Review of X-ray Interactions

Four major interactions are of importance to diagnostic radiology and nuclear medicine:

- Classical (Rayleigh or elastic) scattering
- Compton scattering
- Photoelectric effect
- Pair production
Rayleigh Scattering

- *Excitation of the total complement of atomic electrons* occurs as a result of interaction with the incident photon
- **No ionization** takes place
- The photon is scattered (re-emitted) in a range of different directions, but close to that of the incident photon – *forward scattering*.
- **No loss of** $E$
- Relatively *small probability* $\approx 5\%$
- **Not detectable** by x-ray detectors.
Compton Scattering

- **Dominant interaction** of x-rays with low-Z materials in the diagnostic range and beyond (approx. 30 keV - 30MeV)
- Occurs between the photon and a outer shell e\(^{-}\), which is considered free when \(E_g >> \) binding energy, \(E_b\) of the e\(^{-}\).
- Encounter results in **ionization of the atom** and probabilistic distribution of the incident photon \(E\) to that of the scattered photon and the ejected e\(^{-}\).
- A probabilistic distribution determines the scattering angle.
Compton Scattering

- Compton interaction probability is dependent on the total no. of e\textsuperscript{-} in the absorber vol. \((e^{-}/cm^3 = e^{-}/gm \cdot \text{density})\) or electron density
- With the exception of \(^1\text{H}, e^{-}/gm\) is fairly constant for organic materials \((Z/A \approx 0.5)\), thus the probability of Compton interaction proportional to material density \((\rho)\)
- **Conservation of energy and momentum** yield the following equations:

\[
E_{hv'} = \frac{E_{hv}}{1 + \frac{E_{hv}}{m_e c^2} (1 - \cos \theta)},
\]

where

\[
m_e c^2 = 511\text{keV}
\]

\[
E_{hv}: \text{energy of the incident photon}
\]

\[
E_{hv'}: \text{energy of the scattered photon}
\]
Angular Distribution of the Scattered Gamma Rays

Klein-Nishina formula:

\[
\frac{d\sigma}{d\Omega} = Zr_e^2 \left( \frac{1}{1 + \alpha(1 - \cos(\theta))} \right)^2 \left( 1 + \cos^2(\theta) \right) \left( 1 + \frac{1 + \cos^2(\theta)}{(1 + \cos^2(\theta))[1 + \alpha(1 - \cos(\theta))]^2} \right), \quad \alpha = \frac{hv}{m_0c^2}
\]

Probability of Compton scattering within a unit solid angle around a scattering angle \( \theta \).

Incident photons with higher energy tends to scatter with smaller angle (forward scattering).

Incident photons with lower energy (a few hundred keV) have greater chance of undergo large angle scattering (back scattering).

Radial distance represents the differential cross section.
Photoelectric Effect

- Interaction of incident photon with *inner shell e*⁻, why?
- All E transferred to e⁻ (ejected *photoelectron*) as kinetic energy (Eₑ) less the binding energy: Eₑ = E₀ − Eₐ
- *Empty shell immediately filled with e⁻ from outer orbits* resulting in the emission of characteristic x-rays (Eₓ = differences in Eₐ of orbitals), for example, Iodine: Eₖ = 34 keV, Eₗ = 5 keV, Eₘ = 0.6 keV
Photoelectric Effect

- Photoe\textsuperscript{-} absorption is the preferred interaction for X-ray imaging.
- Rem.: $E_b \propto Z^2$; characteristic x-rays and/or Auger e\textsuperscript{-} \textit{preferred in high Z material}.
- Probability of photoe\textsuperscript{-} absorption $\sim Z^3/E^3$ ($Z = \text{atomic no.}$) \textit{provide contrast according to different Z}.
- Due to the absorption of the incident x-ray without scatter, maximum subject contrast arises with a photoe\textsuperscript{-} effect interaction \textit{No scattering contamination} \textit{better contrast}
- Explains why contrast $\downarrow$ as higher energy x-rays are used in the imaging process
- Increased probability of photoe\textsuperscript{-} absorption just above the $E_b$ of the inner shells cause discontinuities in the \textit{attenuation profiles} (e.g., K-edge)
Photoelectric Effect

Copper ($Z=29$)

Electron binding energies:
- K-edge = 8.98 keV
- L_I-edge = 1.10 keV
- L_F-edge = 0.95 keV
- L_m-edge = 0.93 keV

Not shown all below 10 keV

Linear attenuation coefficients (cm$^{-1}$)

Energy (MeV)

Total

Compton

Photoelectric

Pair production

Tungsten ($Z=74$)

Electron binding energies:
- K-edge = 69.53 keV
- L_I-edge = 12.10 keV
- L_F-edge = 11.54 keV
- L_m-edge = 10.21 keV

Compton

Total

Photoelectric

Pair production
X-ray Cross Section and Linear Attenuation Coefficient

- **Cross section** is a measure of the probability (‘apparent area’) of interaction: $\sigma(E)$ measured in barns ($10^{-24}$ cm$^2$)
- Interaction probability can also be expressed in terms of the thickness of the material – **linear attenuation coefficient**: $\mu(E)$ = *fractional number of photons removed (attenuated) from the beam after traveling through a unit length* in media by absorption or scattering
  
  - $\mu(E)$ [cm$^{-1}$] = $Z$ [e$^-$/atom] $\cdot$ $N_{avg}$ [atoms/mole] $\cdot$ $1/A$ [moles/gm] $\cdot$ $\rho$ [gm/cm$^3$] $\cdot$ $\sigma(E)$ [cm$^2$/e$^-$]
  
  - Multiply by 100% to get % removed from the beam/cm
- $\mu(E)$ ↓ as $E$ ↑, e.g., for soft tissue $\mu(30$ keV$) = 0.35$ cm$^{-1}$ and $\mu(100$ keV$) = 0.16$ cm$^{-1}$
Calculation of the Linear Attenuation Coefficient

To the extent that Compton scattered photons are completely removed from the beam, the attenuation coefficient can be approximated as

\[
\mu = \rho N_e \left[ f(E) + C_p \frac{Z^m}{E^n} \right]
\]

\[
N_s = N_A \frac{Z}{A}
\]

\[
Z = \text{atomic number}
\]

\[
A = \text{atomic mass}
\]

\[
N_A = \text{Avogadro's number}
\]

\[
\rho = \text{density}
\]

\[
f(E) = 0.597 \times 10^{-24} e^{-0.0028(E-30)}
\]

= Compton scattering part for low E

\[
C_p = 9.8 \times 10^{-24}
\]

\[
m = 3.8, n = 3.2
\]
Calculation of Effective Z

• This is quite approximate but does permits simple computation provided that (a) the photon energy is high enough (b) scattered photons are removed from the beam and (c) \( E < 200 \text{keV} \).

• The effective Z (\( Z_{\text{eff}} \)) is given as

\[
Z_{\text{eff}} = \left( \sum \alpha_i Z_i \right) \left( \frac{\lambda}{\sum \alpha_i Z_i} \right)
\]

where \( \alpha_i \) is the electron fraction of the \( i^{th} \) element.

\[
\alpha_i = \frac{N_{g_i}}{\sum_j N_{g_j}}
\]

\[
N_{g_i} = N_A w_i \left( \frac{Z_i}{A_i} \right)
\]

\( \rightarrow \) fraction by weight of the element
Measure of X-ray Beam Strength

- An exponential relationship between the incident radiation intensity ($I_0$) and the transmitted intensity ($I$) with respect to thickness:

\[ I(E) = I_0(E) \cdot e^{-\mu(E) \cdot x} \]

\[ \mu_{\text{total}}(E) = \mu_{\text{PE}}(E) + \mu_{\text{CS}}(E) + \mu_{\text{RS}}(E) + \mu_{\text{PP}}(E) \]

- At low x-ray $E$: $\mu_{\text{PE}}(E)$ dominates and $\mu(E) \propto Z^3/E^3$
- At high x-ray $E$: $\mu_{\text{CS}}(E)$ dominates and $\mu(E) \propto \rho$
- Only at very-high $E$ (> 1MeV) does $\mu_{\text{PP}}(E)$ contribute
- The value of $\mu(E)$ is dependent on the phase state: $\mu_{\text{water vapor}} \ll \mu_{\text{ice}} < \mu_{\text{water}}$
Effective Z – An Example

• The effective Z (Z_{eff}) of H_2O is

\[
N_{g_o} = N_A \left( \frac{16}{18} \right) \left( \frac{8}{16} \right) \quad ; \quad N_{g_H} = N_A \left( \frac{2}{18} \right) \left( \frac{1}{1.008} \right)
\]

\[= 0.444 N_A \quad ; \quad = 0.111 N_A\]

\[
\alpha_o = \frac{0.444}{0.555} = 0.8 \quad ; \quad \alpha_H = 0.2
\]

\[
Z_{eff} = \left[ 0.8(8)^{3.8} + 0.2(1)^{3.8} \right]^{\frac{1}{3.8}}
\]

\[= 7.54\]

\textit{A}_{eff} \textit{ is just a normal weighted sum.}
Measure of X-ray Beam Strength

- Attenuation mechanisms as a function of energy

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>$\mu$ dependence</th>
<th>Energy Range in Soft Tissue</th>
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<tr>
<td>simple scatter</td>
<td>$1/E$</td>
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<tr>
<td>photoelectric</td>
<td>$1/E^3$</td>
<td>1-30 keV</td>
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<tr>
<td>Compton</td>
<td>falls slowly with $E$</td>
<td>independent</td>
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<tr>
<td>pair production</td>
<td>rises slowly with $E$</td>
<td>$Z^2$</td>
</tr>
</tbody>
</table>
Attenuation Mechanism

- Attenuation mechanisms as a function of energy

The optimum photon energy is about 30 keV (tube voltage 80-100 kV) where the photoelectric effect dominates. The $Z^3$ dependence leads to good contrast:

$$
\begin{align*}
Z_{\text{fat}} & = 5.9 \\
Z_{\text{muscles}} & = 7.4 \\
Z_{\text{bone}} & = 13.9
\end{align*}
$$

$\Rightarrow$ Photoelectric attenuation from bone is about 11x that due to soft tissue, which is dominated by Compton scattering.
Attenuation Mechanism

- Attenuation mechanisms as a function of energy

**Figure 4.8**
Linear attenuation coefficient for bone, muscle, and fat as a function of incident x-ray photon energy.
Half-Value Layer

- Thickness of material required to reduce the intensity of the incident beam by $\frac{1}{2}$
- $\frac{1}{2} = e^{-\mu(E) \cdot \text{HVL}}$ or HVL = $\frac{0.693}{\mu(E)}$
- Units of HVL expressed in mm Al for a Dx x-ray beam
- For a monoenergetic incident photon beam (i.e., that from a synchrotron), the HVL is easily calculated
- For each HVL, $I \downarrow$ by $\frac{1}{2}$: $5 \text{ HVL} \rightarrow I/I_0 = 100%/32 = 3.1\%$
X-ray Linear Attenuation Coefficient
Monoenergetic and Narrow Beam Case

• Suppose the slab is not homogeneous $\rightarrow \mu$ dependents on $x$
  (along the beam direction)

\[
\mu = \frac{n/N}{\Delta x},
\]

• Assuming monoenergetic

\[
N = N_0 e^{-\mu \Delta x}, \quad \leftrightarrow \quad I = I_0 e^{-\mu \Delta x},
\]
X-ray Linear Attenuation Coefficient
Polyenergetic and Narrow Beam Case

- The attenuation of a poly energetic X-ray beam by a thin slab of material can be decomposed as x-ray flux at different energies.

\[ S(E) = S_0(E) e^{-\mu(E) \Delta x} . \]

- The attenuation of a poly energetic X-ray beam by a heterogeneous slab is then

\[ S(x; E) = S_0(E) \exp \left\{ - \int_0^x \mu(x'; E) \, dx' \right\} . \]
X-ray Linear Attenuation Coefficient
Polyenergetic and Narrow Beam Case

- The energy dependent X-ray flux can be converted to the intensity as

\[ I = \int_0^\infty S_0(E') E' \exp \left\{ -\mu(E') \Delta x \right\} dE' \]

- Taking into account the spatial variation of the energy-dependent attenuation coefficient, the remaining beam intensity at a given position \( x \) is

\[ I(x) = \int_0^\infty S_0(E') E' \exp \left\{ -\int_0^x \mu(x'; E') dx' \right\} dE' \]
Beam Hardening Effect
Mean Free Path and Beam Hardening

Mean free path (avg. path length of x-ray): \( = 1/\mu = \text{HVL}/0.693 \)

Beam hardening:
- The Bremsstrahlung process produces a wide spectrum of energies, resulting in a polyenergetic (polychromatic) x-ray beam
- As lower E photons have a greater attenuation coefficient, they are preferentially removed from the beam
- Thus the mean energy of the resulting beam is shifted to higher E
Mean Free Path and Beam Hardening

Beam-hardening effect and correction. The upper left picture shows a water-filled cylinder beam-hardening corrected homogeneous cross-sectional image, below the yellow line corresponding profile curve. The upper right is the image of the same cylinder without beam-hardening correction and the corresponding profile curve is shown below.
Filtration of X-ray Generators
Photoelectric Effect

Copper (Z=29)

Electron binding energies:
- K-edge = 8.98 keV
- L$_{1}$-edge = 1.10 keV
- L$_{2}$-edge = 0.95 keV
- L$_{3}$-edge = 0.93 keV

Linear attenuation coefficients (cm$^{-1}$)

Energy (MeV)

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Compton

Pair production

Photoelectric

Tungsten (Z=74)

Electron binding energies:
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- L$_{3}$-edge = 10.21 keV

Linear attenuation coefficients (cm$^{-1}$)

Energy (MeV)

Total

Compton

Photoelectric

Pair production

NPRE 435, Principles of Imaging with Ionizing Radiation, Fall 2016
X-ray Generation – Filtration

Options:
- Molybdenum (Mo)
- Ruthenium (Ru)
- Rhodium (Rh)
- Palladium (Pd)
- Silver (Ag)
- Cadmium (Cd)

**FIGURE 8-7.** The output of a mammography x-ray system is composed of bremsstrahlung and characteristic radiation. The characteristic radiation energies of molybdenum (17.5 and 19.6 keV) are nearly optimal for detection of low-contrast lesions in breasts of 3- to 6-cm thickness.
X-ray Generation – Filtration

**FIGURE 8-8.** (a) The attenuation coefficients of Mo and Rh are plotted as a function of energy. A low-attenuation “window” exists just below the K-edge energy. (b) *Unfiltered* spectra from a molybdenum target are shown for 26- and 30-kVp tube voltages. These spectra contain a relatively large fraction of low- and high-energy photons. (c) The filtered spectra from a molybdenum target at 26 and 30 kVp after transmission through a 30 μm Mo filter. The filter eliminates a majority of the low- and high-energy x-rays.
X-ray Generation – Filtration

- Filtering reduces the x-ray energy photons below the K-shell edge providing a transmission window for characteristic x-rays.

- Typical values – Mo target with 0.03 mm Mo filter (Mo/Mo)
  - Rh target with 0.025 mm Rh filter (Rh/Rh)
  - Mo target with Rh filter

- Note: cannot use Rh target with Mo filter!

Ref: Bushberg
Application of Contrast Agents
Photoelectric Effect and Absorption Edge

- Edges become significant factors for higher Z materials as the $E_b$ are in the diagnostic energy range:
- **Contrast agents** – barium (Ba, Z=56) and iodine (I, Z=53)
- Rare earth materials used for intensifying screens – lanthanum (La, Z=57) and gadolinium (Gd, Z=64)
- **Computed radiography** (CR) and **digital radiography** (DR) acquisition – europium (Eu, Z=63) and cesium (Cs, Z=55)
- Increased absorption probabilities improve subject contrast and quantum detective efficiency
- At photon $E << 50 \text{ keV}$, the photoelectric effect plays an important role in **imaging soft tissue**, amplifying small differences in tissues of slightly different Z, thus improving subject contrast (e.g., in mammography)
Example of Contrast Agents

Bring out the difference between fat and soft tissues

Figure 5.8
Linear attenuation coefficients of bone, muscle, fat, and two contrast agents. (From Johns and Cunningham, 1983.)
Example of Contrast Agents

- Blood vessels are not normally seen in an x-ray image, because they do not contrast sufficiently with the surrounding tissues.
- To increase image contrast, contrast agents, which are dense fluids with elements of high atomic numbers, such as iodine, are injected into a blood vessel during angiography. Because of its higher density and high atomic number, iodine absorbs photons more than blood and tissue.
- This creates detailed images of the blood vessels in real time.
- The first contrast media used for intravascular injection were called highosmolar contrast media (HOCM). (Osmolality is the measure of the particle concentration in a solution.)
Example of Contrast Agents

Angiographic Imaging:

Blood vessels are visualized by injecting a “contrast medium”.

Example:
Blood vessels in kidney.
Example of Contrast Agents

Principles of DSA (Digital Subtraction Angiography)

1. Recording of image $I_1$ without contrast medium ("mask image").
2. Recording of image $I_2$ after contrast injection.
3. Subtraction $I_1 - I_2$.

**DAS of aorta.**
X-ray Generation – Characteristic X-rays

This is a mask image showing the background bone which obscures many of the smaller vessels.

Subtracted image with the background details removed.

Both images from Bushberg et al. 2003
Contrast Enhanced X-ray Imaging

(a) and (b): in vivo projection x-ray tumor images at day 3. (b): the magnified, color-coded view of the rectangular portion of (a), showing that the tumor already developed a vessel network with feeding arterioles (arrow, red color vessels) and venules (arrow head, blue color vessels). The brown area is the inoculated site. (c): projection image similar to (a), at day 7. (d) and (e): tomographic reconstructed images corresponding to (c) with two levels of magnification (supplementary data S2 d and e). Tumor angiogenesis at day 7 is denser than at day 3. Most of inoculated primary tumor cells stayed at the inoculation site and angiogenesis developed around but not inside the central area. (f)-(h): tomography results at day 7 without Au-NPs also show angiogenesis around but not inside the inoculation site (black dotted area). (f): projection image (the dotted circle marks the tumor whereas (g) and (h) are tomographic reconstructed image with low and high magnification (supplementary data S2 g and h). All scale bars are 500 μm.

Fig. from “X-ray imaging of tumor growth in live mice by detecting gold-nanoparticle-loaded cells”, Chia-Chi Chien et al., Scientific Report, 2012.
Fig. 5. Micro-CT liver imaging with hepatocyte-specific contrast agent: vitamin E nano-emulsions, from [41]. (A 1) Left lateral view, 3D rendering, (A 2) sagittal view, maximal intensity projection. (B 1) to (B 5) show 3D rendering of liver sections, emphasizing the clear contrast different between the liver tissues and its vascularization. (B 4 bis) is the transverse view of maximal intensity projection corresponding to (B 4).

From contrast agents for preclinical targeted X-ray imaging, 2014
X-ray Generation – Characteristic X-rays

Figure 5.5
Relative intensity of x-ray photons. (Adapted from Webster, 1998. This material is used by permission of John Wiley & Sons, Inc.)
Effect of Compton Scattering on X-ray Image Quality
Compton Scattering

• Compton interaction probability is dependent on the total no. of e\(^{-}\) in the absorber vol. \((e^{-}/cm^3 = e^{-}/gm \cdot \text{density})\) or electron density

• With the exception of \(^1\text{H}\), \(e^{-}/gm\) is fairly constant for organic materials \((Z/A \approx 0.5)\), thus the probability of Compton interaction proportional to material density \((\rho)\)

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\]

where

\(m_e c^2 = 511\) keV

\(E_{hv} : \) energy of the incident photon

\(E_{hv'} : \) energy of the scattered photon
Scattered Radiation

- \( m_m(CS) \approx m_m(PE) \)
  - Tissue @ 26 keV
  - Bone @ 35 keV
- Most radiographic interactions produce scattered photons
- Scattered photons \( \rightarrow \) violation of the basic principle of projection imaging: misinformation reducing contrast
Scattered Radiation

Scatter-to-Primary ratio (S/P)
- Area of collimated x-ray field
- Object thickness
- kVp of x-ray beam

Field of View (side of square in cm)

100 kVp
no grid

T=30 cm
T=20 cm
T=10 cm
T=5 cm
Scattered Radiation

Loss of contrast
In the absence of scatter:
\[ C_0 = \frac{[A-B]}{A} \]
In the presence of scatter:
\[ C = C_0 \times \frac{1}{1 + \frac{S}{P}} \]
\( S/P \uparrow \rightarrow \) contrast \( \downarrow \)
\( 1/(1+\{S/P\}) \): contrast reduction factor
Scattered Radiation

Diagnostic relevant information contained in primary (unscattered) radiation.

Scattered radiation may be several times higher (typically 4~5) and acts like a veil over the image, reducing the contrast.

primary “image”

primary + scatter “image”
Example of Contrast Agents
Most grid artifacts due to mispositioning

Upside down: severe loss of OD at margins
Crooked & off-center: general decrease of OD across entire image

Off-focus: loss at lateral edges
Scattered Reduction – Air Gap

- **Air gap**
  - Slightly decreases primary radiation
  - Strongly decreases secondary radiation
  - Requires higher kV and-or mA
  - Results in a magnified image
Angular Distribution of the Scattered Gamma Rays

Klein-Nishina formula:

\[
\frac{d\sigma}{d\Omega} (\theta) = Zr_e^2 \left( \frac{1}{1 + \alpha(1 - \cos(\theta))} \right)^2 \left( 1 + \frac{1 + \cos^2(\theta)}{2} \right) \left( 1 + \frac{1 + \cos^2(\theta)}{(1 + \cos^2(\theta))[1 + \alpha(1 - \cos(\theta))]} \right), \quad \alpha = \frac{\hbar v}{m_0c^2}
\]

Probability of Compton scattering within a unit solid angle around a scattering angle \( \theta \).

Incident photons with higher energy tends to scatter with smaller angle (forward scattering)

Incident photons with lower energy (a few hundred keV) have greater chance of undergo large angle scattering (back scattering)

Radial distance represents the differential cross section.
X-Ray Film-screen Detectors

Standard projection radiography geometry

- Projection imaging is the acquisition of a 2D image of a patient’s 3D anatomy
- Projection radiography is a transmission imaging procedure
- The optical density at any location on the film corresponds to the attenuation characteristics \( e^{-\mu x} \) of the patient at that location
X-Ray Film-screen Detectors

X-ray absorption vs. energy by different screens

Spectrum of primary and scattered x-rays from a tube operated at 80 kVp, with a Perspex (clear acrylic resin) phantom $\Rightarrow$ usefulness of Gd screen to suppress scattered x-rays