3. An unbiased die is rolled 10 times.
   (a) What is the probability that exactly 4 threes will occur?
   (b) What is the probability that exactly 4 of any one number alone will occur?
   (c) What is the probability that two numbers occur exactly 4 times?

   a.) Probability of exactly rolling 3, four times?
   \[ Probability = \left( \frac{1}{6} \right)^4 \times \left( \frac{5}{6} \right)^6 \times \left( \frac{10}{4} \right) = 0.0543 \]

   b.) Probability of any one number will occur four times only?
   \[ Probability = 6 \left( \frac{1}{6} \right)^4 \times \left( \frac{5}{6} \right)^6 \times \left( \frac{10}{4} \right) = 0.326 \]

   c.) 2 numbers exactly four times?
   \[ Probability = \binom{6}{2} \times \left( \frac{1}{6} \right)^4 \times \left( \frac{5}{6} \right)^6 \times \left( \frac{10}{8} \right) = 2.791 \times 10^{-4} \]

7. A sample consists of 16 atoms of $^{222}$Rn.
   (a) What is the probability that exactly one-half of the atoms will decay in 2 d?
   (b) In 3 d?
   (c) Calculate the probability that one week could pass without the decay of a single atom.
   (d) What is the probability that all of the atoms will decay in the first day?

   a.) $\frac{1}{2}$ the atoms decay in 2 days?
   \[ \lambda = \frac{\ln(2)}{3+8 \text{days}} = 0.1824 \text{ days}^{-1} \]
   \[ q = e^{-\lambda \times 2 \text{days}} = 0.694 \]
   \[ p = 1 - 0.694 = 0.306 \]
   \[ probability = \binom{16}{8} (0.306)^8 (0.694)^8 = 0.053 \]

   b.) Then 3 days?
   \[ q = e^{-\lambda \times 3 \text{days}} = 0.5786 \]
   \[ p = 1 - 0.5786 = 0.4214 \]
Probability = \( \binom{16}{8} (0.4214)^8 (0.5786)^8 = 0.161 \)

c.) \( q = e^{-\lambda \cdot 7 \text{days}} = 0.2789 \)

Probability = \( \binom{16}{0} (0.2789)^{16} = 1.34 \times 10^{-9} \)

d.) \( q = e^{-\lambda \cdot 1 \text{days}} = 0.83327 \)

\[ p = 1 - 0.83327 = 0.1667 \]

Probability = \( \binom{16}{16} (0.1667)^{16} = 3.55 \times 10^{-13} \)

20. The activities of two sources can be compared by counting them for equal times and then taking the ratio of the two count numbers, \( n_1 \) and \( n_2 \). Show that the standard deviation of the ratio \( Q = n_1 / n_2 \) is given by

\[
\sigma_Q = Q \left( \sigma_1^2 \frac{n_1^2}{n_2^2} + \sigma_2^2 \right)^{1/2},
\]

where \( \sigma_1 \) and \( \sigma_2 \) are the standard deviations of the two individual count numbers.

\[
Q = \frac{n_1}{n_2}
\]

Error Propagations = \( \sigma_Q^2 = \left( \frac{dQ}{dn_1} \right)^2 \sigma_{n_1}^2 + \left( \frac{dQ}{dn_2} \right)^2 \sigma_{n_2}^2 
\]

\[
= \frac{1}{n_2^2} \sigma_{n_1}^2 + \left( \frac{n_2^2}{n_1^2} \right) \sigma_{n_2}^2
\]

\[
= \frac{1}{\left( \frac{n_1}{Q} \right)^2} \sigma_{n_1}^2 + \left( \frac{n_1^2}{n_2^2} \right) \sigma_{n_2}^2
\]

\[
= \frac{Q^2 \sigma_{n_1}}{n_2^2} + \frac{Q^4 \left( \frac{n_1}{Q} \right)^4 \sigma_{n_2}^2}{n_2^2}
\]

\[
= \frac{Q^2 \sigma_{n_1}}{n_2^2} + Q^2 \left( \frac{n_1}{n_2} \right)^2 \sigma_{n_2}^2
\]

\[
= \frac{Q^2 \sigma_{n_1}^2}{n_2^2} + \frac{Q^2 \sigma_{n_2}^2}{n_2^2}
\]

\[
\sigma_Q = Q \left[ \frac{\sigma_{n_1}^2}{n_1^2} + \frac{\sigma_{n_2}^2}{n_2^2} \right]
\]
30. With a certain counting system, a 6-h background measurement registers 6588 counts. A long-lived sample is then placed in the counter, and 840 gross counts are registered in 30 min.

(a) What is the net count rate?

(b) What is the standard deviation of the net count rate?

(c) Without additional background counting, how many gross counts would be needed in order to obtain the net count rate of the sample to within ±5% of its true value with 90% confidence?

(d) Without a remeasurement of background, what is the smallest value obtainable for the standard deviation of the net count rate by increasing the gross counting time?

\[ \text{Net Count Rates} = \frac{840}{30 \text{ min}} - \frac{6588}{360 \text{ min}} \]

\[ r_n = 9.7 \text{ cpm} \]

\[ b) \ R_n = \frac{\eta_s - \eta_b}{t_s - t_b} \]

\[ = \sigma_{R_n}^2 = \left( \frac{dR_n}{dt_s} \right)^2 \sigma_{t_s}^2 + \left( \frac{dR_n}{dt_b} \right)^2 \sigma_{t_b}^2 + \left( \frac{dR_n}{d\eta_s} \right)^2 \sigma_{\eta_s}^2 + \left( \frac{dR_n}{d\eta_b} \right)^2 \sigma_{\eta_b}^2 \]

\[ = \left( \frac{\sigma_{n_s}}{t_s} \right)^2 + \left( \frac{\sigma_{n_b}}{t_b} \right)^2 \]

\[ = \sigma_{R_n} = \sqrt{\frac{\sigma_{n_s}^2}{t_s^2} + \frac{\sigma_{n_b}^2}{t_b^2}}, \sigma_{n_s}^2 = n_s, \sigma_{n_b}^2 = n_b \]

\[ = \sqrt{\frac{n_s}{t_s^2} + \frac{n_b}{t_b^2}} \]

\[ = \sqrt{\frac{r_n}{t_s} + \frac{r_b}{t_b}} \]

\[ = \frac{840}{30^2} + \frac{6588}{360^2} = .992 \]

\[ c) \sigma_{rn} = \frac{n_g + n_b}{\sqrt{t_g, t_b}} \]

\[ k_{\alpha|95\%} = 1.645 \]

\[ 1.645 \sigma_{rn} = 0.05 r_n \]

\[ 1.645 \sigma_{rn} = 0.05(9.7 \text{ cpm}) \]
\[ \sigma_{rn} = \frac{0.05(9.7)}{1.645} \]

29.1 = 1.65 * \[ \sqrt{\frac{r_{gr}}{t_{gr}} + \frac{r_{bg}}{t_{bg}}} = \sqrt{\frac{1680}{t_{gr}} + \frac{1098}{6}} \]

\[ t_{gr} = 13.1 \text{ hrs} \]
\[ n_{gr} = t_{gr} * r_{gr} = 22,000 \text{ counts} \]

d) \[ \sigma_{rn} = \sqrt{\frac{r_{g}}{t_{g}} + \frac{r_{b}}{t_{b}}} \]

\[ t_{g} = \frac{r_{g}}{r_{b}} \]

\[ t_{g} = \frac{28}{\sqrt{18.3}} \]

\[ t_{g} = 445.30 \text{ minutes} \]

\[ \sigma_{rn} = \sqrt{\frac{1680}{7.42} + \frac{1098}{6 \text{ hr}}} \]

\[ \sigma_{rn} = 20.2 \text{ hrs}^{-1} \]

33. A sample containing a short-lived radionuclide is placed in a counter, which registers 57,912 counts before the activity disappears completely. Background is zero.

(a) If the counter efficiency is 28%, how many atoms of the radionuclide were present at the beginning of the counting period?

(b) What is the standard deviation of the number?

\[ \varepsilon = 28\% \]

\[ \mu_c = \varepsilon N (1 - e^{-\lambda t}); \lambda t = \text{ infinity} \]

a.)

\[ \mu = \varepsilon N; N = 206.828 \times 10^3 \text{ atoms} \]
b.)

\[ \sigma_c = \sqrt{57912(1 - 0.28)} = 204.197 \]

\[ \sigma = 730 \]

44. You are shown a counting facility for analyzing samples for radioactivity. The calibration is such that 84 net counts correspond to \(3.85 \times 10^4\) Bq of sample activity. The background count of 1270 is stable and accurately known. You are told that the minimum significant net sample count is 70.

(a) What is the minimum significant measured activity for the facility?

(b) What is the maximum risk of making a type-I error?

(c) What is the minimum detectable true activity, if the maximum risk for making a type-II error is 5%?

(d) Does the minimum detectable true activity increase or decrease, if the risk of a type-II error is decreased from 5%?

(e) If the true activity is exactly equal to the minimum detectable true activity \(A_{II}\), what is the probability that a single measurement will give a result that implies an activity less than \(A_{II}\)?

a.) \[ \frac{70}{84} = \frac{A_I}{3.85 \times 10^4} \]

\[ A_I = 32083 \text{ Bq} \]

b.) \[ \Delta_1 = k_\alpha \sqrt{B} \]

\[ 70 = k_\alpha \sqrt{1270} \]

\[ k_\alpha = 1.964 \]

2.5 % chance

c.) \[ \Delta = \sqrt{B}(k_\alpha + \frac{k_\beta}{2\sqrt{B}} + k_\beta \frac{k_\alpha}{\sqrt{B}} + \frac{k_\beta^2}{4B}) = 131.576 \]

\[ A_{II} = \frac{131.576}{84} \text{ Bq} = 60305.7 \text{ Bq} \]
d.) The minimum activity would increase as $\beta$ decreases. We can view this as shifting the curve in Fig. 11.5 to the right.

![Figure 11.5](image)

**Fig. 11.5** Probability density $P_n(r_n)$ for net count rate $r_n$. When activity $A = 0$, $r_1$ is fixed by choice of probability $\alpha$ for type-I errors. Use of $r_1$ and choice of probability $\beta$ for type-II errors fixes $r_2$, corresponding to the minimum detectable true activity $A_\text{II}$. (If $\alpha = \beta$, then the intersection of the two curves occurs at the value $r_n = r_1$.) (Courtesy James S. Bogard.)

e.) If the true activity is equal to the minimum detectable true activity, then the probability that a single measurement will give a result that implies an activity less than $A_\text{II} = \beta = 0.05$