0.200 $\frac{100\sigma_\varphi}{\varphi}$	0.500 φ	0.500 $\frac{100\sigma_\varphi}{\varphi}$	1.000 φ	1.000 $\frac{100\sigma_\varphi}{\varphi}$	1.500 φ	1.500 $\frac{100\sigma_\varphi}{\varphi}$	2.000 φ	2.000 $\frac{100\sigma_\varphi}{\varphi}$	4.000 φ	4.000 $\frac{100\sigma_\varphi}{\varphi}$	Target Organ
Adrenals	0.352E−04	36.	0.138E−03	35.	0.100E−03	42.	0.107E−03	43.	0.114E−03	43.			Adrenals
Bladder	0.327E−02	5.0	0.341E−02	6.6	0.274E−02	8.3	0.291E−02	8.4	0.231E−02	9.6	0.147E−02	12.	Bladder
GI (stom)	0.218E−02	7.0	0.258E−02	7.7	0.181E−02	9.8	0.199E−02	10.	0.212E−02	10.	0.119E−02	14.	GI (stom)
GI (SI)	0.106–01	3.4	0.114E−01	3.8	0.109E−01	4.2	0.915E−02	4.8	0.820E−02	5.2	0.409E−02	7.3	GI (SI)
GI (ULI)	0.256E−02	6.3	0.306–02	7.0	0.228E−02	8.9	0.209E−02	9.4	0.197E−02	10.	0.160E−02	12.	GI (ULI)
GI (LLI)	0.151E−02	7.6	0.184E−02	8.8	0.178E−02	9.7	0.181E−02	11.	0.157E−02	12.	0.673E−03	18.	GI (LLI)
Heart	0.337E−02	5.8	0.372E−02	6.6	0.301E−02	8.1	0.345E−02	7.8	0.312E−02	8.3	0.145–02	13.	Heart
Kidneys	0.171E−02	7.4	0.142E−02	9.7	0.161E−02	10.	0.152E−0.2	11.	0.154E−02	12.	0.904E−03	16.	Kidneys
Liver	0.111–01	3.4	0.101E−01	4.1	0.896E−02	4.7	0.912E−02	4.9	0.847E−02	5.1	0.560E−02	6.4	Liver
Lungs	0.507E−02	4.3	0.496E−02	5.2	0.466E-02	6.1	0.466E−02	6.5	0.427E−02	6.9	0.568E−02	6.4	Lungs
Marrow	0.221E−01	1.5	0.194E−01	1.0	0.182E-01	2.0	0.164E−01	2.2	0.156E−01	2.3	0.969E−02	3.0	Marrow
Pancreas	0.444E−03	14.	0.382E−03	17.	0.534E−03	19.	0.348E−03	22.	0.358E−03	24.	0.142–03	39.	Pancreas
Sk. (rib)	0.505E−02	4.1	0.435E−02	5.6	0.421E−02	6.3	0.405E−02	7.0	0.350E−02	7.7	0.338E−02	8.0	Sk. (rib)
Sk. (pelvis)	0.668E−02	3.9	0.569E−02	5.0	0.562E−02	5.7	0.511E−02	6.3	0.422E−02	7.0	0.256E−02	9.3	Sk. (pelvis)
Sk. (spine)	0.910E−02	3.6	0.763E−02	4.5	0.751E−02	5.1	0.610E−02	5.7	0.606E−02	5.9	0.341E−02	8.1	Sk. (spine)
Sk. (skull)	0.277E−02	6.3	0.304E−02	7.2	0.280E−02	8.0	0.254E−02	9.0	0.292E−02	8.8	0.224E−02	10.	Sk. (skull)
Skeleton (total)	0.550E−01	1.4	0.488E−01	1.7	0.456E−01	2.0	0.413E−01	2.2	0.396E−01	2.3	0.252E−01	3.0	Skeleton (total)
Skin	0.677E−02	3.5	0.757E−02	4.2	0.745E−02	4.8	0.759E−02	5.0	0.664E−02	5.5	0.123E−01	4.3	Skin
Spleen	0.798E−03	11.	0.116E−02	11.	0.914E−03	14.	0.903E−03	16.	0.740E−03	17.	0.368E−03	24.	Spleen
Thyroid	0.418E−04	42.							0.810E−04	46.			Thyroid
Uterus	0.408E−03	15.	0.473E−03	16.	0.517E−03	18.	0.323E−03	23.	0.364E−03	25.	0.238–03	33.	Uterus
Trunk	0.223	0.81	0.225	0.84	0.210	0.92	0.198	0.99	0.186	1.0	0.156	1.2	Trunk
Legs	0.102	1.3	0.101	1.4	0.965E−01	1.5	0.917E−01	1.6	0.846E−01	1.6	0.710E−01	1.8	Legs
Head	0.134E−01	3.2	0.147E−01	3.5	0.145E−01	3.8	0.130E−01	4.1	0.139E−01	4.1	0.127E−01	4.4	Head
Total body	0.338	0.57	0.340	0.60	0.321	0.67	0.302	0.73	0.284	0.77	0.240	0.90	Total body

[a]The digits following the symbol E indicate the powers of 10 by which each number is to be multiplied; A blank in the table indicates that the coefficient of variation was greater than 50%; Total body = head + trunk + legs.

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Internally Deposited Radioisotope (II)
Effective Half-Life

- The total **dose absorbed by an organ during any given time interval** after the deposition of the isotope in the organ may be calculated by integrating the dose rate over the required time interval. For this purpose, two factors must be considered:

  In situ **radioactive decay** of the isotope → exponential decay

  Biological elimination of the isotope → follows the first-order kinetics → exponential decay

- The equation for the quantity of radioisotope within an organ at any given time after the deposition of a quantity Q_0 is given by

$$Q = (Q_0 e^{-\lambda_R t})(e^{-\lambda_B t})$$

where λ_R is the radioactive decay constant and λ_B is the biological elimination constant.

Internally Deposited Radioisotope (II)
Effective Half-Life

☞ One can define an effective elimination constant $\lambda_E = \lambda_R + \lambda_B$ that represents the combined effects of these two decay processes,

$$Q = Q_0 e^{-\lambda_E t}$$

and

$$T_E = \frac{0.693}{\lambda_E}$$

is called the **effective half-life**.

Internally Deposited Radioisotope (III)
Accumulated Dose and Dose Commitment

- Given the initial dose rate $\dot{D}_0$, the accumulated dose received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda_E t} dt = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda_E t})$$

For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda_E}$$

- For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the **dose commitment**.

Internally Deposited Radioisotope (III)
Total Dose: Dose Commitment

Generally, if there is more than one compartment, the body burden at any time t after deposition of q_0 units of a radionuclide is given by

$$q(t) = f_1 q_0 e^{-\lambda_1 t} + f_2 q_0 e^{-\lambda_2 t} + \cdots + f_n q_0 e^{-\lambda_n t}, \tag{6.60}$$

where $f_1, f_2, \ldots, f_n$ = fraction of the total activity deposited in compartments 1, 2, . . . , n, and $\lambda_1, \lambda_2, \ldots, \lambda_n$ = effective clearance rates for compartments 1, 2, . . . , n.

Since the activity in each compartment contributes to the dose to that organ or tissue, Eq. (6.57) becomes, for the multicompartment case,

$$D = \frac{\dot{D}_{10}}{\lambda_{1E}}\left(1 - e^{-\lambda_{1E} t}\right) + \frac{\dot{D}_{20}}{\lambda_{2E}}\left(1 - e^{-\lambda_{2E} t}\right) + \cdots + \frac{\dot{D}_{n0}}{\lambda_{nE}}\left(1 - e^{-\lambda_{nE} t}\right), \tag{6.61}$$

and when the radionuclide has completely been eliminated, Eq. (6.61) reduces to

$$D(t) = \frac{\dot{D}_{10}}{\lambda_{1E}} + \frac{\dot{D}_{20}}{\lambda_{2E}} + \cdots + \frac{\dot{D}_{n0}}{\lambda_{nE}}. \tag{6.62}$$

Overall dose commitment

Internally Deposited Radioisotope (III)
Total Dose: Dose Commitment

first-order kinetics and is emptied at its own clearance rate. Thus, for example, cesium is found to be uniformly distributed throughout the body, although the body behaves as if the cesium were stored in two compartments. One compartment contains 10% of the total body burden and has a retention half-time of 2 days, while the second compartment contains the other 90% of the body's cesium content and has a clearance half-time of 110 days. The retention curve for cesium, therefore, is given by the equation

$$q(\mathrm{t}) = 0.1\, q_0\, \mathrm{e}^{-(0.693\ \mathrm{t}/2\,\mathrm{days})} + 0.9\, q_0\, \mathrm{e}^{-(0.693\ \mathrm{t}/110\,\mathrm{days})}, \qquad \textbf{(6.59)}$$

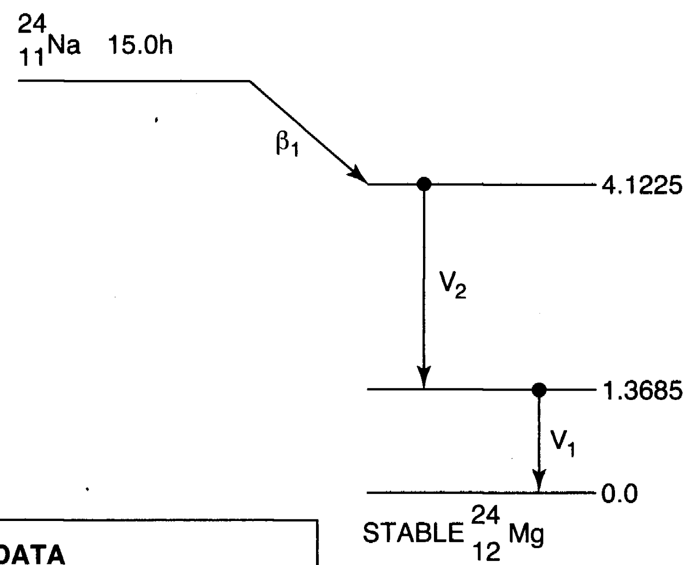
where $q(t)$ is the body burden at time t after deposition of q_0 amount of cesium in the body. Ten percent of the total is deposited in compartment 1 and 90% is deposited in compartment 2.

MIRD Method – General Treatment of Internal Dose

An Example

Calculate the total dose and initial dose rate to a 70-kg, 160-cm-tall reference man who is intraveneously injected with 1-MBq ^{24}NaCl. Assume the ^{24}NaCl to become uniformly distributed within a very short time and to have a biological half-life of 11 days (264 hours).

SODIUM-24 BETA-MINUS DECAY



INPUT DATA

Radiation	%/disin-tegration	Transition energy (MeV)	Other nuclear parameters
Beta-1	99.9	1.392*	Allowed
All other betas	<0.1	—	—
Gamma-1	100.	1.3685	E2, $\alpha x < 0.00001$ (T)
Gamma-2	99.9	2.7539	E2, $\alpha x < 0.00001$ (T)
All other gammas	<0.1	—	—

Ref. Lederer, C. M. et al, *Table of isotopes*, 6th ed.
*Endpoint energy (MeV). (T) = Theoretical value.

OUTPUT DATA

Radiation [1]	Mean number disinte-gration (n_i)	.n energy (MeV) [$\bar{E}_i$]	$\Delta i \left(\frac{\text{g-rad}}{\mu\text{Ci-h}}\right)$
Beta-1	0.999	0.5547	1.1803
Gamma-1	0.999	1.3685	2.9149
Gamma-2	0.999	2.7539	5.8599

MIRD Method – General Treatment of Internal Dose

An Example

absorption fraction (→ φ_i)

amount of E absorbed per decay (→ $\varphi_i \Delta_i$)

RADIATION	E_i (MeV)	φ_i	Δ_i, $\frac{\text{kg-fGy}}{\text{Bq-s}}$	$\varphi_i \Delta_i$
Beta 1	0.555	1.000	88.64	88.64
Gamma 1	1.369	0.31	218.91	67.86
Gamma 2	2.754	0.265	440.08	116.62
				$\sum = 273.12 (\text{kg} \cdot \text{fGy})/(\text{Bq} \cdot \text{s})$

amount of E emitted per decay (→ Δ_i)

The decay scheme and the accompanying table of input data show one beta (actually >0.999) particle whose maximum energy is 1.392 MeV, and one 1.3685-MeV gamma per decay. The output data list the integral dose in an infinite medium, per unit of cumulated activity, in units of $\frac{\text{g} \cdot \text{rads}}{\mu\text{Ci} \cdot \text{h}}$ for each radiation. To convert from the old system of units found in the MIRD publications to the SI system, that is, to go from $\frac{\text{g} \cdot \text{rads}}{\mu\text{Ci} \cdot \text{h}}$ to $\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}}$, we use the following relation:

$$\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = \frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} \times \frac{10^{-3}\,\dfrac{\text{kg}}{\text{g}} \times 10^{-2}\,\dfrac{\text{Gy}}{\text{rad}}}{3.7 \times 10^{4}\,\dfrac{\text{Bq}}{\mu\text{Ci}} \times 3.6 \times 10^{3}\,\dfrac{\text{s}}{\text{h}}}$$

$$\frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = \frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} \times 75.1 \times 10^{-14}. \qquad \textbf{(6.87)}$$

To convert from SI units to traditional units:

$$\frac{\text{g} \cdot \text{rad}}{\mu\text{Ci} \cdot \text{h}} = \frac{1}{7.51 \times 10^{-14}} \times \frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}} = 1.33 \times 10^{13} \times \frac{\text{kg} \cdot \text{Gy}}{\text{Bq} \cdot \text{s}}. \qquad \textbf{(6.88)}$$

Now let us return to the problem. Since the ^{24}Na is cleared exponentially at an effective rate λ_E, the amount of activity in the source organ is given by

$$A_s(t) = A_s(0) \times e^{-\lambda_E t}, \tag{6.89}$$

where $A_s(0)$ is the initial activity in the source.

$$\tilde{A} = \int_0^\infty A_s(t)\,dt = A_s(0)\int_0^\infty e^{-\lambda_E t}dt = \frac{A_s(0)}{\lambda_E}. \tag{6.90}$$

***$\tilde{A}$, organ burden:* total number of decays that would happen in the organ.**

Since

$$\lambda_E = \frac{0.693}{T_E} = \frac{0.693}{(T_R \times T_B)/(T_R + T_B)}$$

the biological half-life T_B is found in International Commission on Radiological Protection (ICRP) Publication 2 to be 264 hours, and the radioactive half-life T_R is 15 hours, therefore

$$\tilde{A} = \frac{10^6\ \text{Bq}}{1.36 \times 10^{-5}\ \text{s}^{-1}} = 7.35 \times 10^{10}\ \text{Bq} \cdot \text{s}.$$

The absorbed fractions, φ_i, in a number of target organs and tissues, for photons ranging in energy from 0.01 to 4 MeV that originate in a number of different source organs and tissues, are tabulated in Appendix A of the *Journal of Nuclear Medicine*, Supplement No. 3, August 1969.[2] Table 6-8 shows the absorbed fractions from a photon emitter that is uniformly distributed throughout the body, as in the case of ^{24}Na. The values of φ_i for the 1.369-MeV and 2.754-MeV gammas were found by interpolation between values in Table 6-8 and are listed below, together with Δ_i, which was found in the output data listing in Figure 6-12 and was converted to $\text{kg} \cdot \text{fGy/Bq} \cdot \text{s}$, where 1 fGy $= 10^{-15}$ Gy:

For each particle emitted in the source, the amount of energy absorbed in the target

RADIATION	E_i (MeV)	φ_i	Δ_i, $\frac{\text{kg-fGy}}{\text{Bq-s}}$	$\varphi_i \Delta_i$
Beta 1	0.555	1.000	88.64	88.64
Gamma 1	1.369	0.31	218.91	67.86
Gamma 2	2.754	0.265	440.08	116.62
				$\sum = 273.12 (\text{kg} \cdot \text{fGy})/(\text{Bq} \cdot \text{s})$

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation). Gamma Emitter Uniformly Distributed Throughout the Body[a]

	PHOTON ENERGY (MeV)												
	0.010		0.015		0.020		0.030		0.050		0.100		
TARGET ORGAN	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	Target Organ
Adrenals	0.270E−03	35.	0.228E−03	34.	0.175E−03	37.	0.209E−03	28.	0.131E−03	23.	0.101E−03	26.	Adrenals
Bladder	0.757E−02	6.6	0.762E−02	6.5	0.683E−02	6.6	0.625E−02	6.1	0.445E−02	5.6	0.352E-02	5.2	Bladder
GI (stom)	0.570E−02	7.6	0.507E−02	8.0	0.573E−02	7.1	0.560E−02	6.4	0.391E−02	5.8	0.273E−02	5.9	GI (stom)
GI (SI)	0.254E−01	3.6	0.236E−01	3.7	0.234E−01	3.6	0.209E−01	3.4	0.163E−01	3.1	0.120E−01	3.2	GI (SI)
GI (ULI)	0.541E−02	7.8	0.561E−02	7.5	0.647E−02	6.6	0.533E−02	5.9	0.374E−02	5.4	0.262E−02	5.7	GI (ULI)
GI (LLI)	0.350E−02	9.7	0.441E−02	8.5	0.457E−02	7.7	0.285E−02	7.9	0.256E−02	6.2	0.187E−02	6.3	GI (LLI)
Heart	0.756E−02	6.6	0.804E−02	6.3	0.769E-02	6.2	0.635E−02	6.0	0.469E−02	5.4	0.420E−02	5.0	Heart
Kidneys	0.410E−02	9.0	0.446E−02	8.5	0.412E-02	8.3	0.338E−02	7.4	0.233E−02	6.4	0.183E−02	6.6	Kidneys
Liver	0.260E−01	3.5	0.244E−01	3.6	0.249E-01	3.5	0.221E−01	3.3	0.154E−01	3.2	0.120E-01	3.2	Liver
Lungs	0.127E−01	5.1	0.142E−01	4.7	0.138E−01	4.4	0.122E−01	3.8	0.808E−02	3.4	0.551E−02	3.6	Lungs
Marrow	0.560E−01	1.4	0.594E−01	1.4	0.655E−01	1.3	0.740E−01	1.1	0.613E−01	1.1	0.329E−01	1.3	Marrow
Pancreas	0.134E−02	16.	0.103E−02	18.	0.828E−03	17.	0.780E−03	14.	0.567E−03	12.	0.449E−03	12.	Pancreas
Sk. (rib)	0.168E−01	4.4	0.206E−01	3.9	0.247E−01	3.4	0.263E−01	2.9	0.176E−01	2.9	0.764E−02	3.3	Sk. (rib)
Sk. (pelvis)	0.147E−01	4.7	0.160E−01	4.51	0.163E−01	4.3	0.224E−01	3.4	0.199E−01	3.0	0.103E−01	3.3	Sk. (pelvis)
Sk. (spine)	0.186E−01	4.2	0.190E−01	4.1	0.234E−01	3.7	0.253E−01	3.3	0.229E−01	3.0	0.144E−01	3.2	Sk. (spine)
Sk. (skull)	0.103E−01	5.6	0.115E−01	5.3	0.123E−01	5.	0.128E−01	4.6	0.722E−02	5.1	0.313E−02	6.0	Sk. (skull)
Skeleton (total)	0.144	1.4	0.153	1.3	0.167	1.2	0.188	1.1	0.153	1.1	0.810E−01	1.3	Skeleton (total)
Skin	0.258E−01	3.5	0.227E−01	3.5	0.169E−01	3.7	0.116E−01	3.3	0.758E−02	2.9	0.585E−02	3.1	Skin
Spleen	0.260E−02	11.	0.237E−02	12.	0.242E−02	11.	0.223E−02	9.1	0.149E−02	8.5	0.111E−02	8.7	Spleen
Thyroid	0.265E−03	35.	0.263E−03	34.	0.602E−04	48.	0.111E−03	36.	0.114E−03	27.	0.873E−04	29.	Thyroid
Uterus	0.999E−03	18.	0.109E−02	17.	0.122E−02	15.	0.924E−03	13.	0.712E−03	12.	0.611E−03	11.	Uterus
Trunk	0.604	0.47	0.589	0.48	0.566	0.50	0.500	0.55	0.358	0.67	0.245	0.79	Trunk
Legs	0.309	0.86	0.299	0.88	0.285	0.90	0.242	0.97	0.171	1.1	0.113	1.3	Legs
Head	0.488E−01	2.5	0.474E−01	2.5	0.440E−01	2.6	0.342E−01	2.7	0.200E−01	3.1	0.127E−01	3.1	Head
Total body	0.959	0.11	0.933	0.15	0.892	0.19	0.774	0.27	0.548	0.43	0.370	0.56	Total body

(Continued)

TABLE 6-8. Absorbed Fractions (and Coefficients of Variation), Gamma Emitter Uniformly Distributed Throughout the Body[a] (Continued)

TARGET ORGAN	PHOTON ENERGY (MeV) 0.200		0.500		1.000		1.500		2.000		4.000		Target Organ
	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	φ	$100\sigma_\varphi/\varphi$	
Adrenals	0.352E−04	36.	0.138E−03	35.	0.100E−03	42.	0.107E−03	43.	0.114E−03	43.			Adrenals
Bladder	0.327E−02	5.0	0.341E−02	6.6	0.274E−02	8.3	0.291E−02	8.4	0.231E−02	9.6	0.147E−02	12.	Bladder
GI (stom)	0.218E−02	7.0	0.258E−02	7.7	0.181E−02	9.8	0.199E−02	10.	0.212E−02	10.	0.119E−02	14.	GI (stom)
GI (SI)	0.106−01	3.4	0.114E−01	3.8	0.109E−01	4.2	0.915E−02	4.8	0.820E−02	5.2	0.409E−02	7.3	GI (SI)
GI (ULI)	0.256E−02	6.3	0.306−02	7.0	0.228E−02	8.9	0.209E−02	9.4	0.197E−02	10.	0.160E−02	12.	GI (ULI)
GI (LLI)	0.151E−02	7.6	0.184E−02	8.8	0.178E−02	9.7	0.181E−02	11.	0.157E−02	12.	0.673E−03	18.	GI (LLI)
Heart	0.337E−02	5.8	0.372E−02	6.6	0.301E−02	8.1	0.345E−02	7.8	0.312E−02	8.3	0.145−02	13.	Heart
Kidneys	0.171E−02	7.4	0.142E−02	9.7	0.161E−02	10.	0.152E−0.2	11.	0.154E−02	12.	0.904E−03	16.	Kidneys
Liver	0.111−01	3.4	0.101E−01	4.1	0.896E−02	4.7	0.912E−02	4.9	0.847E−02	5.1	0.560E−02	6.4	Liver
Lungs	0.507E−02	4.3	0.496E−02	5.2	0.466E-02	6.1	0.466E−02	6.5	0.427E−02	6.9	0.568E−02	6.4	Lungs
Marrow	0.221E−01	1.5	0.194E−01	1.0	0.182E-01	2.0	0.164E−01	2.2	0.156E−01	2.3	0.969E−02	3.0	Marrow
Pancreas	0.444E−03	14.	0.382E−03	17.	0.534E−03	19.	0.348E−03	22.	0.358E−03	24.	0.142−03	39.	Pancreas
Sk. (rib)	0.505E−02	4.1	0.435E−02	5.6	0.421E−02	6.3	0.405E−02	7.0	0.350E−02	7.7	0.338E−02	8.0	Sk. (rib)
Sk. (pelvis)	0.668E−02	3.9	0.569E−02	5.0	0.562E−02	5.7	0.511E−02	6.3	0.422E−02	7.0	0.256E−02	9.3	Sk. (pelvis)
Sk. (spine)	0.910E−02	3.6	0.763E−02	4.5	0.751E−02	5.1	0.610E−02	5.7	0.606E−02	5.9	0.341E−02	8.1	Sk. (spine)
Sk. (skull)	0.277E−02	6.3	0.304E−02	7.2	0.280E−02	8.0	0.254E−02	9.0	0.292E−02	8.8	0.224E−02	10.	Sk. (skull)
Skeleton (total)	0.550E−01	1.4	0.488E−01	1.7	0.456E−01	2.0	0.413E−01	2.2	0.396E−01	2.3	0.252E−01	3.0	Skeleton (total)
Skin	0.677E−02	3.5	0.757E−02	4.2	0.745E−02	4.8	0.759E−02	5.0	0.664E−02	5.5	0.123E−01	4.3	Skin
Spleen	0.798E−03	11.	0.116E−02	11.	0.914E−03	14.	0.903E−03	16.	0.740E−03	17.	0.368E−03	24.	Spleen
Thyroid	0.418E−04	42.							0.810E−04	46.			Thyroid
Uterus	0.408E−03	15.	0.473E−03	16.	0.517E−03	18.	0.323E−03	23.	0.364E−03	25.	0.238−03	33.	Uterus
Trunk	0.223	0.81	0.225	0.84	0.210	0.92	0.198	0.99	0.186	1.0	0.156	1.2	Trunk
Legs	0.102	1.3	0.101	1.4	0.965E−01	1.5	0.917E−01	1.6	0.846E−01	1.6	0.710E−01	1.8	Legs
Head	0.134E−01	3.2	0.147E−01	3.5	0.145E−01	3.8	0.130E−01	4.1	0.139E−01	4.1	0.127E−01	4.4	Head
Total body	0.338	0.57	0.340	0.60	0.321	0.67	0.302	0.73	0.284	0.77	0.240	0.90	Total body

[a]The digits following the symbol E indicate the powers of 10 by which each number is to be multiplied; A blank in the table indicates that the coefficient of variation was greater than 50%; Total body = head + trunk + legs.

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Substituting the values (and 10^{15} femto Gy(fGy) per Gy):

$$\tilde{A} = 7.35 \times 10^{10}\,\text{Bq} \cdot \text{s}, \quad \sum \varphi_i \Delta_i = 273.12 \frac{\text{kg} \cdot \text{fGy}}{\text{Bq} \cdot \text{s}}, \quad \text{and } m = 70\,\text{kg}$$

into Eq. (6.86) yields

$$D = \frac{7.35 \times 10^{10}\,\text{Bq} \cdot \text{s}}{70\,\text{kg}} \times 273.12 \frac{\text{kg} \cdot \text{fGy}}{\text{Bq} \cdot \text{s}}$$

$$= 2.868 \times 10^{11}\,\text{fGy} \qquad (29\,\text{mrads}).$$

The total dose that the organ will be receiving from the internally administrated radioactivity

The initial dose rate may be found by substituting 10^6 Bq for A_s in Eq. (6.83):

$$\dot{D} = \frac{10^6\,\text{Bq}}{70\,\text{kg}} \times 273.12 \frac{\text{kg} \cdot \text{fGy}}{\text{Bq} \cdot \text{s}}$$

$$= 3.9 \times 10^6\,\text{fGy/s} \quad (1.4\,\text{mrads/h}).$$

6.18 A child drinks 1 liter of milk per day containing ^{131}I at a mean concentration of 33.3 Bq (900 pCi) per liter over a period of 30 days. Assuming that the child has no other intake of ^{131}I, calculate the dose to the thyroid at the end of the 30 days ingestion period, and the dose commitment.

MIRD Method – Another Example, Cember, 6.18

Step 1: Derive the effective half-live of I-131

First calculate the effective half life of the I–131 in the body, use equation 6.54:

$T_R = 8.05$ d
$T_B = 138$ d (ICRP 28)

$$T_E = \frac{T_R \times T_B}{T_R + T_B} = \frac{8.05 \text{ d} \times 138 \text{ d}}{8.05 \text{ d} + 138 \text{ d}} = 7.6 \text{ d effective half life of } {}^{131}\text{I in body.}$$

Converting to effective elimination constant, using equation 6.52:

$$\lambda = \frac{0.693}{T} = \frac{0.693}{7.6 \text{ d}} = 0.091 \text{ d}^{-1}$$

Note that we will use λ to symbolize the effective decay constant in the following derivations.

Step 2: Derive the absorbed dose to the thyroid per I-131 decay

The average energy of each ^{131}I beta particle is found (Figure 6.11), and the yield from each decay is also tabulated:

Energy, MeV/t	Yield, f
0.0701	0.016
0.0955	0.069
0.1428	0.005
0.1917	0.904
0.2856	0.006

The mean β energy/transformation is:

$$\overline{E}_e(\beta) = \Sigma \overline{E} \times f_{\beta i} = (0.0701 \times 0.016) + (0.0955 \times 0.069) + (0.1428 \times 0.005) + (0.1917 \times 0.904) + (0.2856 \times 0.006)$$

$$\overline{E}_e(\beta) = 0.184 \text{ MeV/t}$$

beta dose per decay

Absorption fraction

MeV	*f*	Spec. Abs	MeV/t
0.723	0.016	0.00166	1.92E-05
0.637	0.069	0.00166	7.3E-05
0.503	0.003	0.00166	2.5E-06
0.326	0.002	0.00155	1.01E-06
0.177	0.002	0.00155	5.49E-07
0.365	0.853	0.00155	0.000483
0.284	0.051	0.00155	2.25E-05
0.08	0.051	0.0429	0.000175
0.164	0.006	0.00155	1.53E-06
		Sum	0.000778

Gamma dose to the thyroid per I-131 decay

Calculating the contribution due to the γ, the specific absorbed fraction is found in appendix 4 for an adult. Assume all the ^{131}I is deposited in the thyroid. So for this case, the contribution from the γ is not significant and can be ignored, especially since the child's thyroid is small (~2-5g, ICRP 53).

Step 3: derive the I-131 activity in the thyroid as a function of t

The intake is $q = 33.3$ Bq/day, however, only one–third of the iodine is directly deposited in the thyroid (ICRP 30).

$$K = 33.3\frac{\text{Bq}}{\text{d}} \times \frac{1 \text{ deposited}}{3 \text{ ingested}} = 11.1\frac{\text{Bq}}{\text{d}}$$ ← ***rate of intake***

Some of the iodine is eliminated daily, so find the concentration in the thyroid at any time:

$$\frac{dq}{dt} = \text{deposition} - \text{disappearance}$$

$$\frac{dq}{dt} = K - \lambda \quad q$$

Separating the variables, we have

$$\int_0^q \frac{dq}{(K - \lambda \quad q)} = \int_0^t dt$$

Note that

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b) + C,$$

and consider the following conditions:

$$\begin{cases} a = -\lambda_{eff} \\ b = K \\ q(t=0) = 0 \end{cases}$$

After integration, and solving for q as a function of t;

$$q(t)=\frac{K}{\lambda}\left(1-e^{-\lambda t}\right)$$

$\Rightarrow$

If the uptake of I-131 continues, the dose rate as a function of time is

$$\dot{D}(t)=\frac{\bar{E}}{m}\cdot\frac{K}{\lambda}\cdot\left(1-e^{-\lambda t}\right)$$
$$=\dot{D}_{\infty}\cdot\left(1-e^{-\lambda t}\right)$$

As $t\rightarrow\infty$, q approaches

$$q_{\infty}=\frac{K}{\lambda}=\frac{11.1\frac{\text{Bq}}{\text{day}}}{0.091\ \text{day}}=122\ \text{Bq}$$

$\Rightarrow$

If the uptake of I-131 continues, the saturation dose rate is given by

$$\dot{D}_{\infty}=\frac{\bar{E}}{m}\cdot\frac{K}{\lambda}$$

$m=20$ g (for adult, from appendix C), for a child, assume 10% of adult mass (10 CFR 20), 2 g.

Step 4: Derive the accumulated dose received within the first 30 days.

If the uptake continues, the accumulated dose received by a given time t is

$$D = \int_0^t \dot{D}(t') \cdot dt'$$

where

$$\dot{D}(t) = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t}\right),$$

$$\int_0^t (1 - e^{-\lambda t})\,dt$$

$$= \left[t - \left(-\frac{1}{\lambda}\right)e^{-\lambda t}\right]\Big|_0^t$$

$$= \left[t + \frac{1}{\lambda}e^{-\lambda t}\right] - \left[0 + \frac{1}{\lambda}\right]$$

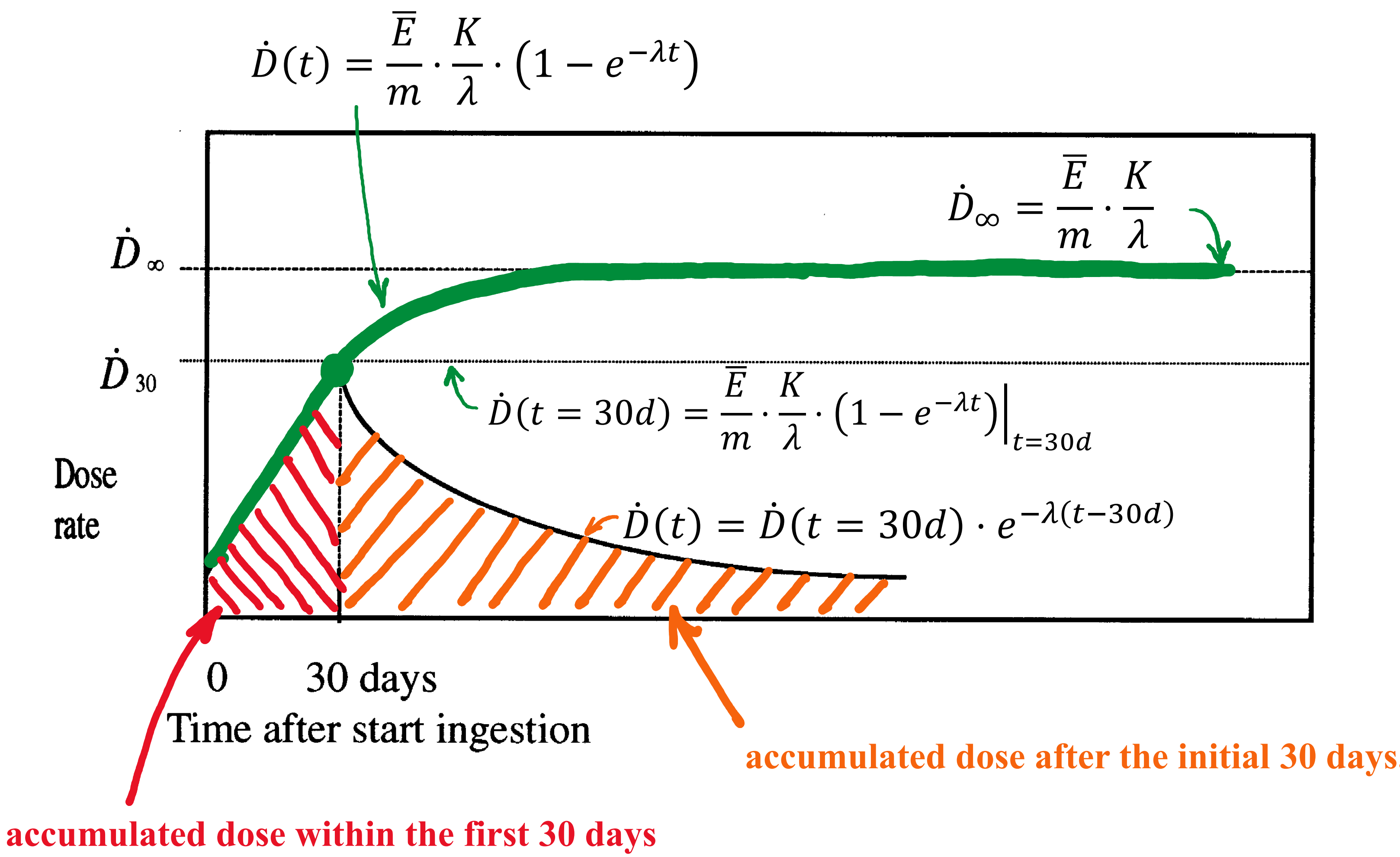
$$= \left[t + \frac{1}{\lambda}(e^{-\lambda t} - 1)\right]$$

and $\overline{E}$ is the mean absorbed energy in the organ per decay of I-131 in the thyroid.

So the **accumulated dose** is given by

$$D = \int_0^t \dot{D}(t') \cdot dt' = \int_0^t \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot \left(1 - e^{-\lambda t'}\right) \cdot dt'$$

$$= \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot \left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right]$$



$\dot{D}(t) = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot (1 - e^{-\lambda t})$
$\dot{D}_\infty = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda}$
$\dot{D}_\infty$
$\dot{D}_{30}$
$\dot{D}(t = 30d) = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda} \cdot (1 - e^{-\lambda t})\Big|_{t=30d}$
Dose rate
$\dot{D}(t) = \dot{D}(t = 30d) \cdot e^{-\lambda(t-30d)}$
0
30 days
Time after start ingestion
accumulated dose after the initial 30 days.
accumulated dose within the first 30 days

Step 4: Derive the accumulated dose received within the first 30 days (continued)

Using the following equation for accumulated dose till time t,

$$D = \dot{D}_{\infty}\left[t + \frac{1}{\lambda}\left(e^{-\lambda t} - 1\right)\right] \quad \textit{and} \quad \dot{D}_{\infty} = \frac{\overline{E}}{m} \cdot \frac{K}{\lambda},$$

the accumulated dose at 30 days is:

$$\dot{D}_{\infty} = 0.155 \text{ mGy/day}$$
$$\lambda = 0.091 \text{ d}^{-1}$$
$$t = 30 \text{ d}$$

$$D = 0.155\frac{\text{mGy}}{\text{d}} \times \left[30 \text{ d} + \frac{1}{0.091\text{d}^{-1}} \times \left(\text{e}^{-0.091\times(30)} - 1\right)\right] = 3 \text{ mGy}$$

3 mGy is the accumulated dose at the end of the 30 day period.

Internally Deposited Radioisotope: Total Dose or Dose Commitment (Revisited)

- The **total dose** received during a time interval t after the deposition of the isotope is

$$D = \dot{D}_0 \int_0^t e^{-\lambda\ t} dt\ = \frac{\dot{D}_0}{\lambda_E}(1 - e^{-\lambda\ t})$$

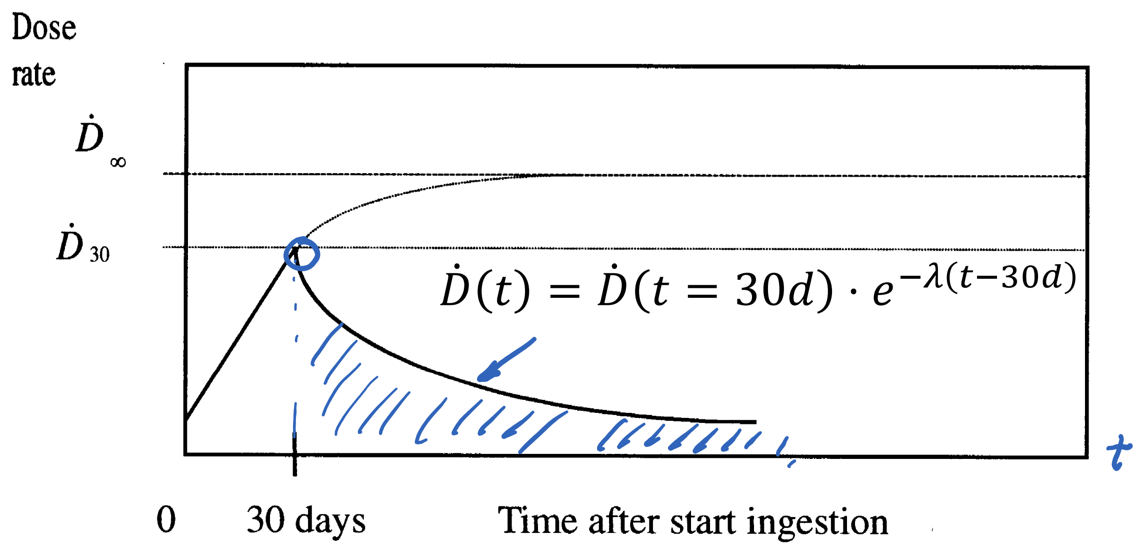
For an infinitely long time—that is, when the isotope is completely gone-

$$D = \frac{\dot{D}_0}{\lambda}$$

- For practical purpose, an infinitely long time corresponding to about 6 half-lives. The total dose received from complete decay is called the **dose commitment**.

Step 5: Derive the total dose (dose commitment) received after t=30d.

The dose commitment is the sum of the dose accumulated during intake, and then during elimination (washout).

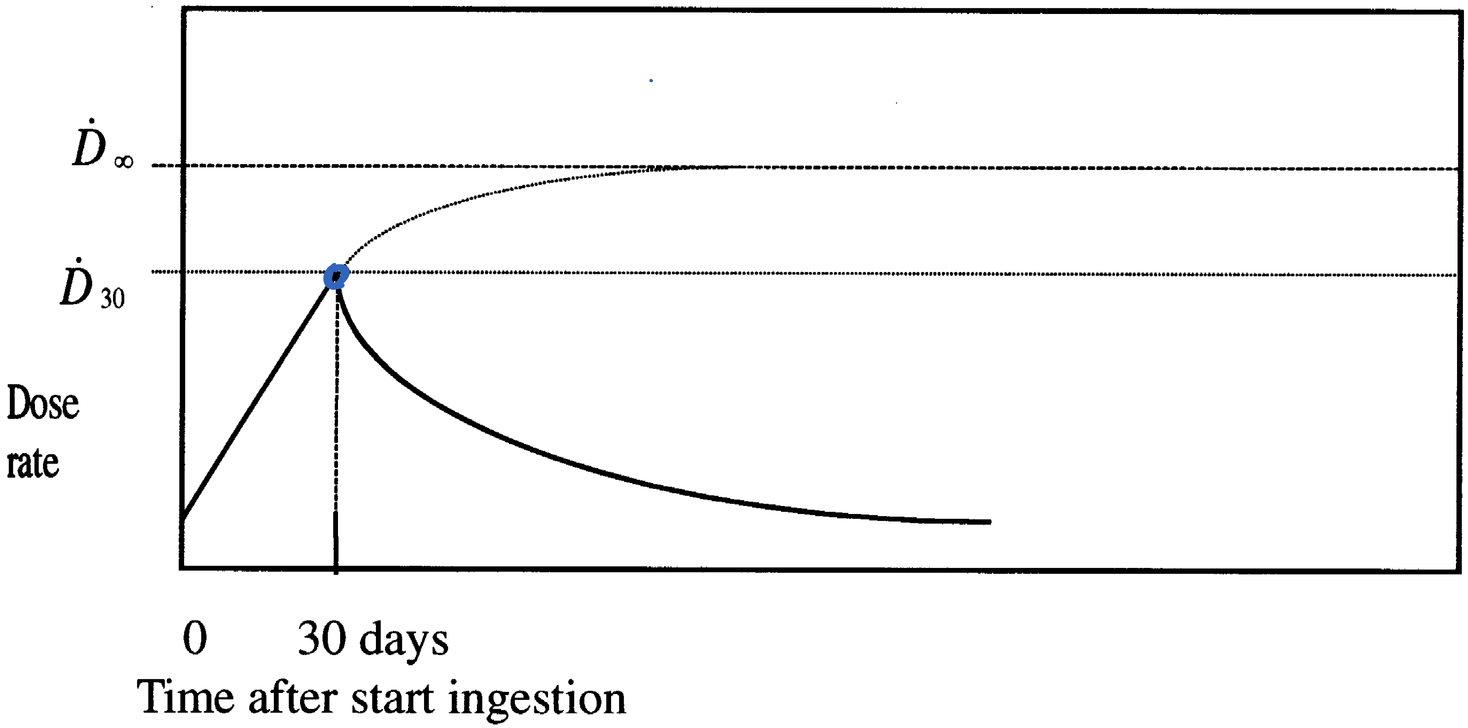


Find the dose rate at $t = 30$ days

$$\dot{D}_{30} = \dot{D}_{\infty}(1 - e^{-\lambda t}) = 0.155\frac{\text{mGy}}{\text{d}} \times \left(1 - e^{-0.091\times(30)}\right) = 0.145\ \frac{\text{mGy}}{\text{day}}$$

$$D = \frac{\dot{D}_{30}}{\lambda} = \frac{0.145\frac{\text{mGy}}{\text{d}}}{0.091\frac{1}{\text{d}}} = 1.59\ \text{mGy}$$ is the dose after ingestion stops.

(Final) Step 6: Derive the total dose (dose commitment) from the initial intake of I-131



The total dose, from the time intake started to the end of the first 30 days, plus the dose after the intake stopped will be;

3 mGy + 1.6 mGy = 4.6 mGy total dose to the childs thyroid.