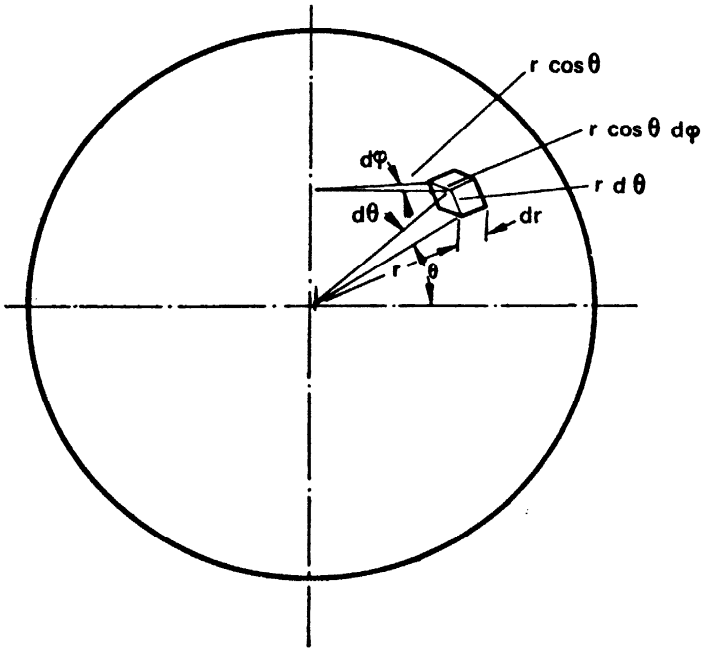
Radiation Dose Induced by Gamma Radiation

Specific Gamma-Ray Constant

- **Specific Gamma Ray Constant (Γ):** The gamma ray ***exposure rate*** from a point source of a unit activity at a unit distance. It is given in unit of coulombs per kilogram per hour at 1m from a 1 MBq point source.
- It is a useful numerical quantity for predicting the exposure resultant from a given gamma ray emitting radioisotope.



Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x<<1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident gamma ray beam is $\dot{\Psi}_0\left(\mathrm{J}\cdot\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\right)$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0\mu_{en}x\ \ (J\cdot cm^{-2}\cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness x is

$$A\dot{\Psi}_0\mu_{en}x\ (J\cdot s^{-1})$$

Given the density of the material is ρ, the rate of energy absorption per unit mass (Dose Rate) in the slab is

$$\dot{D}=\frac{A(cm^2)\cdot\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\cdot\mu_{en}(cm^{-1})\cdot x\ (cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$Dose\ rate\ in\ the\ absorber:\ \ \dot{D}(J\cdot g^{-1}\cdot s^{-1})=\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\frac{\mu_{en}(cm^{-1})}{\rho(g/cm^3)}$$

Specific Gamma-Ray Constant

TABLE 6.3. Specific Gamma-ray Constant of Some Radioisotopes

Isotope	Γ	
	$\frac{R \cdot m^2}{Ci \cdot h}$ [a]	$\frac{X \cdot m^2}{MBq \cdot h}$ [b]
Antimony 122	0.24	1.67E—09
Cesium 137	0.33	2.30E—09
Chromium 51	0.016	1.11E—10
Cobalt 60	1.32	9.19E—09
Gold 198	0.23	1.60E—09
Iodine 125	0.07	4.87E—10
Iodine 131	0.22	1.53E—09
Iridium 192	0.48	3.34E—09
Mercury 203	0.13	9.05E—10
Potassium 42	0.14	1.39E—09
Radium 226	0.825	5.75E—09
Sodium 22	1.20	8.36E—09
Sodium 24	1.84	12.80E—09
Zinc 65	0.27	1.88E—09

[a] From *Radiological Health Handbook*, rev. ed., U.S. Public Health Service, Bureau of Radiological Health, Rockville, MD, 1970.
[b] 1 X unit = 1 C/kg.

Cember, pp. 226.

Specific Gamma-Ray Constant

Given the specific gamma ray constant for an isotope, the exposure rate induced at a location at a distance, r, is simply

$$\dot{X} = \Gamma \frac{A}{r^2}$$

activity

Example

(a) Estimate the specific gamma-ray constant for ^{137}Cs. (b) Estimate the exposure rate at a distance of 1.7 m from a 100-mCi point source of ^{137}Cs.

Solution

(a) The isotope emits only a 0.662-MeV gamma ray in 85% of its transformations (Appendix D). The average energy per disintegration released as gamma radiation is therefore $0.85 \times 0.662 = 0.563$ MeV. The estimated specific gamma-ray constant for ^{137}Cs is therefore $\Gamma =$ $0.28\ \mathrm{R\,m^2\,Ci^{-1}\,h^{-1}}$.

(b) From Eq. (12.28), the exposure rate at a distance $r = 1.7$ m from a point source of activity $C = 100$ mCi $= 0.1$ Ci is

$$\dot{X} = 0.28 \frac{\mathrm{R\ m^2}}{\mathrm{Ci\ h}} \times \frac{0.1\ \mathrm{Ci}}{(1.7\ \mathrm{m})^2} = 9.7 \times 10^{-3}\ \mathrm{R\,h^{-1}} = 9.7\ \mathrm{mR\,h^{-1}}. \qquad (12.29)$$

Turner, pp. 382.

Specific Gamma-Ray Constant

- **Specific Gamma Ray Constant (Γ):** The exposure rate from a gamma ray point source of unit activity and positioned at a unit distance. It is given in unit of coulombs per kilogram per hour at 1 m from a 1 MBq point source.
- It is a numerical quantity used to predict the exposure resultant from a given gamma ray emitting radioisotope – A measure of the ability of a gamma-ray source to deliver exposure and dose.

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

- For gamma rays, one can no longer assume that the organ is infinitely large.
- For a uniformly distributed gamma ray emitting isotope, the dose rate from the isotope in an infinitesimal volume *dV to* Point ***p*** at a distance ***r*** away is

Mass energy absorption coefficient of the dose receiving media

$$d\dot{D} = C \cdot \Gamma \cdot \frac{e^{-\mu r}}{r^2} \cdot dV \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

where C is the concentration of the isotope, Γ is the specific gamma-ray emission, and μ is the linear energy absorption coefficient.

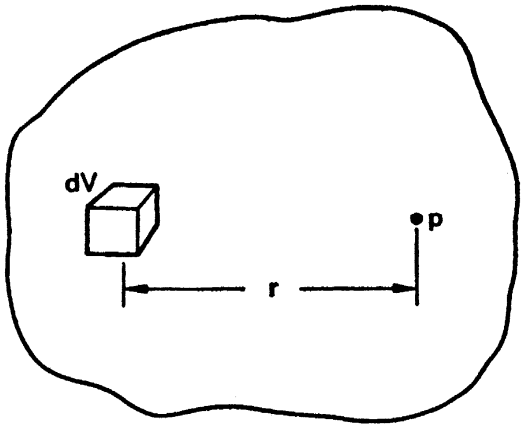


FIGURE 6.8. Diagram for calculating dose at point *p* from the gamma rays emitted from the volume element d*V* in a tissue mass containing a uniformly distributed isotope.

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

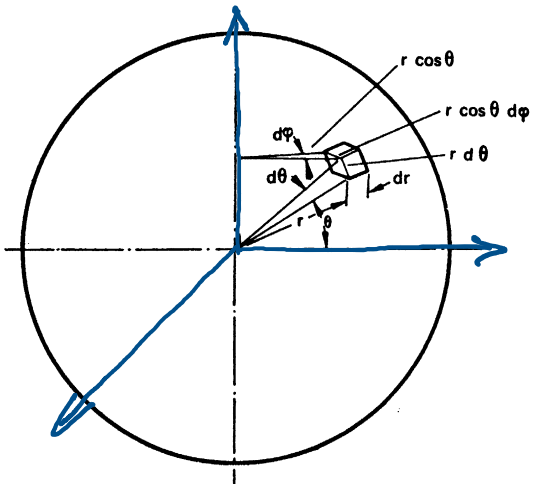
- As an example, for a uniform spherical source, the dose rate at the center is given by

$$\dot{D} = 4C\Gamma \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi/2} \int_{\varphi=0}^{\varphi=\pi} \frac{e^{-\mu r}}{r^2} \cdot r \, d\theta \cdot r\cos\theta \, d\varphi \cdot dr \cdot (34 \frac{J/kg}{Coulomb/kg}) \cdot (\frac{\mu_m/\rho_m}{\mu_a/\rho_a})$$

$$= C\Gamma \cdot \frac{4\pi}{\mu}(1 - e^{-\mu R}) \cdot (34 \frac{J/kg}{Coulomb/kg}) \cdot (\frac{\mu_m/\rho_m}{\mu_a/\rho_a})$$

- And the dose rate at the surface of the spherical source volume is given by

$$\dot{D}_{surface} = 0.5 \, \dot{D}_{center}$$



Example 6.12

A spherical tank, capacity 1 m^3 and radius 0.62 m, is filled with aqueous ^{137}Cs waste containing a total activity of 37,000 MBq (1 Ci). What is the dose rate at the tank surface if we neglect absorption by the tank wall?

Example 6.12

A spherical tank, capacity 1 m^3 and radius 0.62 m, is filled with aqueous ^{137}Cs waste containing a total activity of 37,000 MBq (1 Ci). What is the dose rate at the tank surface if we neglect absorption by the tank wall?

From Table 6.3 we find $\Gamma = 2.3 \times 10^{-9}$ X units/h/MBq at 1 m. Since water absorbs 38 Gy/X unit, the dose rate is 8.74×10^{-8} Gy/h/MBq at 1 m. The absorption coefficient of water for the 0.661 MeV gammas from ^{137}Cs is listed in Table 5.3 as 0.0327 cm^2/g. Since the density of water is 1 g/cm^3, the linear absorption coefficient is 0.0327/cm, or 3.27/m. The dose rate at the center of the sphere is found by substituting the respective values into Eq. (6.66):

$$\dot{D}_0 = C\Gamma \cdot \frac{4\pi}{\mu}(1 - e^{-\mu R}) \cdot \left(34 \frac{J/kg}{C/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_{air}/\rho_{air}}\right)$$

$$\dot{D}_0 = 3.7 \times 10^4 \text{ MBq/m}^3 \times 8.74 \times 10^{-8} \frac{\text{Gy} \cdot \text{m}^2}{\text{MBq} \cdot \text{h}} \times \frac{4\pi}{3.27 \text{ m}^{-1}}(1 - e^{-3.27 \times 0.62})$$

$$= 1.08 \times 10^{-2} \text{ Gy/h (1.08 rad/h).}$$

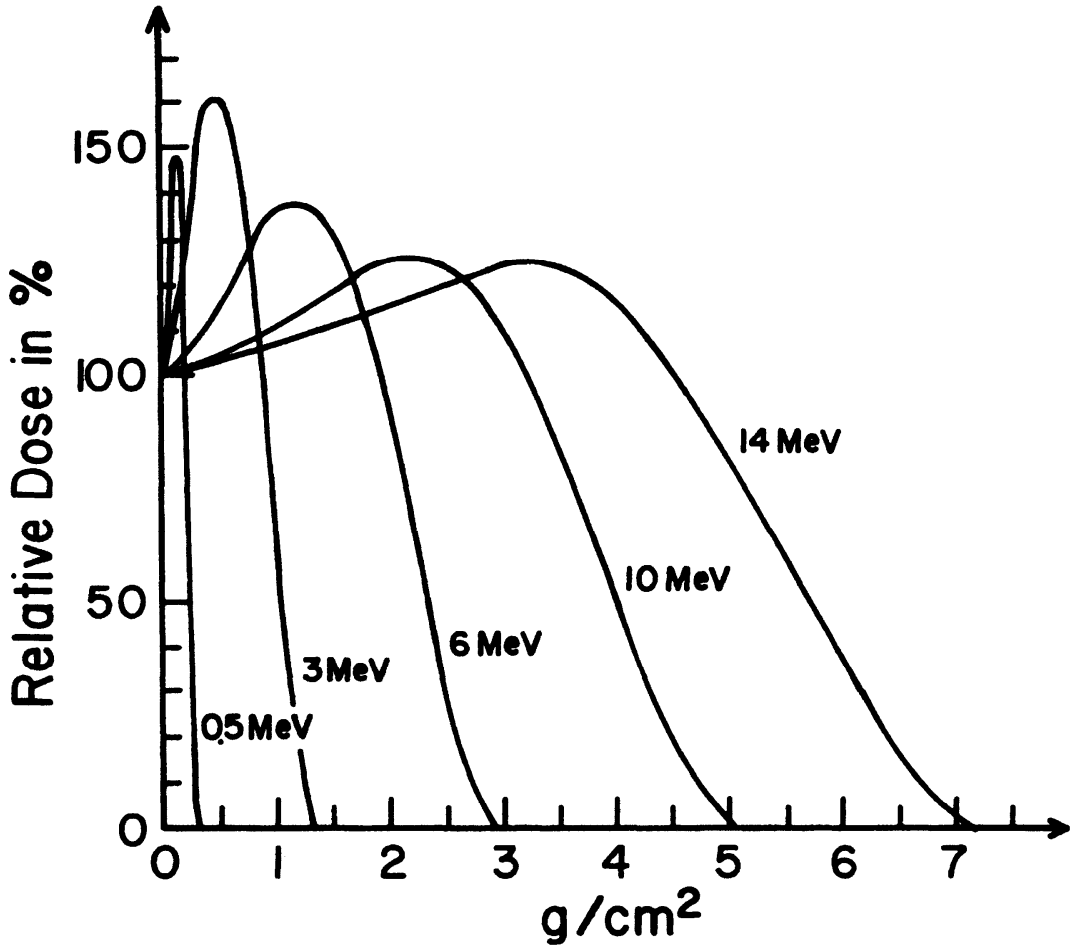
From Eq. (6.71), we see that the surface dose rate is $0.5 \times \dot{D}_0$.

Therefore

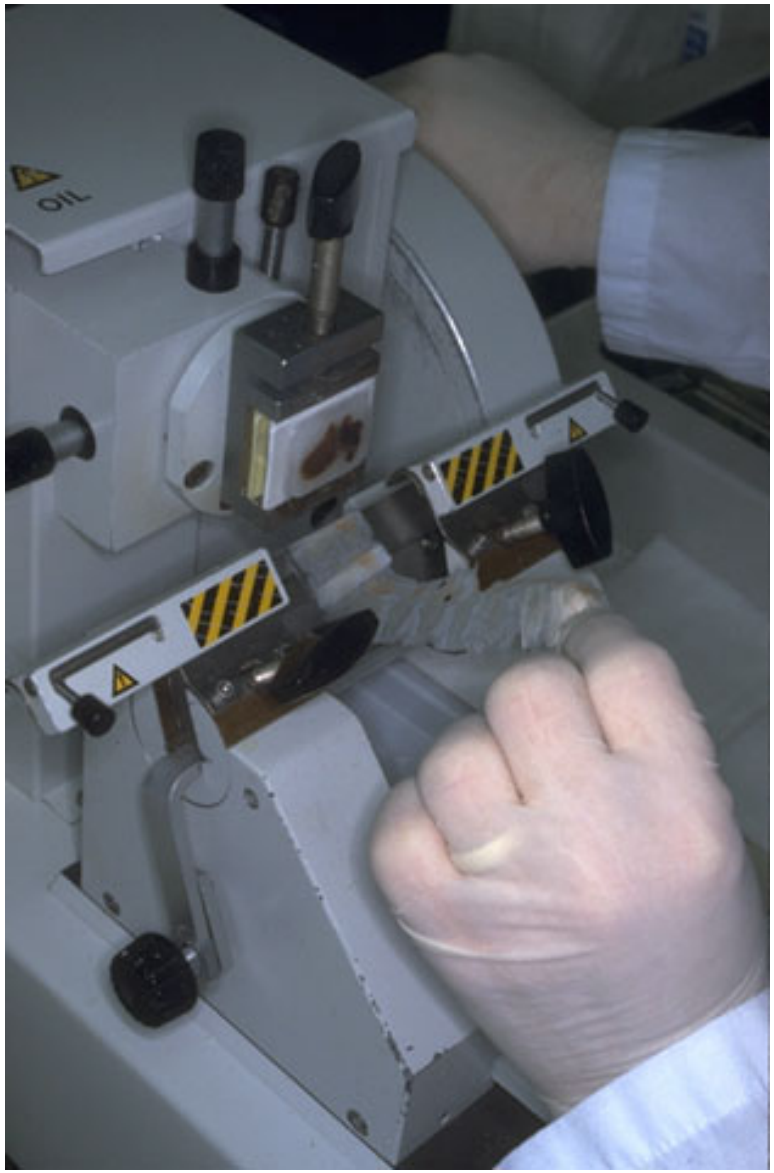
$$\dot{D}_{\text{surface}} = 0.5 \times 1.08 \times 10^{-2} = 0.54 \times 10^{-2} \text{ Gy/h (0.54 rad/h).}$$

Radiation Dose Induced by Beta Radiation

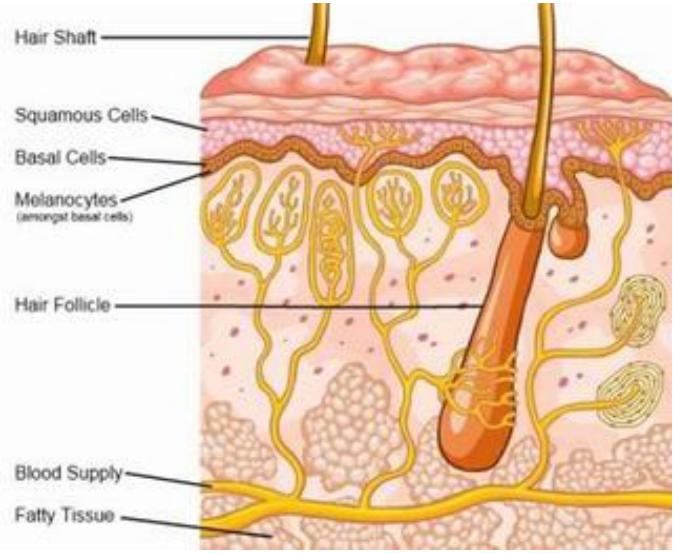
Dose from Monochromatic Electron Beam as a Function of Depth



URE 8.14*a*. Dose vs. depth in water for broad electron beams of the indicated incident rgies. (After Holm, 1969. Reproduced with permission of N. W. Holm and Academic Press.)



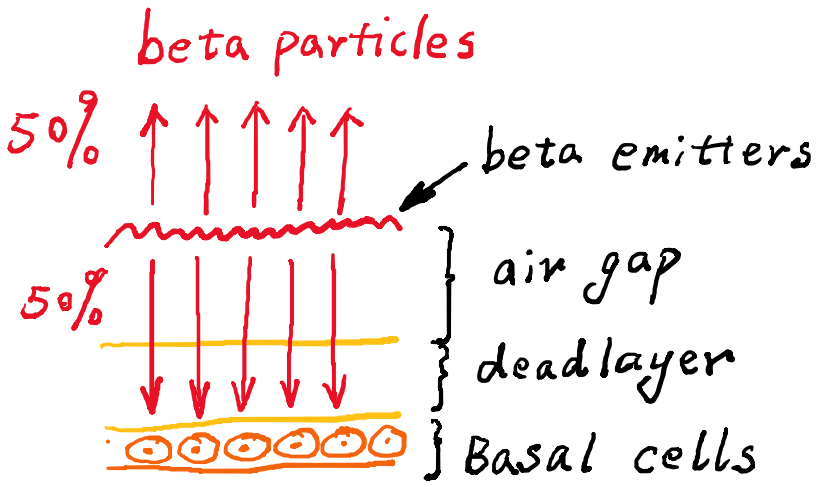
Handling Beta-Emitting Radioisotope by hand

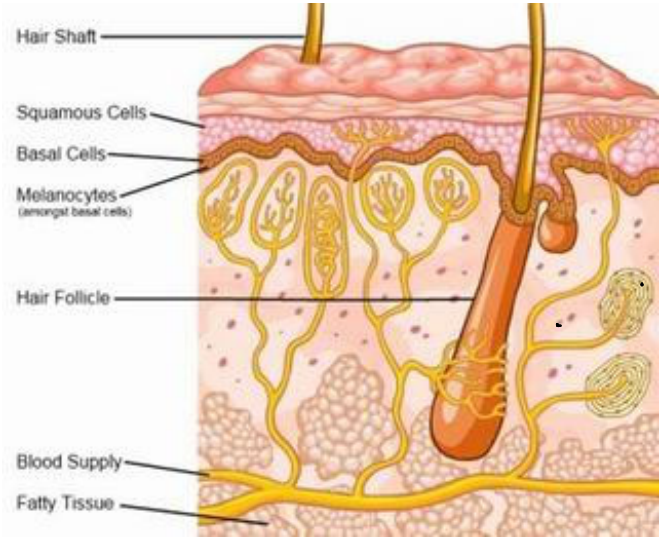


http://www.ehealthzine.com/what-is-skin-cancer-what-causes-skin-cancer.html

Skin Dose from Surface Contamination

1. Beta particles are very easy to attenuate, for beta particles to contribute to skin dose, the source must be very close to the skin.
2. If we consider a thin layer of beta emitters placed parallel to the skin at a very small distance, we could assume that beta particles are traveling in parallel beams along two opposite directions only – 50% going up and 50% going down.
3. For skin dose, we only consider the dose deliver to the thin layer of Basel cells under the dead-layer (density thickness: 0.007g/cm^2) of the skin.





Beta Dose

4. The intensity of beta particles in a parallel beam decreases approximately exponentially with absorber thickness:

$$\varphi = \varphi_0 e^{-\mu_\beta t},$$

where φ = intensity at depth t
φ_0 = initial intensity
μ_β = beta-ray absorption coefficient.

5. For beta particles emitted by a beta emitter of maximum energy E_m (MeV), the corresponding beta-ray absorption coefficients for air and for tissue could be approximately given as

$$\mu_{\beta,a}\ (\text{air}) = 16(E_m - 0.036)^{-1.4}\ \frac{\text{cm}^2}{\text{g}}$$

$$\mu_{\beta,t}\ (\text{tissue}) = 18.6(E_m - 0.036)^{-1.37}\ \frac{\text{cm}^2}{\text{g}}$$

6. If a beta particle of energy *E* is attenuated in the absorption media, we assume that all the energy originally carried by the beta particle will be locally absorbed and contributing to the absorbed dose.

Calculation of Energy Transfer and Energy Absorption

Assuming $\mu_{en}x<<1$, which is consistent with the thin slab approximation and the energy fluence rate carried by the incident beam of particles is $\dot{\Psi}_0\left(\mathrm{J}\cdot\mathrm{cm}^{-2}\cdot\mathrm{s}^{-1}\right)$. Then the energy absorbed in the thin slab per second over a unit cross section area is given by

$$\dot{\Psi}_0\mu_{en}x\ \ (J\cdot cm^{-2}\cdot s^{-1})$$

The rate of energy absorbed in the slab of area A (cm^2) and thickness x is

$$A\dot{\Psi}_0\mu_{en}x\ (J\cdot s^{-1})$$

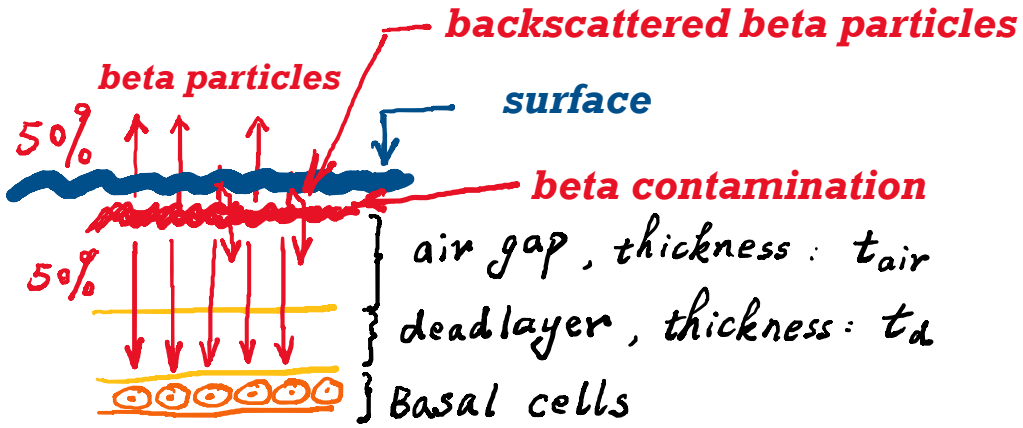
Given the density of the material is $\boldsymbol{\rho}\left(g/cm^3\right)$, and the linear energy absorption coefficient is $\boldsymbol{\mu_{en}}(cm^{-1})$, the **rate of energy absorption per unit mass (Dose Rate)** in the slab is

$$\dot{D}=\frac{A\left(cm^2\right)\cdot\dot{\Psi}_0\left(J\cdot cm^{-2}\cdot s^{-1}\right)\cdot\mu_{en}(cm^{-1})\cdot x(cm)}{\rho(g\cdot cm^{-3})\cdot A(cm^2)\cdot x(cm)},$$

$$Dose\ rate\ in\ the\ absorber:\ \ \dot{D}(J\cdot g^{-1}\cdot s^{-1})=\dot{\Psi}_0(J\cdot cm^{-2}\cdot s^{-1})\frac{\mu_{en}(cm^{-1})}{\rho(g/cm^3)}$$

Skin Dose from Surface Contamination

Skin Dose from Surface Contamination



For a planar beta emitting surface, the surface dose rate may be easily calculated. Suppose the surface concentration is C_a Bq/cm^2, the dose rate to the basal cell region is

surface concentration is C_a (Bq/cm^2)

Average energy of beta particles

Mass attenuation coefficient of tissue (cm^2/g)

$$\dot{D} = C_a \cdot 0.5 \cdot f_b \cdot \bar{E} \cdot e^{-\mu_{air} \cdot t_{air}} \cdot e^{-\mu_d \cdot t_d} \cdot \mu_\beta$$

Backscattering correction, 1.25

Attenuation by air

Attenuation by dead skin layer

energy fluence rate, (J/cm^2/sec)

Beta Radiation – Dose from Surface Contamination
An Example (Cember, p. 190)

Example 6.7

A solution of ^{32}P is spilled, and contaminates a large surface to an areal concentration of 37 Bq/cm^2. What is the estimated beta-ray-contact dose rate to the skin and the dose rate at a height of 1 m above the contaminated area?

For ^{32}P:

$E_m = 1.71$ MeV $\quad \bar{E} = 0.7$ MeV.

The beta absorption coefficients in air and in tissue are calculated by substituting 1.71 for the value of E_m in Eqs. (6.20) and (6.21):

$$\mu_{\beta,a} = 16(1.71 - 0.036)^{-1.4} \frac{\text{cm}^2}{\text{g}} = 7.78 \frac{\text{cm}^2}{\text{g}}$$

$$\mu_{\beta,t} = 18.6(1.71 - 0.036)^{-1.37} \frac{\text{cm}^2}{\text{g}} = 9.18 \frac{\text{cm}^2}{\text{g}},$$

Beta Radiation – Dose from Surface Contamination
An Example (Cember, p. 190)

and the dose rate to the skin in contact with the contaminated area is calculated with Eq. (6.26):

energy fluence rate *attenuation in the dead layer*

$$\dot{D}_\beta, \frac{\text{Gy}}{\text{h}} = \frac{\left(3.6 \times 10^{-10} \times C_a \times \overline{E}\right) \frac{\text{J}}{\text{cm}^2} \Big/ \text{h} \times \mu_{\beta,t} \frac{\text{cm}^2}{\text{g}}}{0.001 \frac{\text{J}}{\text{g}} \Big/ \text{Gy}} \times e^{-.007 \frac{\text{g}}{\text{cm}^2} \times \mu_{\beta,t} \frac{\text{cm}^2}{\text{g}}}$$

$$= \frac{3.6 \times 10^{-10} \times 37 \times 0.7 \times 9.18}{0.001} \times e^{-0.007 \times 9.18} = 8 \times 10^{-5} \frac{\text{Gy}}{\text{h}} = 0.08 \frac{\text{mGy}}{\text{h}}.$$

The dose rate to the skin, at a height of 1 m above the contaminated surface, is calculated with Eq. (6.31):

extra attenuation by the air gap

$$\dot{D}_b = 3.6 \times 10^{-4} \times C_a \times \overline{E} \times e^{-\mu_{\beta,a} d} \times e^{-\mu_{\beta,t} \times 0.007} \times \mu_{\beta,t} \frac{\text{mGy}}{\text{h}}$$

$$= 3.6 \times 10^{-4} \times 37 \times 0.7 \times e^{-7.78 \times 0.129} \times e^{-9.18 \times 0.007} \times 9.18 = 2.9 \times 10^{-2} \frac{\text{mGy}}{\text{h}}.$$

Beta Radiation – Submersion Dose

Exposure Measurement: The Free Air Chamber

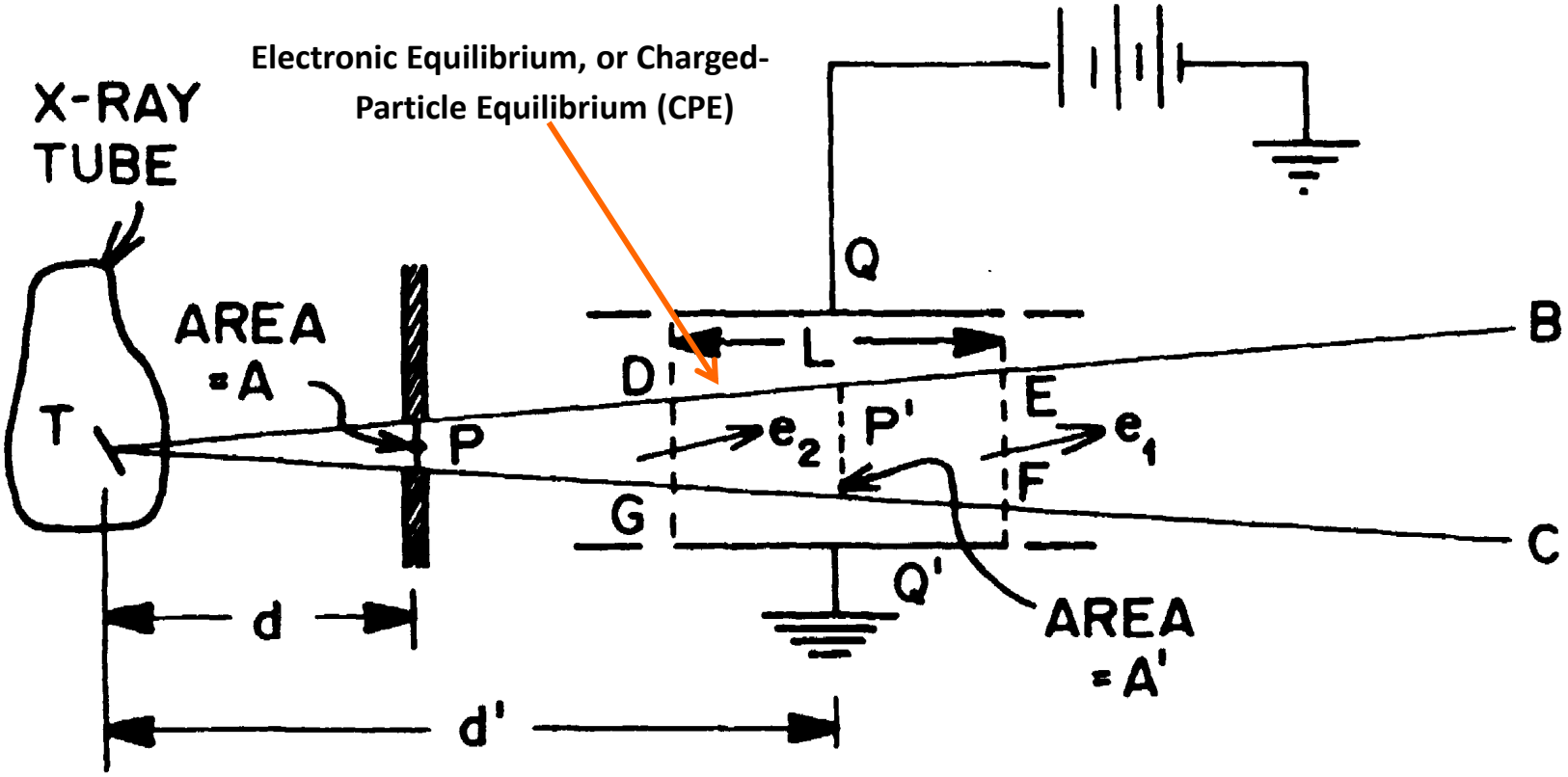


Fig. 12.1 Schematic diagram of the "free-air" or "standard" ionization chamber.

Turner, pp. 365.

Charged-Particle Equilibrium (CPE)

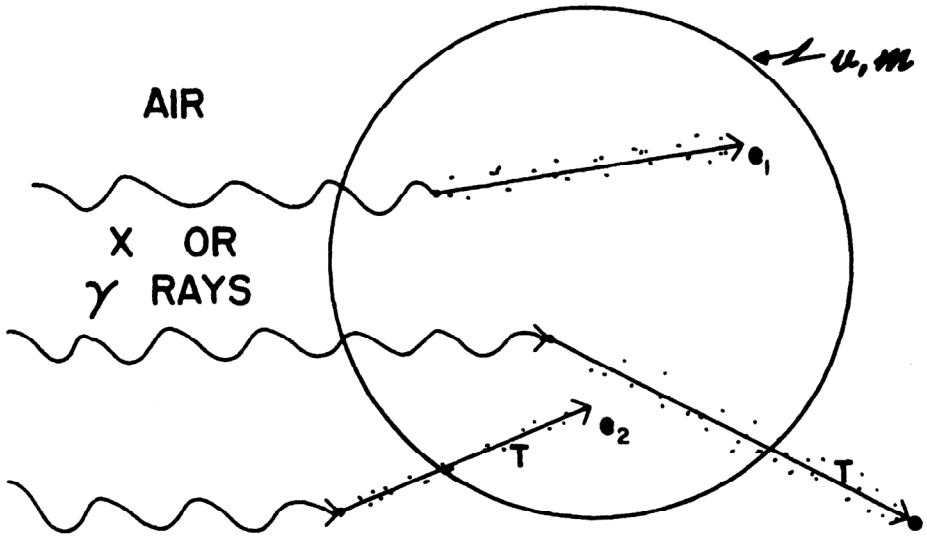


FIGURE 4.5. The role of CPE in the measurement of exposure X. The average exposure in the finite air volume v equals the total charge of either sign released in air by all electrons (e_1) that originate in v, divided by the air mass m in v. If CPE exists, each electron carrying an energy (say, T) out of v is compensated by another electron (e_2) carrying the same energy in. Thus the same ionization occurs in v as if all electrons e_1 remained there. The measurement of that charge divided by m is thus equivalent to a measurement of the average exposure in v. Radiative losses are assumed to escape from v, and any ionization they produce is not to be included in X.

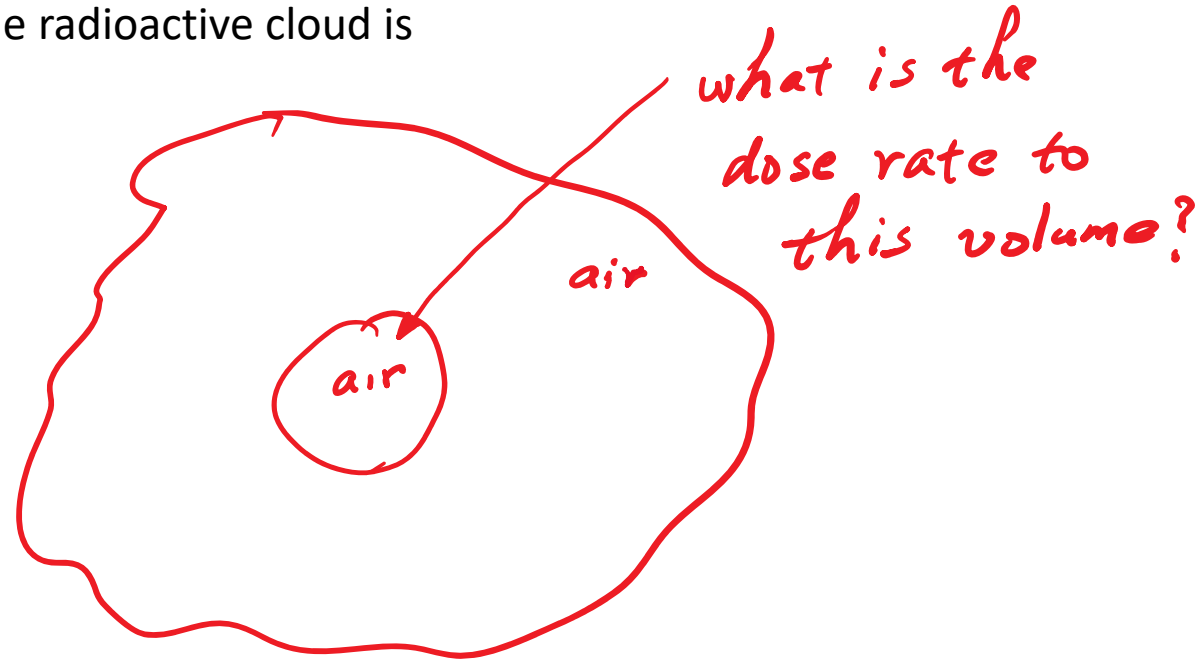
From <<Introduction to Radiological Physics and Radiation Dosimetry>>

Beta Radiation – Submersion Dose

- For **a small volume of air inside an infinite cloud of beta emitting radionuclide**, we have

$$\text{Rate of energy emission} = \text{rate of energy absorption}$$

- In an infinite cloud containing C Bq/m^3 of a beta emitter, the dose rate in a small volume inside the radioactive cloud is



Beta Radiation – Submersion Dose

- For **a small volume of air inside an infinite cloud of beta emitting radionuclide**, we have

$$\text{Rate of energy emission} = \text{rate of energy absorption}$$

- In an infinite cloud containing C Bq/m^3 of a beta emitter, the dose rate in a small volume inside the radioactive cloud is

$$\dot{D}_{\text{inf}}, \frac{\text{mGy}}{\text{h}} = \frac{C\frac{\text{Bq}}{\text{m}^3} \times 1\frac{\text{tps}}{\text{Bq}} \times \overline{E}\frac{\text{MeV}}{\text{t}} \times 1.6 \times 10^{-13}\frac{\text{J}}{\text{MeV}} \times 3.6 \times 10^{3}\frac{\text{s}}{\text{h}}}{1.293\frac{\text{kg}}{\text{m}^3} \times 1\frac{\text{J}}{\text{kg}}\Big/\text{Gy} \times \frac{1\ \text{Gy}}{10^3\ \text{mGy}}}$$

$$\dot{D}_{\text{inf}} = 4.45 \times 10^{-7} \times C \times \overline{E}\frac{\text{mGy}}{\text{h}}$$

Understanding the Radiation from Cs-137

Decay scheme:

$$^{137}_{55}\mathrm{Cs} \rightarrow {}^{137}_{56}\mathrm{Ba} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}.$$

What will happen to the excited Ba-137 nucleus?

http://faithandsurvival.com/wp-content/uploads/2023/04/fukushima-cesium-137-spread.jpg

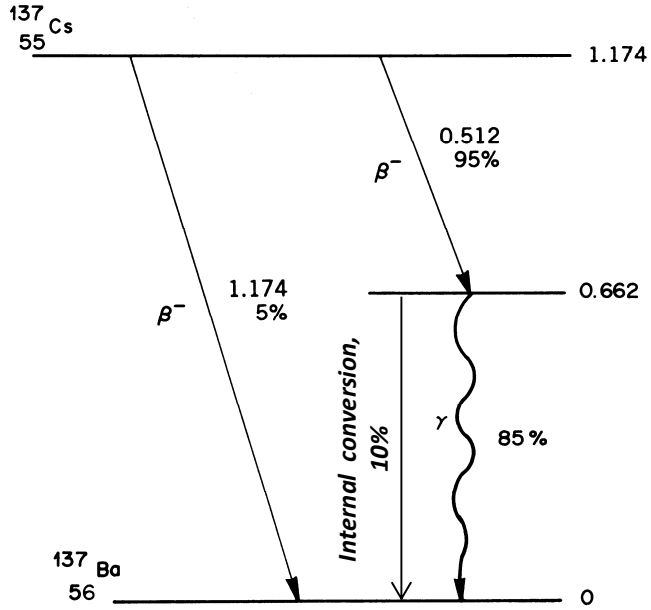
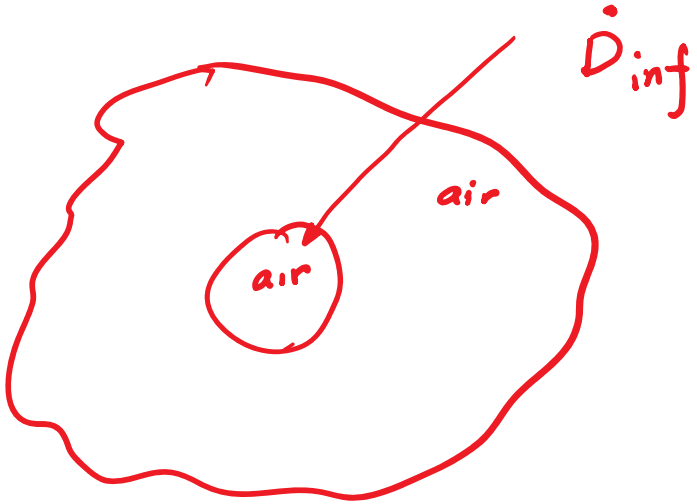


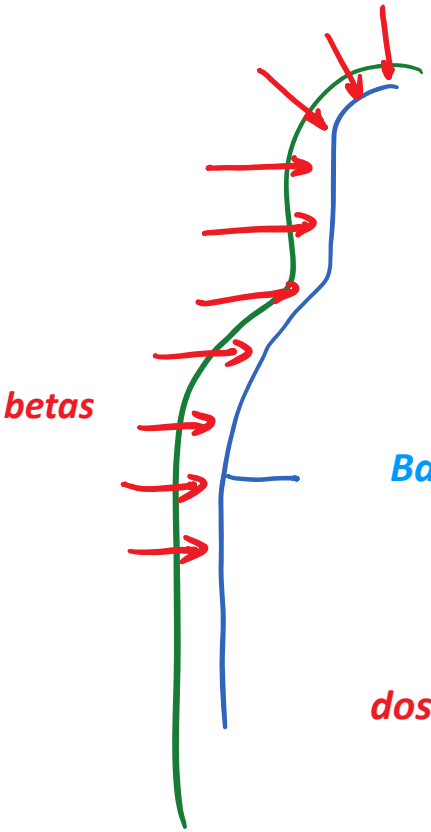
FIGURE 3.8. Decay scheme of $^{137}_{55}$Cs.

Skin Dose for Human Submersed in Radioactive Cloud Containing Beta Emitters

Human Submersed in Radioactive Cloud Containing Beta Emitters



Rate of energy emission = rate of energy absorption



dose rate to the skin, D_b ??

$$\dot{D}_b = 0.5 \times 1.1 \times \dot{D}_{\text{inf}}\ (\text{air}) \times e^{-\mu_{\beta,t} \times 0.007}$$

Human Submersed in Radioactive Cloud Containing Beta Emitters

- Because (a) the skin is irradiated from one side only, and (b) soft tissue absorbs about 10% more energy per kilogram than dose in air, the dose rate to the basal cells of the skin in a semi-infinite medium is

$$\dot{D}_{\text{inf}} = 4.45 \times 10^{-7} \times C \times \overline{E}\ \frac{\text{mGy}}{\text{h}}$$

$$\dot{D}_b = 0.5 \times 1.1 \times \dot{D}_{\text{inf}}\ (\text{air}) \times e^{-\mu_{\beta,t} \times 0.007}$$

- Therefore, the dose rate to the skin of a person immersed in a large cloud of concentration C Bq/m^3 is

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \overline{E} \times e^{-\mu_{\beta,t} \times 0.007}\ \frac{\text{mGy}}{\text{h}}$$

- Generally, if the cloud consists of several groups of betas with different maximum energies, the beta dose rate is

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \sum_i f_i \overline{E}_i e^{-\mu_{\beta i,t} \times 0.007}\ \frac{\text{mGy}}{\text{h}}$$

f_i : Fraction of the *i' th* group of beta particles

$\overline{E}_i$: Average energy of beta particles

$\mu_{\beta i,\ t}$: Linear energy absorption coefficient of tissue.

Beta Radiation – Submersion Dose

An example. Cember, pp. 231.

Calculate the dose rate to the skin of a person immersed in a large cloud of ^{85}Kr at a concentration of 37 kBq/m^3 (10^{-6} μCi/mL).

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \overline{E} \times e^{-\mu_{\beta,t} \times 0.007} \frac{\text{mGy}}{\text{h}}$$

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \sum_i f_i \overline{E}_i e^{-\mu_{\beta i,t} \times 0.007} \frac{\text{mGy}}{\text{h}}$$

Solution

Krypton-85 is a pure beta emitter that is transformed to ^{85}Rb by the emission of a beta particle whose maximum energy is 0.672 MeV and whose average energy is 0.246 MeV. The tissue absorption coefficient is calculated with Eq. (6.21):

$$\mu_{\beta,t} = 18.6(0.672 - 0.036)^{-1.37} = 34.6 \text{ cm}^2/\text{g},$$

and the skin dose is calculated with Eq. (6.38):

$$\dot{D}_b = 2.45 \times 10^{-7} \times C \times \bar{E} \times e^{-(\mu_{\beta,t} \times 0.007)} \text{ mGy/h}$$

$$\dot{D}_b = 2.45 \times 10^{-7} \times 3.7 \times 10^4 \times 0.246 \times e^{-(34.6 \times 0.007)}$$

$$\dot{D}_b = 1.8 \times 10^{-3} \text{ mGy/h (0.18 mrads/h)} .$$

The Fukushima nuclear **disaster** is estimated to have released between 20-200 megacuries of **Krypton 85** from three melted down reactors

Organ Dose from Internally Deposited Beta Emitters

Internally Deposited Radioisotope (I)
Corpuscular Beta Radiation

- For an infinitely large medium containing a uniformly distributed radioisotope, **the concentration of absorbed energy must be equal to the concentration of energy emitted by the isotope**.
- The energy absorbed per unit mass per transformation in a given organ is called the **specific effective energy** (SEE).
- For practical health physics purposes, "infinitely large" may be approximated by a tissue mass whose dimension exceed the range of the radiation.
- For alpha and beta radiation, this condition can be easily met, so that the SEE is simply the average energy of the radiation divided by the mass of the tissue in which it is distributed.

$$SEE\ (\alpha \text{ or } \beta) = \frac{\bar{E}(\alpha \text{ or } \beta)}{m} \frac{\text{MeV}}{t} \Big/ \text{kg}$$

Internally Deposited Beta Emitters – An Example Cember, pp. 234.

Calculate the daily dose rate to a testis that weighs 18 g and has 6660 Bq of ^{35}S uniformly distributed throughout the organ.

Sulfur is a pure beta emitter whose maximum-energy beta particle is 0.1674 MeV and whose average energy is 0.0488 MeV. The beta-ray dose rate from q Bq uniformly dispersed in m kg of tissue, if the specific effective energy is SEE MeV per transformation per kg, is

$$\dot{D}(\beta) = \frac{q \text{ Bq} \times 1 \text{ tps/Bq} \times \bar{E} \text{ MeV}/t \times 1.6 \times 10^{-13} \text{ J/MeV} \times 8.64 \times 10^{4} \text{ s/day}}{m \text{ kg} \times 1 \frac{\text{J}}{\text{kg}} \Big/ \text{Gy}}$$

$$= \frac{6.66 \times 10^{3} \times 1 \times 4.88 \times 10^{-2} \times 1.6 \times 10^{-13} \times 8.64 \times 10^{4}}{0.018 \times 1}$$

$$= 2.5 \times 10^{-4} \text{ Gy/day (0.025 rad/day).}$$

Neutron Dose

Neutron dose to tissue:

- Fast neutron dose from elastic scattering (mostly from first-collision dose).
- Thermal neutron dose
  - neutron capture by H → gamma ray dose.
  - neutron capture by N → dose from the recoil nucleus.

Thermal Neutron Capture Radiation Therapy

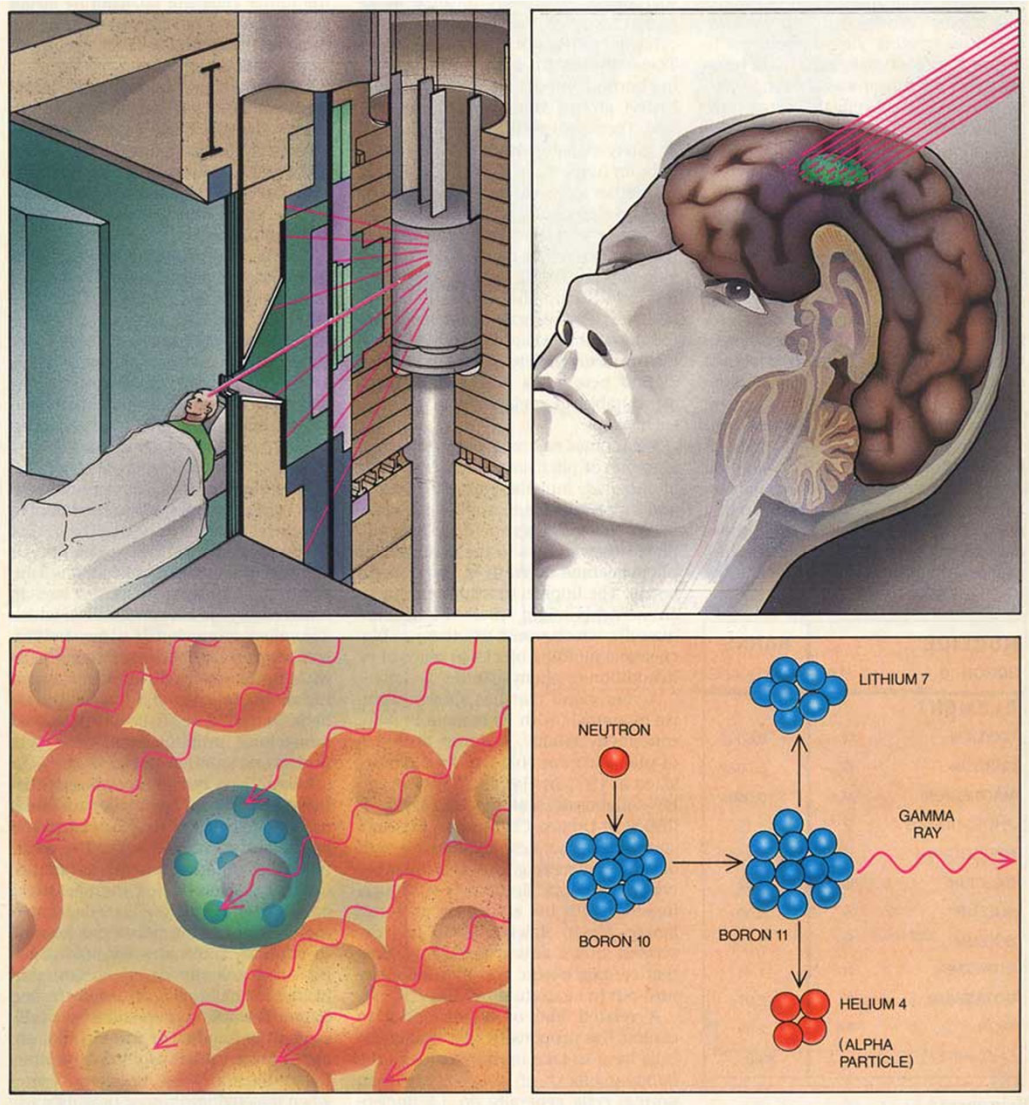


Figure from: http://en.wikipedia.org/wiki/File:NeutronCaptureTherapyImage.jpg

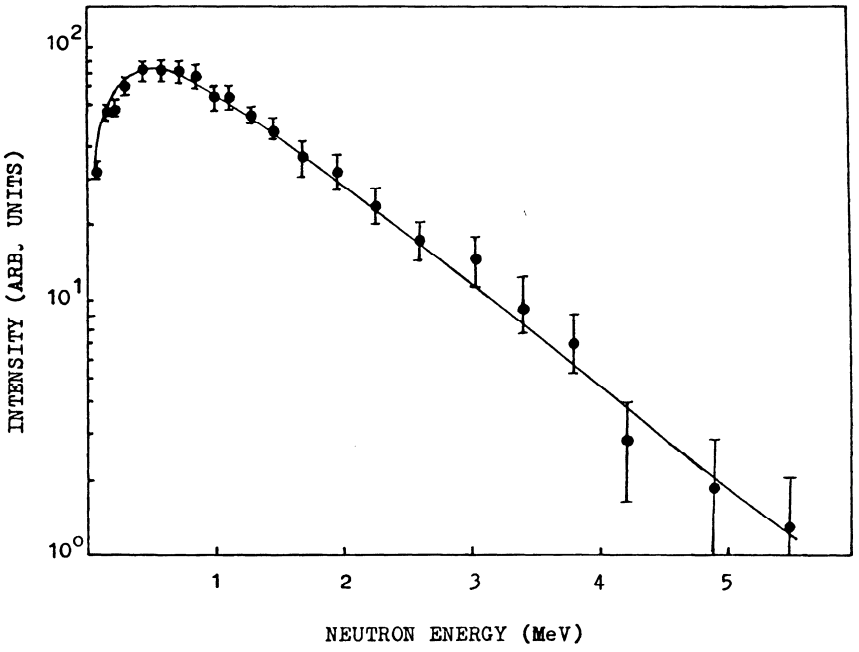
Fig.1 boron neutron capture therapy (BNCT) can be performed at a facility with a nuclear reactor or at hospitals that have developed alternative neutron sources. A beam of epithermal neutrons penetrates the brain tissue, reaching the malignancy. Once there the epithermal neutrons slow down and these low-energy neutrons combine with boron-10 (delivered beforehand to the cancer cells by drugs or antibodies) to form boron-11, releasing lethal radiation (alpha particles and lithium ions) that can kill the tumor.[1]

Barth, Rolf F.; Soloway, Albert H.; Fairchild, Ralph G. (1990). "Boron Neutron Capture Therapy for Cancer". Scientific American 263 (4): 100–3, 106–7. Bibcode:1990SciAm.263d.100B. doi:10.1038/scientificamerican1090-100. PMID 2173134.

Neutron Sources – Spontaneous Fission

***Spontaneous fission of tarnsuranic heavy nuclides**, such as ^{252}Cf, produces several fast neutrons, in addition to heavy fission products, prompt fission gamma rays and beta and gamma ray activities.*

- ***Half-life**: 2.65 years*
- *Neutron yield: 0.116n/s per Bq, or 2.3 ×10^6 n/s per mg*
- ***Neutron energy** peaking at 0.5MeV and extends beyond 10MeV.*



Measured neutron energy spectrum from spontaneous fission of ^{252}Cf

Page 19-27, Radiation Detection and Measurements, Third Edition, G. F. Knoll, John Wiley & Sons, 1999.

Radiation Dose from Fast Neutrons

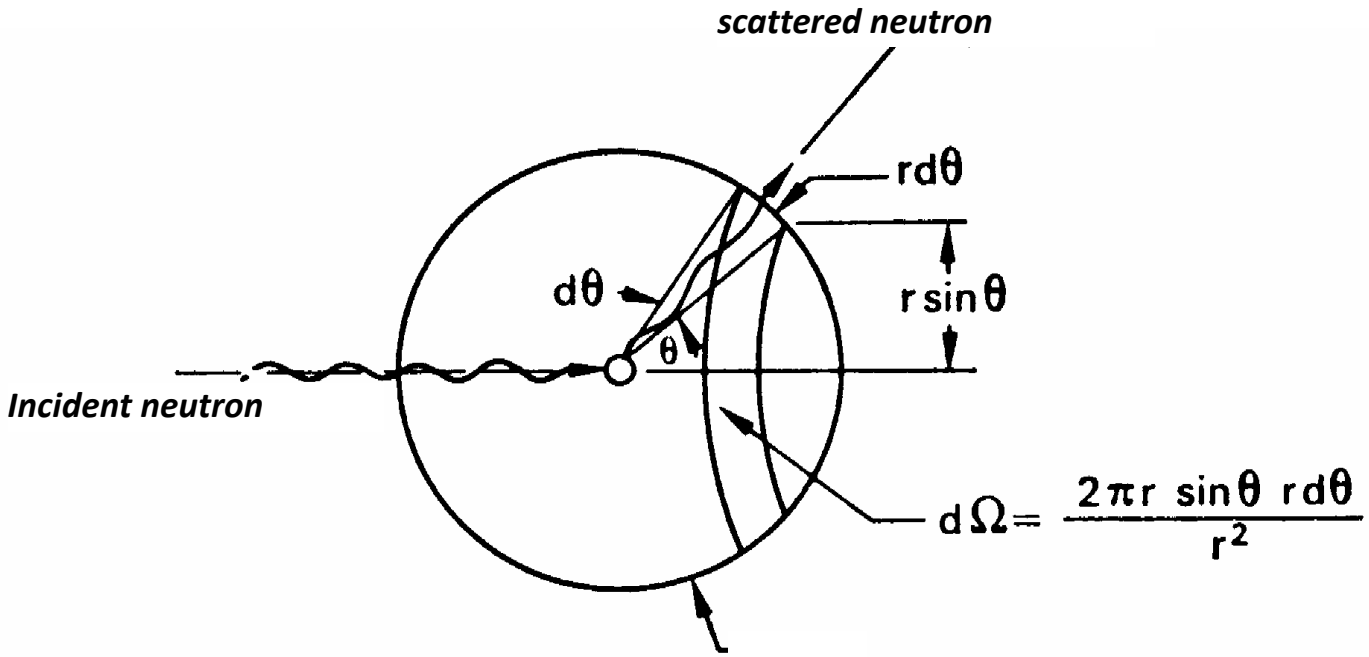
Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

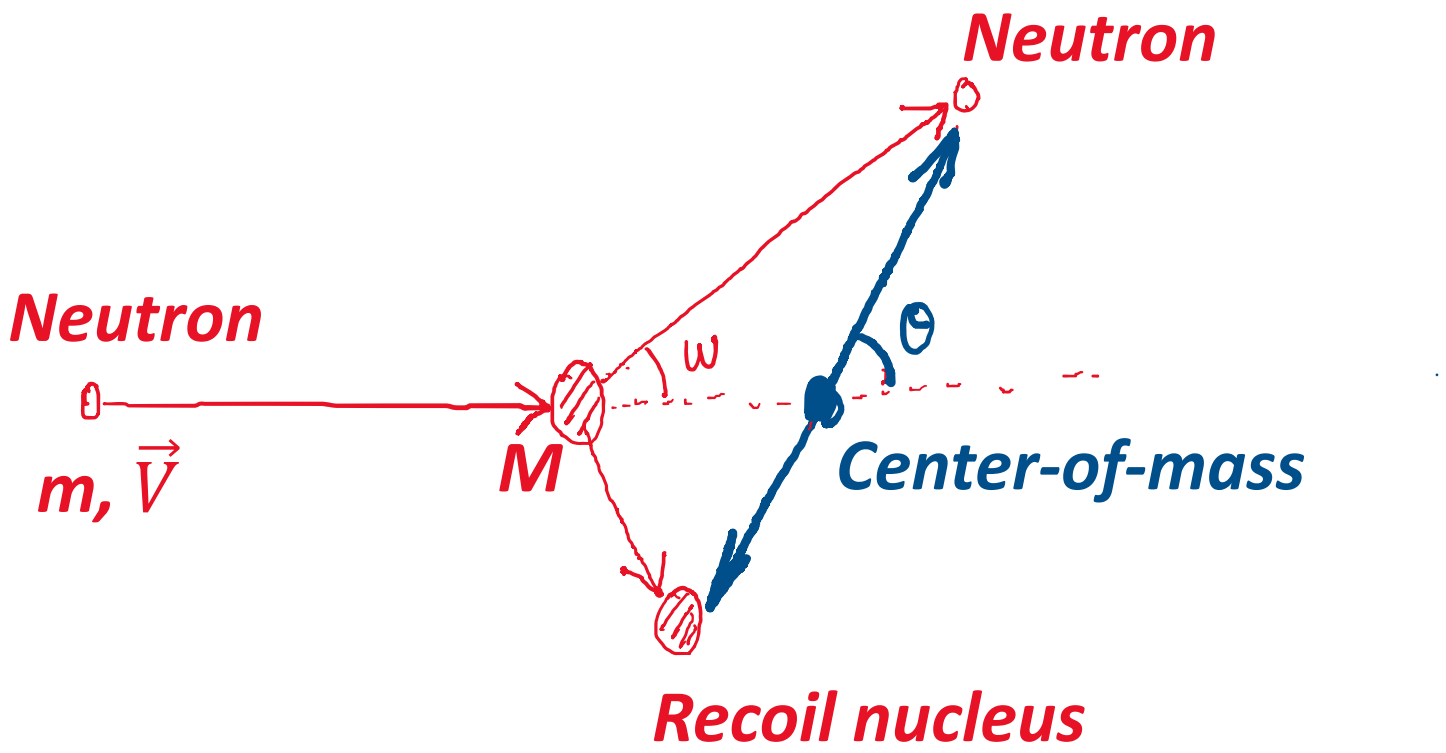
How do Fast Neutrons Deposit Energy in Tissue?

Angular Distributions of the Scattered Neutrons

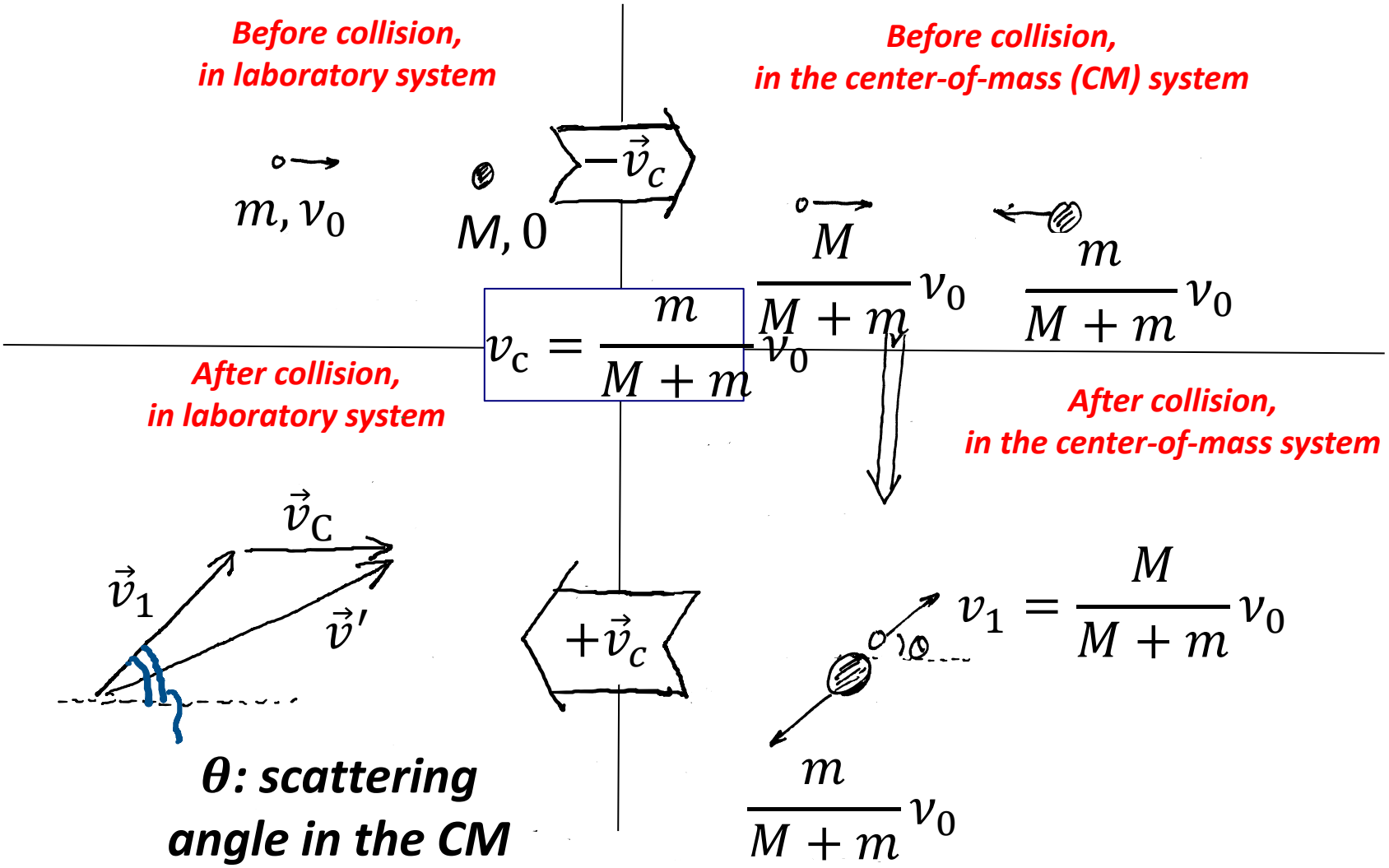
- For neutron energies up to 10MeV, it is experimentally observed that the scattering of neutrons is isotropic in the center-of-mass coordinate system. The neutron and the recoil nuclei are scattered with equal probability in any direction in this 3-D coordinate system.



Angular Distributions of the Scattered Neutrons



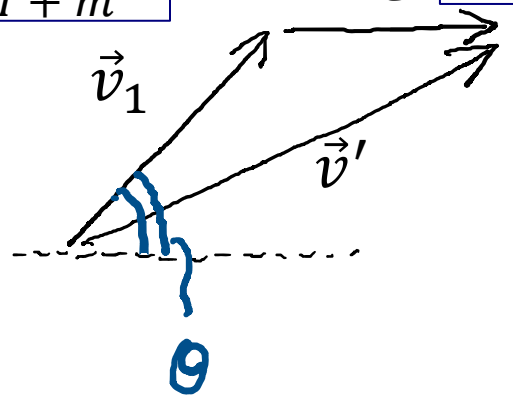
Angular Distributions of the Scattered Neutrons



Angular Distributions of the Scattered Neutrons

$$v_1 = \frac{M}{M+m} v_0$$

$$v_c = \frac{m}{M+m} v_0$$



The speed of the scattered neutron is

$$\vec{v}' = \vec{v}_1 + \vec{v}_c.$$

Therefore

$$v'^2 = v_1^2 + v_c^2 - 2v_1 v_c \cos(\pi - \theta)$$
$$= v_1^2 + v_C^2 + 2v_1 v_C \cos\theta.$$

The kinetic energy of the scattered neutron, E', is

$$E' = \frac{1}{2} m v'^2,$$

and

$$\frac{E'}{E_0} = \frac{1}{v_0^2}\left[\left(\frac{M}{M+m} v_0\right)^2 + \left(\frac{m}{M+m} v_0\right)^2 + 2\frac{M}{M+m}\frac{m}{M+m} v_0^2 \cos\theta\right].$$

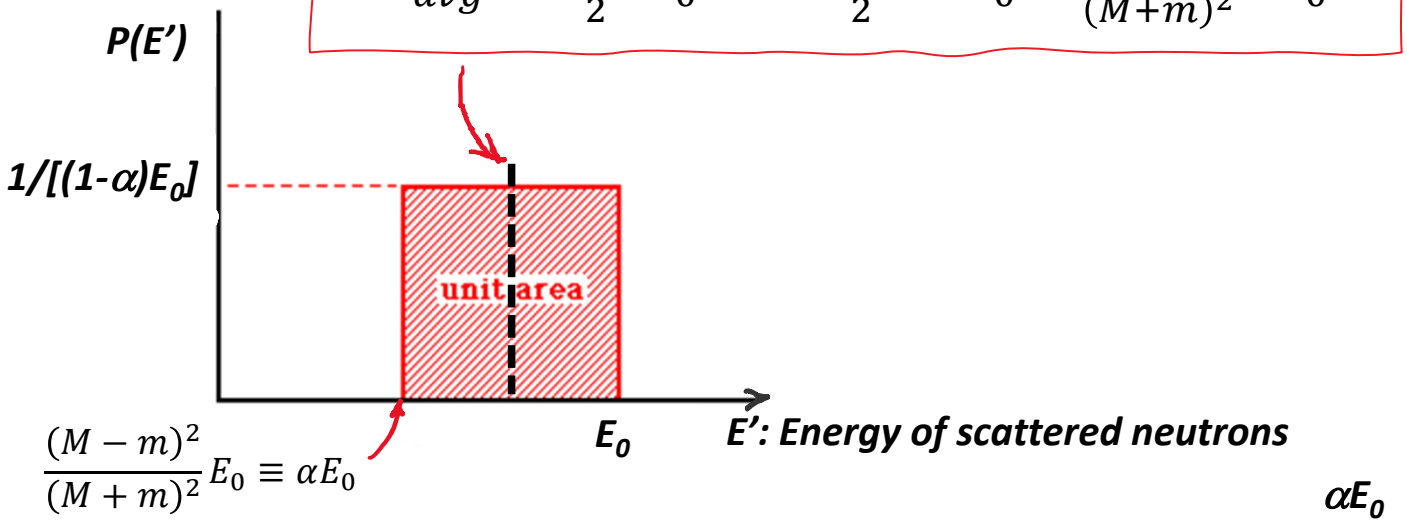
Therefore,

$$\frac{E'}{E_0} = \frac{M^2+m^2+2Mm\cos\theta}{(M+m)^2}.$$

Energy Spectrum of Scattered Neutrons

Average energy carried by the scattered neutron:

$$E'_{avg} = \frac{1+\alpha}{2} E_0 = \frac{1+\frac{(M-m)^2}{(M+m)^2}}{2} E_0 = \frac{M^2+m^2}{(M+m)^2} \cdot E_0$$



Average energy transferred to the recoil nucleus:

$$E_{avg_energy_loss} = E_0 - E'_{avg} = \frac{2Mm}{(M+m)^2} \cdot E_0$$

Radiation Dose from Fast Neutrons

- For isotropic scattering, the average fraction of energy transferred in an elastic scattering with a nucleus of atomic mass number M is

$$f = \frac{2M}{(M+1)^2}$$

- The composition of soft tissue is shown below

TABLE 6.12. Synthetic Tissue Composition

Element	% Mass	N, atoms/kg	f
Oxygen	71.39	2.69×10^{25}	0.111
Carbon	14.89	6.41×10^{24}	0.142
Hydrogen	10.00	5.98×10^{25}	0.500
Nitrogen	3.47	1.49×10^{24}	0.124
Sodium	0.15	3.93×10^{22}	0.080
Chlorine	0.10	1.70×10^{22}	0.053

Source: Adapted from G. L. Brownell, W. H. Ellett, and A. R. Reddy, Absorbed Fractions for Photon Dosimetry. *J Nuclear Medicine*, Supplement No. 1, MIRD Pamphlet No. 3, February 1968. By permission.

Radiation Dose from Fast Neutrons

- Neutron dose is deposited through scattering and neutron induced nuclear reactions.
- In cases of elastic scattering, the scattered nuclei dissipate their energy in the immediate vicinity of the primary neutron interaction. The radiation dose absorbed locally in this way is called the **first collision dose**. The scattered neutron is **not** considered after this primary interaction.
- For fast neutrons, the first collision dose rate is given by

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1\ \text{J/kg}\cdot\text{Gy}}, \qquad (6.103)$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

The scattering cross sections of each of the tissue elements for 5-MeV neutrons are listed below:

ELEMENT	σ, $\times 10^{-24}$ cm^2	$N_i\sigma_i f_i$
O	1.55	4.628
C	1.65	1.502
H	1.50	4.485×10^{1}
N	1.00	1.848×10^{-1}
Na	2.3	7.231×10^{-3}
Cl	2.8	2.523×10^{-3}
		$\sum N_i\sigma_i f_i = 5.117 \times 10^{1} \frac{\text{cm}^2}{\text{kg}}$

$$\dot{D}_n(E) = \frac{\phi(E)E\sum_i N_i\sigma_i f}{1 \text{ J/kg}\cdot\text{Gy}},$$

where $\phi(E)$ = flux of neutrons whose energy is E, in neutrons/cm^2. s,
E = neutron energy, in joules,
N_i = atoms per kilogram of the ith element,
σ_i = scattering across section of the ith element for neutrons of energy E, in barns $\times 10^{-24}$ cm^2,
f = mean fractional energy transferred from neutron to scattered atom during collision with neutron.

Radiation Dose from Fast Neutrons

Example 6.16

What is the absorbed dose rate to soft tissue in a beam of 5-MeV neutrons whose intensity is 2000 neutrons per square centimeter per second?

Substituting the appropriate values into Eq. (6.103) yields

$$\dot{D}_n = \frac{2 \times 10^3\ n/\text{cm}^2 \cdot \text{s} \times 5\ \text{MeV}/n \times 1.6 \times 10^{-13}\ \text{J/MeV} \times 51.17\ \text{cm}^2/\text{kg}}{1\ \text{J/kg} \cdot \text{Gy}}$$

$$= 8.19 \times 10^{-8}\ \text{Gy/s}\,(8.19 \times 10^{-6}\ \text{rad/s}),$$

or

$$8.19 \times 10^{-8}\ \text{Gy/s} \times 10^6\ \mu\text{Gy/Gy} \times 3.6 \times 10^3\ \text{s/h}$$

$$= 295\ \mu\text{Gy/h}\ (29.5\ \text{mrad/h}).$$

How do Thermal Neutrons Deposit Energy in Tissue?

Interaction of Slow and Thermal Neutrons (E<0.5eV) (Revisited)

The most important interactions between slow neutrons and absorbing materials are neutron-induced reactions, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

$$\text{neutron} + \text{target nucleus} \Rightarrow \begin{cases} \text{recoil nucleus} \\ \text{protons} \\ \text{alpha particles} \\ \text{fission fragments} \end{cases}$$

Neutron Induced Reactions

$$^{1}_{0}n + ^{14}_{7}N \rightarrow ^{14}_{6}C + ^{1}_{1}p$$

- Cross section for thermal neutron is 1.70 barns.
- Q=0.626MeV.
- Since the range of the proton and the ^{14}C nucleus are relatively small, their energy is deposited locally at the site where the neutron was captured.
- Capture by hydrogen and nitrogen are the only two processes through which neutron deliver a significant does to soft tissue.

Thermal Neutron Dose from the $^{14}N(n,p)^{14}C$ Reaction

- Two reactions are normally considered, namely $^{14}N(n,p)^{14}C$ and $^{1}H(n,r)^{2}H$ reactions.

- For the $^{14}N(n,p)^{14}C$ reaction, the dose is given by

$$\dot{D}_{np} = \frac{\phi N_N \sigma_N Q \times 1.6 \times 10^{-13}\ \text{J/MeV}}{1\ \text{J/kg} \cdot \text{Gy}},$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_N = number of nitrogen atoms per kg tissue, 1.49×10^{24},
σ_N = absorption cross section for nitrogen, 1.75×10^{-24} cm^2,
Q = energy released by the reaction = 0.63 MeV.

Neutron Induced Reactions

$$ {}_0^1n + {}_1^1H \rightarrow {}_1^2H + {}_0^0\gamma $$

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy Q=2.22MeV released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.

Thermal Neutron Dose from the $^1H(n, \gamma)^2H$ Reaction

- For the $^1H(n, \gamma)^2H$ reaction, the dose is deposited by the gamma rays emitted throughout the entire volume. The number of reaction per second per gram is governed by the neutron flux and is given by

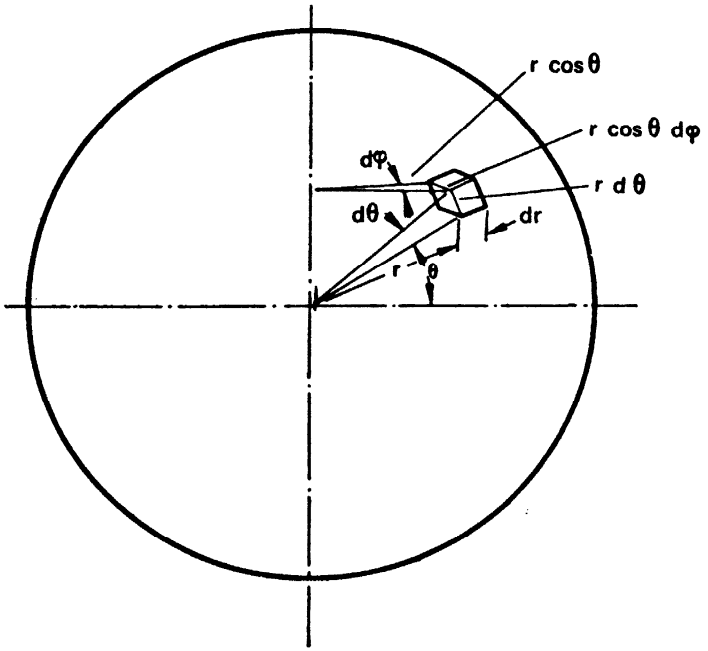
$$A = \phi N_H \sigma_H \text{ “Bq”/kg,}$$

where ϕ = thermal flux, neutrons per cm^2 per second,
N_H = number of hydrogen atoms per kg tissue = 5.98×10^{25},
σ_H = absorption cross section for hydrogen = 0.33×10^{-24} cm^2.

- The resulting gamma ray dose is illustrated with the following example.

Specific Gamma-Ray Constant

- **Specific Gamma Ray Constant (Γ):** The gamma ray ***exposure rate*** from a point source of a unit activity at a unit distance. It is given in unit of coulombs per kilogram per hour at 1m from a 1 MBq point source.
- It is a useful numerical quantity for predicting the exposure resultant from a given gamma ray emitting radioisotope.



Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

- For gamma rays, one can no longer assume that the organ is infinitely large.
- For a uniformly distributed gamma ray emitting isotope, the dose rate from the isotope in an infinitesimal volume *dV to* Point **p** at a distance **r** away is

Mass energy absorption coefficient of the dose receiving media

$$d\dot{D} = C \cdot \Gamma \cdot \frac{e^{-\mu r}}{r^2} \cdot dV \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

where C is the concentration of the isotope, Γ is the specific gamma-ray emission, and μ is the linear energy absorption coefficient.

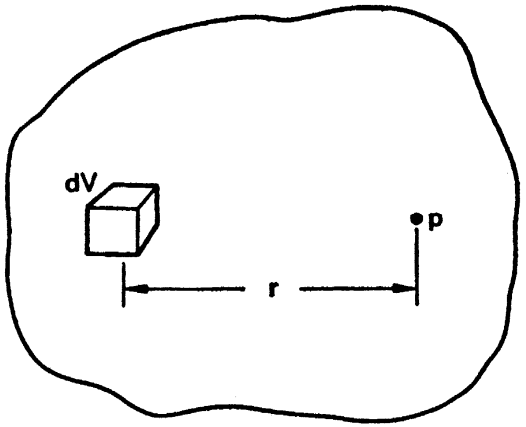


FIGURE 6.8. Diagram for calculating dose at point *p* from the gamma rays emitted from the volume element d*V* in a tissue mass containing a uniformly distributed isotope.

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

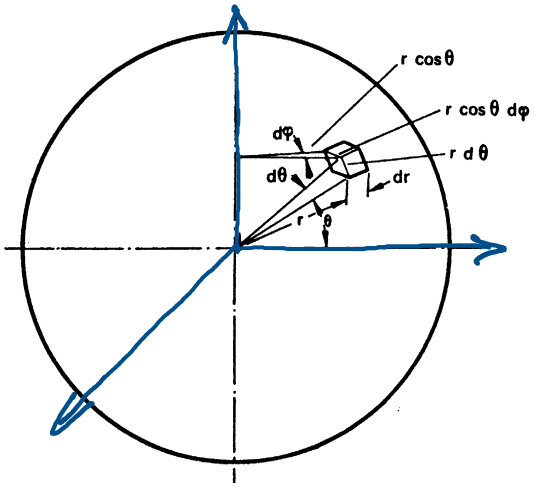
- The **dose rate due to the gamma ray emitter uniformly distributed through the entire source volume** is therefore

$$d\dot{D} = C \cdot \Gamma \cdot \frac{e^{-\mu r}}{r^2} \cdot dV \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

- In case of a uniform spherical source, the dose rate at the center is given by

$$\dot{D} = 4C\Gamma \int_{r=0}^{r=R} \int_{\theta=0}^{\theta=\pi/2} \int_{\varphi=0}^{\varphi=\pi} \frac{e^{-\mu r}}{r^2} \cdot r \, d\theta \cdot r \cos\theta \, d\varphi \cdot dr \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$

$$= C\Gamma \cdot \frac{4\pi}{\mu}\left(1 - e^{-\mu R}\right) \cdot \left(34 \frac{J/kg}{Coulomb/kg}\right) \cdot \left(\frac{\mu_m/\rho_m}{\mu_a/\rho_a}\right)$$



- And the dose rate at the surface of the spherical source volume is given by

$$\dot{D}_{surface} = 0.5 \, \dot{D}_{center}$$

Internally Deposited Radioisotope (IV)
Gamma Ray Emitters

- One can define a **geometrical factor**

$$g = \int_0^V \frac{e^{-\mu r}}{r^2} \mathrm{d}V$$

- The total dose rate at the center of the sphere is simplified to

$$\dot{D} = C\Gamma g \cdot (34 \frac{J/kg}{Coulomb/kg}) \cdot (\frac{\mu_m/\rho_m}{\mu_a/\rho_a})$$

- For many instances, we are interested in the **average dose rate** across the entire internal volume rather than the dose rate at a given point. It may be useful to define the average geometrical factor as follows:

$$\bar{g} = \frac{1}{V} \int g \, \mathrm{d}V \,, \qquad \bar{g} = \frac{3}{4}(g)_{\text{center}} \cdot \quad \text{For a sphere}$$

Radiation Dose from Neutrons as a Function of Depth

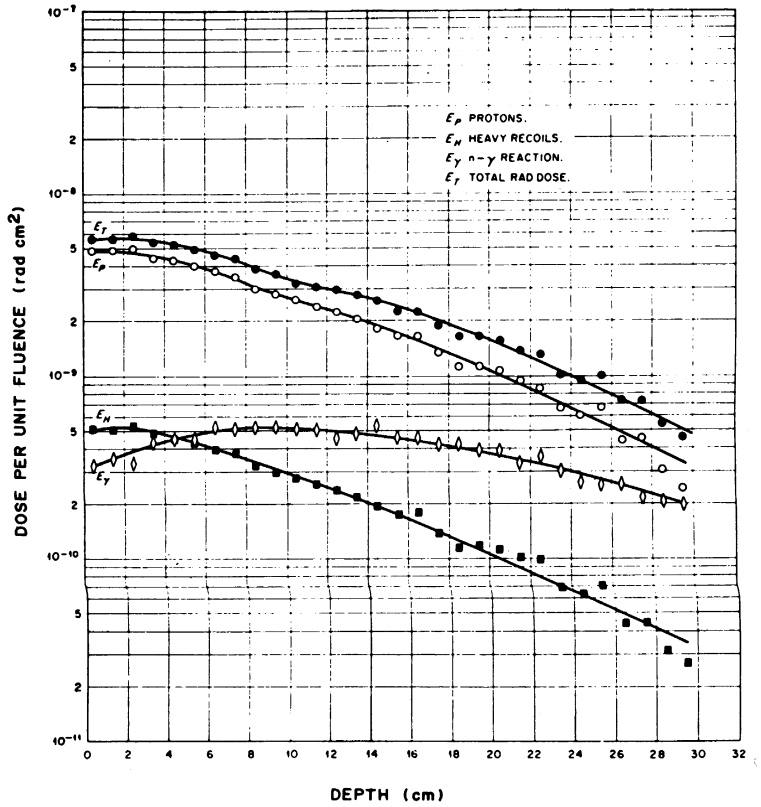


FIGURE 12.10. Depth–dose curves for a broad beam of 5-MeV neutrons incident normally on a soft-tissue slab. Ordinate gives dose per unit fluence at different depths shown by the abscissa. [From "Protection Against Neutron Radiation Up to 30 Million Electron Volts," in *National Bureau of Standards Handbook 63*, p. 44, Washington, D.C. (1957).]

Tuner, p373.

Radiation Dose from Neutrons as a Function of Depth

Figure 12.10 shows the results of Monte Carlo calculations carried out for 5-MeV neutrons incident normally on a 30-cm soft-tissue slab, approximating the thickness of the body. (The geometry is identical to that shown for the charged particles in Fig. 12.9.) The curve labeled E_T is the total dose, E_p is the dose due to H recoil nuclei (protons), E_γ is the dose from gamma rays from the $^1H(n,\gamma)^2H$ slow-neutron capture reaction, and E_H is the dose from the heavy (O, C, N) recoil nuclei. The total dose builds up somewhat in the first few cm of depth and then decreases as the beam becomes degraded in energy and neutrons are absorbed. The proton and heavy-recoil curves, E_p and E_H, show a similar pattern. As the neutrons penetrate, they are moderated and approach thermal energies. This is reflected in the rise of the gamma-dose curve, E_γ, which has a broad maximum over the region from about 6 cm to 14 cm. Note

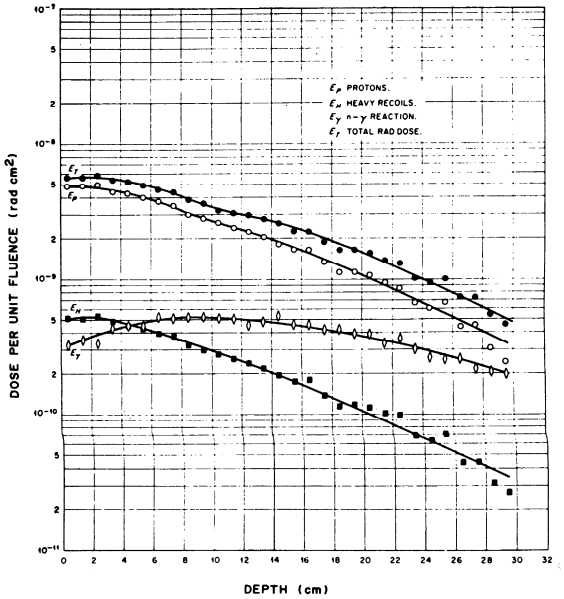


FIGURE 12.10. Depth–dose curves for a broad beam of 5-MeV neutrons incident normally on a soft-tissue slab. Ordinate gives dose per unit fluence at different depths shown by the abscissa. [From "Protection Against Neutron Radiation Up to 30 Million Electron Volts," in *National Bureau of Standards Handbook 63*, p. 44, Washington, D.C. (1957).]