Radiological Imaging

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Course Components

- 37 lectures.
- 1-2 (TBD) term project(s).
- 2 lab tours:
  - Molecular Imaging Laboratory (MIL) SPECT/PET/CT imaging at the Biomedical Imaging Center, Beckman Institute.
  - Microscopy Suit (Micro- and Nano-CT) at Beckman Institute.
- 2 in-class quizzes, one mid-term exam and one final exams.
- 7 homework’s.
Lecture Topics

Introduction
Chapter 1: Ionizing Radiation and Interaction of Radiation with Matter
• Radiation Source
• Radiation Interactions

Chapter 2: Mathematical Preliminaries for Imaging Sciences
• Signals and Systems
• Fourier Transform Basics
• Analytical Image Reconstruction Methods
• Iterative Reconstruction Methods
• Image Quality

Chapter 3: X-Ray Computed Tomography
• X-ray Physics
• Detector Systems
• Image Reconstruction Methods
• Future Developments
• Other X-ray based imaging techniques.

Chapter 4: Single Photon Emission Computed Tomography (SPECT)
• Introduction
• Detector Technologies
• Mechanical Collimation Techniques – General Considerations
• Coded Aperture Imaging
• Compton Imaging Techniques

Chapter 5: Positron Emission Tomography (PET)
• Basic Principles of PET
• Advantages of PET
• Computation Models of the Detected Events
• Detector Technologies for PET
• Future Development – (Time-of-flight, DOI capabilities etc.)

Chapter 6: Emerging Radiological Imaging Techniques
Radiation exists all around us ...
Ionizing Radiation
Sources of Ionizing Radiation Around Us

Cosmic rays and naturally-occurring radioactive elements
Medical X-rays and nuclear medicine studies
Manmade radioactive materials and radiation sources
Cosmic Rays Radiation

For example, muons are the most numerous energetic charged particles at sea level. Muons lose energy at a fairly constant rate of about 2 MeV per g/cm². Since the vertical depth of the atmosphere is about 1000 g/cm², muons will lose about 2 GeV to ionization before reaching the ground. The mean energy of muons at sea level is still 4 GeV.

http://cosmic.lbl.gov/SKliewer/Cosmic_Rays/Muons.htm
Ionizing Radiation

If radiation carries sufficient energy to knock an electron out of its orbital shell around an atom, then it is ionizing radiation. The minimum energy to do this is between 2 and 35 eV, depending upon the atom. For example,

Alpha - $\alpha$ - He$^{++}$ - Very short-ranged radiation, a few microns in water
Beta - $\beta^-$ or $e^-$ - Medium Range, a few hundred microns to a few mm.
Positron - $\beta^+$ - Medium Range, similar
Photon - $\gamma$ or X-rays – penetrating, a few mm to tens of cm.
Proton - p$^+$ - Short range, a few microns.
Neutron - n – Penetrating, energy-dependent, and varies with material.
Imaging Techniques based on “EM” Radiation

MRI: Essentially proton density, phenotyping, physiological imaging and cell tracking

Ultrasound: Mechanical properties

Optical microscopy

Nuclear imaging: metabolism of molecules, e.g. glucose, antibodies, peptides thymidine ...

Fluorescence and Bioluminescence imaging: Quantitative imaging of targeted or ‘smart’ fluorochrome reporters in deep tumors, gene expression, cell tracking

X-ray: Electron density, Lung, bone, tumor imaging, etc
Basic Ideas Behind Radiological Imaging – Planar vs. Tomographic Imaging
Nov. 1895 – Announces X-ray discovery
Jan. 13, 1896 – Images needle in patient’s hand
– X-ray used presurgically
1901 – Receives first Nobel Prize in Physics for the discovery and use of X-rays.
X-ray Planar Radiography

January 1896 - First x-ray made in public

Routine x-ray current technology
Planar X-ray Radiography

- Bone
- Soft Tissue
- Fat
- Air
X-ray Planar Radiography

What are we measuring with planar X-ray radiography?

\[ I = I_o e^{-\mu \Delta x} \]

\[ I = I_o e^{-\mu_1 \Delta x} e^{-\mu_2 \Delta x} \ldots e^{-\mu_n \Delta x} = I_o e^{-(\mu_1 + \mu_2 + \ldots + \mu_n) \Delta x} \]

\[ \Delta x \rightarrow 0, \quad P = -\ln \left( \frac{I}{I_o} \right) = \int_{-\infty}^{\infty} \mu(x) \, dx \]
X-ray Computed Tomography (CT)
X-ray Computed Tomography (CT)

0.2 % attenuation change detectable in CT Images !!
Sir Godfrey Newbold Hounsfield, CBE, FRS, was an English electrical engineer who shared the 1979 Nobel Prize for Physiology or Medicine with Allan McLeod Cormack for his part in developing the diagnostic technique of X-ray computed tomography.
Modern X-ray Computed Tomography (CT)
X-ray Computed Tomography (CT)

Planar X-Ray

Computed Tomography

Separates Objects on Different Planes

Images courtesy of Robert McGee, Ford Motor Company
What Can We See With X-ray CT?

http://fanz.dvrlists.com/about-human-anatomy/

http://www.indiana.edu/~busey/Q301/BrainStructure.html
The Tracer Principle
George de Hevesy, who received the Nobel Prize in Chemistry in 1943.
What is the Tracer Principle?

What is the tracer principle?

• Appropriately chosen radioactive compounds participate in an organism’s physiological processes in the same way as non-radioactive materials.

• These compounds can be detected through the detection of their radiation signatures, such as gamma rays.
Emission Tomography

- Drug is labeled with radioisotopes that emit gamma rays.
- Drug localizes in patient according to metabolic properties of that drug.
- Trace (pico-molar) quantities of drug are sufficient.
- Radiation dose fairly small (<1 rem).

Drug Distributes in Body
Emission Tomography

(Left and above) Typical emission tomography images

Is there anything that we do not like??

http://imaging.cancer.gov/patientsandproviders/cancerimaging/nuclearimaging
Single Photon Emission Computed Tomography (SPECT)

Siemens Symbia SPECT/CT

Philips Precedence SPECT/CT
Single Photon Emission Computed Tomography (SPECT)

Collimator in front of the detector to select gamma rays from certain directions only ...

Collimator

Rotated around the object for collecting multiple projections ...

Pinhole

Coded Aperture

Compton
Single Photon Emission Computed Tomography (SPECT)

\[ I = I_0 e^{-\mu_1 \Delta x} e^{-\mu_2 \Delta x} \ldots e^{-\mu_n \Delta x} \]

\[ = I_0 e^{-(\mu_1 + \mu_2 + \ldots + \mu_n) \Delta x} \]

\[ \Delta x \rightarrow 0, \quad P = -\ln \left( \frac{I}{I_0} \right) = \int_{-\infty}^{\infty} \mu(x) dx \]
Single Photon Emission Computed Tomography (SPECT)

Endocrine tumor visualized with a commercial SPECT system.

SPECT Imaging of Radiolabeled Nano-Engineered Neural Stem Cells for Glioblastoma Targeting and Therapy

Application of SPECT for Small Animal Studies

Imaging of dopamine transporter in mouse brain

In vivo tracking of radiolabeled T cells in mouse brain.
Neutron-deficient isotopes can decay by emitting positrons – $\beta^+$ decay

Net effect: one proton replaced by

- neutron
- anti-neutrino
- positron
Positron Emission Tomography

Beta-plus decay or positron decay:

\[ ^{A}ZX \rightarrow ^{A}Z-1Y + ^{0}e^+ + \nu \]

Example of positron annihilation

Radiolabeling
Clinical Applications of PET – Functional Brain Imaging

Functional Brain Imaging with Fluorodeoxyglucose (FDG)

http://www.osti.gov/accomplishments/pet.html
Clinical Applications of PET – Diagnosis of Cancer

FDG study: lung or ribs?

I-124 study: bone or soft tissue?
Clinical Applications of PET – Treatment Planning

Treatment planning for radiation therapy
Autoradiography

with $^{14}$C, $^3$H, $^{35}$S, $^{125}$I labeled tracers
Bioimaging of copper alterations in the aging mouse brain by autoradiography, laser ablation inductively coupled plasma mass spectrometry and immunohistochemistry

Li-Ming Wang*, J. Sabine Becker*, Qi Wu*, Marcus F. Oliveira*, Fernando A. Bozza*, Andrea L. Schwager†, John M. Hoffman† and Kathryn A. Morton†
Nuclear Imaging based on Stimulated Emission
Drug is labeled with radioisotopes that emit gamma rays.

Drug localizes in patient according to metabolic properties of that drug.

Trace (pico-molar) quantities of drug are sufficient.

Radiation dose fairly small (<1 rem).

Drug Distributes in Body
Emission Tomography

(Left and above) Typical emission tomography images

Is there anything that we do not like??

http://imaging.cancer.gov/patientsandproviders/cancerimaging/nuclearimaging
X-ray Fluorescence Emission
Re-Engineering the Way that Ionizing Radiation Interacting with Tissue

Combined chemical and physical targeting?

Nano-materials that preferably receive radiation and react in various ways?

Specific targeting uptake

Deep-penetrating radiation

UV, \text{NIR}, \text{X-rays}, \gamma\text{-rays}, ...

Localized \text{PDT}

Imaging, Sensing, Detection

Precisely modulate therapeutic delivery?

Center figure from L-W Luo, Dept. Radiology, U Chicago
Typical Apertures for X-ray Fluorescence Emission Tomography (XFET)

- Sample
- X-ray Beam (~2 mm)
- Translation Stage (100 nm resolution)
- Rotation Stage (10 mdeg resolution)
- Fluorescence Detector
- Silica Fiber (~5 mm)

Figure courtesy Mark Rivers, ANL GSECARS

Meng and La Riviere
XFET

Meng and La Riviere
Mapping the Distribution of Metal Elements in β-cells

**UNTREATED**

<table>
<thead>
<tr>
<th>Element</th>
<th>max</th>
<th>min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca</td>
<td>0.557</td>
<td>0.010</td>
</tr>
<tr>
<td>Mn</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Zn</td>
<td>0.100</td>
<td>0.000</td>
</tr>
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</table>

**50 uM Mn and 16 Mm Glucose**

<table>
<thead>
<tr>
<th>Element</th>
<th>max</th>
<th>min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca</td>
<td>0.800</td>
<td>0.000</td>
</tr>
<tr>
<td>Mn</td>
<td>0.200</td>
<td>0.010</td>
</tr>
<tr>
<td>Zn</td>
<td>0.300</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**50 uM Mn, 16 Mm Glucose and 30 mM KCl**

<table>
<thead>
<tr>
<th>Element</th>
<th>max</th>
<th>min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca</td>
<td>4.330</td>
<td>0.020</td>
</tr>
<tr>
<td>Mn</td>
<td>0.970</td>
<td>0.000</td>
</tr>
<tr>
<td>Fe</td>
<td>0.165</td>
<td>0.000</td>
</tr>
<tr>
<td>Zn</td>
<td>0.113</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Unpublished results by Patrick La Riviere
XFET Imaging of Tracer Metals

(Below) XFET Imaging of Os in Zebrafish

(Below) 3-D Elemental Mapping of Trace Metals in Plants

“Emission tomography and its place in the matrix of molecular imaging technologies”
An Overview of Mouse Imaging Systems


<table>
<thead>
<tr>
<th>Technique</th>
<th>Resolution</th>
<th>Depth</th>
<th>Time</th>
<th>Imaging agents</th>
<th>Cost</th>
<th>Primary use</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic resonance imaging (MRI)</td>
<td>10–100 µm</td>
<td>No limit</td>
<td>Min/hours</td>
<td>Gadolinium, dysprosium, iron oxide particles</td>
<td>$$$</td>
<td>phenotyping, physiological imaging and cell tracking</td>
</tr>
<tr>
<td>X-ray computed tomography (CT)</td>
<td>50 µm</td>
<td>No limit</td>
<td>Min</td>
<td>Iodine</td>
<td>$$</td>
<td>Lung, bone, tumour imaging</td>
</tr>
<tr>
<td>Ultrasound imaging</td>
<td>50 µm</td>
<td>mm</td>
<td>Min</td>
<td>Microbubbles</td>
<td>$$</td>
<td>Vascular and interventional imaging</td>
</tr>
<tr>
<td>Positron emission Tomography (PET)</td>
<td>0.8–2 mm</td>
<td>No limit</td>
<td>Min</td>
<td>18F, 11C, 15O</td>
<td>$$$</td>
<td>Imaging metabolism of molecules, such as glucose, thymidine ...</td>
</tr>
<tr>
<td>Single photon emission tomography (SPECT)</td>
<td>0.1-1 mm</td>
<td>No limit</td>
<td>Min</td>
<td>99mTc, 111In</td>
<td>$$</td>
<td>Imaging of probes such as antibodies, peptides ...</td>
</tr>
<tr>
<td>Fluorescence reflectance imaging (FRI)</td>
<td>1–2 mm</td>
<td>&lt; 1 cm</td>
<td>Sec/min</td>
<td>Fluorescent proteinsNIR fluorochromes</td>
<td>$</td>
<td>Rapid screening of molecular events in surface-based tumours</td>
</tr>
<tr>
<td>Fluorescence-mediated tomography (FMT)</td>
<td>1–2 mm</td>
<td>&lt; 10 cm</td>
<td>Sec/min</td>
<td>NIR fluorochromes</td>
<td>$$</td>
<td>Quantitative imaging of targeted or ‘smart’ fluorochrome reporters in deep tumours</td>
</tr>
<tr>
<td>Bioluminescence imaging (BLI)</td>
<td>Several mm</td>
<td>cm</td>
<td>Min</td>
<td>Luciferin</td>
<td>$$</td>
<td>Gene expression, cell tracking</td>
</tr>
</tbody>
</table>
Other Examples for “Nuclear” Imaging
“Nuclear” Imaging in Astrophysics

Chandra view of the galactic center

http://chandra.harvard.edu/
X-ray Emission from the Sun

Using ionizing radiation as a probe to study the invisible properties of an object ...

The Sun as seen in X-rays (image taken by the Yohkoh satellite)

X-ray detection of the Moon, taken by ROSAT.
http://wave.xray.mpe.mpg.de/rosat/mission/rosat/introduction
Imaging With Neutrons

The 1994 Nobel Prize in Physics – Shull & Brockhouse

Neutrons show where the atoms are....

...and what the atoms do.

From http://www.dep.anl.gov/nx/lectrnotes.pdf
Imaging Fuel Cell With Neutrons

Key Components of an Imaging System

Based on Ionizing Radiation

- Sources of Radiation
- Objects to be Examined
- Ionizing Radiation as Detectable Signal
- Detection System
- Data Processing System
- Specific Information about the Object-of-interest

A typical medical imaging system

Sources of Ionizing Radiation and Radiation Interactions

Reading Material:

Chapter 1, Radiation Detection and Measurements,

Forth Edition, by G. F. Knoll
Table of Contents

I. Radiation Sources
   • Sources of Fast Electron and Heavy Charged Particles
   • Gamma Rays Sources
   • X-Rays Sources
   • Neutron Sources

II. Radiation Interactions
   • Interactions of X ray and Gamma Rays
   • Interactions of Fast Electrons and Heavy Charged Particles.
   • Interactions of Neutrons
<table>
<thead>
<tr>
<th>isotope</th>
<th>symbol</th>
<th>Z</th>
<th>T$_{1/2}$</th>
<th>decay</th>
<th>gamma (keV)</th>
<th>positron (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaging</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluorine-18</td>
<td>$^{18}$F</td>
<td>9</td>
<td>109.77 m</td>
<td>$\beta^+$</td>
<td>511 (193%)</td>
<td>249.8 (97%)$^{[11]}$</td>
</tr>
<tr>
<td>gallium-67</td>
<td>$^{67}$Ga</td>
<td>31</td>
<td>3.26 d</td>
<td>e$c$</td>
<td>93 (39%), 185 (21%), 300 (17%)</td>
<td>-</td>
</tr>
<tr>
<td>krypton-81m</td>
<td>$^{81m}$Kr</td>
<td>36</td>
<td>13.1 s</td>
<td>IT</td>
<td>190 (68%)</td>
<td>-</td>
</tr>
<tr>
<td>rubidium-82</td>
<td>$^{82}$Rb</td>
<td>37</td>
<td>1.27 m</td>
<td>$\beta^+$</td>
<td>511 (191%)</td>
<td>3.379 (95%)</td>
</tr>
<tr>
<td>nitrogen-13</td>
<td>$^{13}$N</td>
<td>7</td>
<td>9.97 m</td>
<td>$\beta^+$</td>
<td>511 (200%)</td>
<td>1190 (100%)$^{[12]}$</td>
</tr>
<tr>
<td>technetium-99m</td>
<td>$^{99m}$Tc</td>
<td>43</td>
<td>6.01 h</td>
<td>IT</td>
<td>140 (89%)</td>
<td>-</td>
</tr>
<tr>
<td>indium-111</td>
<td>$^{111}$In</td>
<td>49</td>
<td>2.80 d</td>
<td>e$c$</td>
<td>171 (90%), 245 (94%)</td>
<td>-</td>
</tr>
<tr>
<td>iodine-123</td>
<td>$^{123}$I</td>
<td>53</td>
<td>13.3 h</td>
<td>e$c$</td>
<td>159 (83%)</td>
<td>-</td>
</tr>
<tr>
<td>xenon-133</td>
<td>$^{133}$Xe</td>
<td>54</td>
<td>5.24 d</td>
<td>$\beta^-$</td>
<td>81 (31%)</td>
<td>0.364 (99%)</td>
</tr>
<tr>
<td>thallium-201</td>
<td>$^{201}$Tl</td>
<td>81</td>
<td>3.04 d</td>
<td>e$c$</td>
<td>69–83$^*$ (94%), 167 (10%)</td>
<td>-</td>
</tr>
<tr>
<td>Therapy</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yttrium-90</td>
<td>$^{90}$Y</td>
<td>39</td>
<td>2.67 d</td>
<td>$\beta^-$</td>
<td>-</td>
<td>2.280 (100%)</td>
</tr>
<tr>
<td>iodine-131</td>
<td>$^{131}$I</td>
<td>53</td>
<td>8.02 d</td>
<td>$\beta^-$</td>
<td>364 (81%)</td>
<td>0.807 (100%)</td>
</tr>
</tbody>
</table>

Z = atomic number, the number of protons
T$_{1/2}$ = half-life, decay = mode of decay
photons = principle photon energies in kilo-electron volts, keV, (abundance/decay)
$\beta$ = beta maximum energy in mega-electron volts, MeV, (abundance/decay)
$\beta^+$ = $\beta^+$ decay; $\beta^-$ = $\beta^-$ decay; IT = isomeric transition; e$c$ = electron capture
$^*$ X-rays from progeny, mercury, Hg
Beta Decay

Things to consider from imaging viewpoint

• Types of particles emitted, gamma ray, X-ray, fast electrons, Auger electrons, and their corresponding
  • half-life,
  • energy, and
  • relative intensity.
Beta Emission

• Beta particle is an ordinary electron. Many atomic and nuclear processes result in the emission of beta particles.

• One of the most common source of beta particles is the beta decay of nuclides, in which

Beta decay
\[ ^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + ^{0}_{-1}\beta + \bar{\nu} \]

Beta-plus decay
\[ ^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + ^{0}_{1}\beta + \nu \]

Electron capture
\[ ^{A}_{Z}X + e^{-} \rightarrow ^{A}_{Z-1}Y + \nu \]
“Regular” Beta Decay
Energy Release of Beta Decay

The energy release in a beta decay is given as

\[ Q = M_p - (M_d + M_e) \]

- The energy release is once again given by the conversion of a fraction of the mass into energy. Note that atomic electron bonding energy is neglected.
- For a beta decay to be possible, the energy release has to be positive.
• The energy release is shared by all three daughter products. Due to the relatively large mass of the daughter nucleus, it attains only a small fraction of the energy. Therefore, the kinetic energy of the beta particle is

\[ E_{\beta^-} \approx Q - E_\nu \]
An Example of Cancer Therapy with Radiopharmaceuticals

**DIAGNOSIS**
68 Gallium-**Octreotate** PET/CT

**THERAPY**
177 Lutetium-**Octreotate** SPECT-CT

\[ ^{177}\text{Lu} \]

\[ T_{1/2} = 6.716 \text{ days} \]

\[ \beta^- \]

\( Q_\beta = 0.4971 \text{ MeV} \)

\[ ^{177}\text{Hf} \]
Positron Decay
The energy release $Q$ associated with the positron emission process is given by

$$Q \approx M_p - M_d - M_e - M_{e^+} = M_p - (M_d + 2M_e)$$

where the atomic electron binding energy is ignored.
Orbital Electron Capture and Positron Decay

- Electron capture and positron decay are normally competing processes through which a neutron deficient nucleus may attain an increased stability.

- Both the emission of a positron and the capture of an electron, a neutrino is always emitted in order to conserve energy.

- In positron decay, the neutrino carries the difference between the energy release and the energy of the resultant positron. In electron capture, however, the neutrino must be mono-energetic.
Positron Emission Tomography

Beta-plus decay or positron decay:

\[ {}^A_{z+1}X \rightarrow {}^{A-1}_{z-1}Y + {}^0_1\beta + \nu \]

Example of positron annihilation

What about
- positron range?
- Acollinearity? and ...
Positron range in PET imaging: non-conventional isotopes

L Jødal¹, C Le Loirec² and C Champion³

¹ Department of Nuclear Medicine, Aalborg University Hospital, Aal ² CEA, LIST, F-91191 Gif-sur-Yvette, France ³ Centre d’Études Nucléaires de Bordeaux Gradignan, CNRS/IN2P3, Bordeaux, Gradignan, France

Table 1. Studied isotopes. Values of $E_{\text{mean}}$ and $E_{\text{max}}$ are determined from the initial spectra used in the Monte Carlo simulations (Le Loirec and Champion 2007a, 2007b, 2007c). Positron ranges have been determined in water.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Allowed decay</th>
<th>$T_{\text{b}}$</th>
<th>$E_{\text{mean}}$ (keV)</th>
<th>$E_{\text{max}}$ (keV)</th>
<th>$R_{\text{mean}}$ (mm)</th>
<th>$R_{\text{max}}$ (mm)</th>
<th>Example of application</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{64}$Cu</td>
<td>Yes, $\beta^-$ and $\beta^+$</td>
<td>12.7 h</td>
<td>728.2$^c$</td>
<td>653.1</td>
<td>0.56</td>
<td>2.9</td>
<td>Detection of small colorectal tumors (Philpott et al 1995)</td>
</tr>
<tr>
<td>$^{22}$Na</td>
<td>Yes$^d$</td>
<td>2.6 year</td>
<td>220.3</td>
<td>673.5</td>
<td>5.3$^e$</td>
<td>2.2$^e$</td>
<td>Scanner calibration (Topping et al 2013)</td>
</tr>
<tr>
<td>$^{52}$Mn</td>
<td>Yes</td>
<td>5.6 d</td>
<td>244.6</td>
<td>575.8</td>
<td>0.63</td>
<td>2.5</td>
<td>Candidate for bone scanning (Link et al 2006)</td>
</tr>
<tr>
<td>$^{89}$Zr</td>
<td>Yes</td>
<td>78.4 h</td>
<td>402.7</td>
<td>902</td>
<td>1.27</td>
<td>4.2</td>
<td>Quantifying the deposition of monoclonal antibodies in the tissues of tumors (Sasti et al 1981)</td>
</tr>
<tr>
<td>$^{45}$Ti</td>
<td>Yes</td>
<td>3.08 h</td>
<td>442.3</td>
<td>1040.4</td>
<td>1.47</td>
<td>5.2</td>
<td>Uptake measurements to provide insight into the mechanism of the action of titanocene dichloride (Vavere and Welch 2005)</td>
</tr>
<tr>
<td>$^{51}$Mn</td>
<td>Yes</td>
<td>46.2 min</td>
<td>970.2</td>
<td>2185.8</td>
<td>4.3</td>
<td>12.1</td>
<td>Diagnosis and treatment of blood diseases (Sasti et al 1981)</td>
</tr>
<tr>
<td>$^{94m}$Tc</td>
<td>Yes</td>
<td>52.0 min</td>
<td>1076.6</td>
<td>2362.9</td>
<td>4.7</td>
<td>12.8</td>
<td>PET substitute for $^{99m}$Tc (Liu and Laforest 2007)</td>
</tr>
<tr>
<td>$^{52}$Cr</td>
<td>Yes</td>
<td>21.1 min</td>
<td>1179</td>
<td>2630</td>
<td>5.3</td>
<td>14.5</td>
<td>Myocardial imaging (Hui et al 1979, Daube and Nickles 1985, Tolmachov et al 1994)</td>
</tr>
<tr>
<td>$^{38}$K</td>
<td>Yes</td>
<td>7.64 min</td>
<td>1218.8</td>
<td>2728</td>
<td>5.7</td>
<td>15.3</td>
<td>Myocardial perfusion (Bol et al 1993)</td>
</tr>
<tr>
<td>$^{88}$Y</td>
<td>No</td>
<td>14.7 h</td>
<td>640</td>
<td>2010.4</td>
<td>2.5</td>
<td>11.1</td>
<td>Uptake measurements prior to therapy using $^{90}$Y (Ritsch et al 1996)</td>
</tr>
<tr>
<td>$^{124}$I</td>
<td>No</td>
<td>4.176 d</td>
<td>825.9</td>
<td>2095</td>
<td>3.4</td>
<td>11.7</td>
<td>Uptake of monoclonal antibodies in solid tumors (Snyder et al 1975, Larson et al 1992, Daghighian et al 1993)</td>
</tr>
<tr>
<td>$^{120}$I</td>
<td>No</td>
<td>81 min</td>
<td>1747</td>
<td>4600</td>
<td>8.3</td>
<td>27</td>
<td>Shorter lifetime and higher positron yield than $^{124}$I (Zweig et al 1996)</td>
</tr>
</tbody>
</table>

Figure 5. Functions $\rho_{3D}(\xi)$ obtained from Monte Carlo calculations (broken lines) compared with the expressions defined by (3) using the values of $A$ and $B$ provided in table 2 (smooth lines).

---

$^a$ (Le Loirec and Champion 2007b).  
$^b$ (Le Loirec and Champion 2007c).  
$^c$ Mean value for positron emission only.  
$^d$ Decay to the ground state of $^{22}$Ne is forbidden ($3^+ \rightarrow 0^+$); instead, nearly 100% of the decays are to an excited state, $^{22}$Na ($3^+ \rightarrow ^{22}$Ne* ($2^+$)) (Lederer et al 1967).  
$^e$ (Le Loirec 2007).
A Few Details on Positron Decay and Their Implications to PET

<table>
<thead>
<tr>
<th>Isotope</th>
<th>half-life (min)</th>
<th>Maximum positron energy (MeV)</th>
<th>Positron range in water (FWHM in mm)</th>
<th>Production method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}$C</td>
<td>20.3</td>
<td>0.96</td>
<td>1.1</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{13}$N</td>
<td>9.97</td>
<td>1.19</td>
<td>1.4</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{15}$O</td>
<td>2.03</td>
<td>1.70</td>
<td>1.5</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{18}$F</td>
<td>109.8</td>
<td>0.64</td>
<td>1.0</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{68}$Ga</td>
<td>67.8</td>
<td>1.89</td>
<td>1.7</td>
<td>generator</td>
</tr>
<tr>
<td>$^{82}$Rb</td>
<td>1.26</td>
<td>3.15</td>
<td>1.7</td>
<td>generator</td>
</tr>
</tbody>
</table>

Table 2. Properties of commonly used positron emitting radio-isotopes

Element of life.
Negative and positive [C-11]PiB PET images. [C-11]PiB PET images taken in the axial plane at levels indicated in the sagittal MRI image at the far left. The scans in the top row illustrate the 79-year-old [C-11]PiB(−) DLB subject that is the focus of the current study. There is no evidence of [C-11]PiB retention except for nonspecific retention in the white matter. The scans in the bottom row illustrate a 65-year-old [C-11]PiB(+) AD subject showing high [C-11]PiB retention throughout the neocortex and the striatum.
What is $[^{18}\text{F}]$FDG?

**GENERAL INFORMATION**

Chemical name of $[^{18}\text{F}]$FDG is $[^{18}\text{F}]-2$-Fluoro-2-deoxy-$\beta$-D-glucopyranose, Chemical Abstract Service registry number 63503-12-8. More commonly it is called $[^{18}\text{F}]$Fluorodeoxyglucose or simply FDG.

This compound is a radioactive derivative of 2-deoxy-D-glucose labelled with positron-emitting isotope $^{18}\text{F}$ in the position 2 of the glucose core structure.

Relative molecular mass: 181.15 g/mol

**NAMES:**
- FDG
- $[^{18}\text{F}]$FLUORODEOXYGLUCOSE
- $[^{18}\text{F}]-2$-Fluoro-2-deoxy-$\beta$-D-glucopyranose

For more details please refer to [review article](#) on radiohalogenated sugars.
PET studies of glucose metabolism to map human brain's response in performing different tasks.


**PET-labeled probes for biological imaging:** (from S. Cherry, UC Davis)

- Hemodynamic parameters ($H_2^{15}O$, $^{15}O$-butanol, $^{11}CO$, $^{13}NH_3$....)
- Substrate metabolism($^{18}F$-FDG, $^{15}O_2$, $^{11}C$-palmitic acid....)
- Protein synthesis ($^{11}C$-leucine, $^{11}C$-methionine, $^{11}C$-tyrosine)
- Enzyme activity ($^{11}C$-deprenyl, $^{18}F$-deoxyuracil...)
- Drugs ($^{11}C$-cocaine, $^{13}N$-cisplatin, $^{18}F$-fluorouracil...)
- Receptor affinity ($^{11}C$-raclopride, $^{11}C$-carfentanil, $^{11}C$-scopalamine)
- Neurotransmitter biochemistry ($^{18}F$-fluorodopa, $^{11}C$-ephedrine...)
- Gene expression ($^{18}F$-penciclovir, $^{18}F$-antisense oligonucleotides)
Electron Capture
Energy Release Through Orbital Electron Capture

For positron decay to be possible, we need

\[ Q = M_p - M_d - M_e - M_{e^+} > 0, \]

so

\[ M_p > M_d + M_e + M_{e^+} = M_d + 2M_e \]

\( M_p \) and \( M_d \) are the atomic masses of the parent and daughter atoms.

For Electron Capture to occur,

\[ Q = M_p - M_d - \phi > 0 \]

so that

\[ M_p > M_d + \phi \]

where \( \phi \) is the binding energy of the orbital electron.
Internal Conversion

- Conversion electrons can originate from several different electron shells within the atom, a single excited state generally leads to several groups of electrons with different energies.
- The only practical laboratory scale source of mono-energetic electron groups in high keV to MeV energy range.
Auger Electrons

- The excitation energy of the atom may be transferred to one of the outer electrons, causing it to be ejected from the atom.
- Auger electrons are roughly the analogue of internal conversion electrons when the excitation energy originates in the atom rather than in the nucleus.

![Diagram of Auger Electron Process](image)

**Figure 3.7** (A) The usual emission of a K characteristic X-ray, $h\nu$, energy equal to $E_K - E_L$, the difference in binding energy for the two orbital electrons, K and L. (B) $h\nu$ has been absorbed and a monoenergetic Auger electron is emitted, in the example shown, from the M shell, the energy of which is $E_K - E_L - E_M$. (C) In its final state the atom has vacancies in the L and M orbitals.
Special SPECT Imaging Applications: Single Photon Emission Microscopy

Imaging of mouse kidney I-125, Data from Center for Gamma Ray Imaging, University of Arizona

In vivo imaging of (~1500) radiolabeled T cells labeled with I-125 in mouse brain using SPECT (Meng, 2009).

Electron capture: 100%
What would be an ideal gamma ray emitter for nuclear imaging?

- Reasonably penetrative...
- Half-life comparable to the biological process we are trying to visualize...
- Easy radiochemistry...
- Biologically safe...
- Clean ...
Metastable Nuclear States and Gamma Ray Emission

The lifetimes of nuclear excited states vary, but $\sim 10^{-10}$ s can be regarded as typical. Thus, gamma rays are usually emitted quickly after radioactive decay to an excited daughter state.

In some cases, however, selection rules prevent photon emission for an extended period of time. The excited state of $^{137}_{56}$Ba following the decay of $^{137}_{55}$Cs has a half-life of 2.55 min. Such a long-lived nuclear state is termed metastable and is designated by the symbol m: $^{137m}_{56}$Ba.

Another example of a metastable nuclide is $^{99m}_{43}$Tc, which results from the beta decay of the molybdenum isotope $^{99}_{42}$Mo. an isomeric transition (IT) to the ground

Decay Scheme for $^{99}$Mo

![Decay Scheme for $^{99}$Mo](image)
Metastable Nuclear States and Gamma Ray Emission

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Another example of a metastable nuclide is \(^{99m} \text{Tc}\), which results from the beta decay of the molybdenum isotope \(^{99} \text{Mo}\). An isomeric transition (IT) to the ground
Tc-99m Sestamibi Myocardial Perfusion Imaging (MPI)

Exercise stress Tc-99m Sestamibi single day myocardial perfusion SPECT images of the female patient with a significant (80%) distal left main coronary artery disease. Classic features of the high-risk scan are present: severe partially reversible perfusion defect, involvement of the LAD and LCX territory, visual transient ischemic dilation and abnormal TID ratio (1.21). The patient presented with symptoms of stable atypical angina. No significant ECG or hemodynamic changes were noted during the stress portion of the test.

https://www.researchgate.net/figure/Exercise-stress-Tc-99m-Sestamibi-single-day-myocardial-perfusion-SPECT-images-of-the_fig1_283448406
What do MPI images look like?

In a typical nuclear cardiac imaging exam, the physician reviews:

- Static “Summed Perfusion Images”
- Dynamic “Gated Images”

Perfusion Images are viewed in three orientations:
SA – Short Axis
VLA – Vertical Long Axis
HLA - Horizontal Long Axis
Therapeutic Beta Emitters
### Common isotopes used in nuclear medicine

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Symbol</th>
<th>Z</th>
<th>T&lt;sub&gt;1/2&lt;/sub&gt;</th>
<th>Decay</th>
<th>Gamma (keV)</th>
<th>Positron (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Imaging:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluorine-18</td>
<td>¹⁸F</td>
<td>9</td>
<td>109.77 m</td>
<td>β⁺</td>
<td>511 (193%)</td>
<td>249.8 (97%)</td>
</tr>
<tr>
<td>gallium-67</td>
<td>⁶⁷Ga</td>
<td>31</td>
<td>3.26 d</td>
<td>eC</td>
<td>93 (39%), 185 (21%), 300 (17%)</td>
<td>-</td>
</tr>
<tr>
<td>krypton-81m</td>
<td>⁸¹Kr</td>
<td>36</td>
<td>13.1 s</td>
<td>IT</td>
<td>190 (68%)</td>
<td>-</td>
</tr>
<tr>
<td>rubidium-82</td>
<td>⁸²Rb</td>
<td>37</td>
<td>1.27 m</td>
<td>β⁺</td>
<td>511 (191%)</td>
<td>3.379 (95%)</td>
</tr>
<tr>
<td>nitrogen-13</td>
<td>¹³N</td>
<td>7</td>
<td>9.97 m</td>
<td>β⁺</td>
<td>511 (200%)</td>
<td>1190 (100%)</td>
</tr>
<tr>
<td>technetium-99m</td>
<td>⁹⁹mTc</td>
<td>43</td>
<td>6.01 h</td>
<td>IT</td>
<td>140 (89%)</td>
<td>-</td>
</tr>
<tr>
<td>indium-111</td>
<td>¹¹¹In</td>
<td>49</td>
<td>2.80 d</td>
<td>eC</td>
<td>171 (90%), 245 (94%)</td>
<td>-</td>
</tr>
<tr>
<td>iodine-123</td>
<td>¹²³I</td>
<td>53</td>
<td>13.3 h</td>
<td>eC</td>
<td>159 (83%)</td>
<td>-</td>
</tr>
<tr>
<td>xenon-133</td>
<td>¹³³Xe</td>
<td>54</td>
<td>5.24 d</td>
<td>β⁻</td>
<td>81 (31%)</td>
<td>0.364 (99%)</td>
</tr>
<tr>
<td>thallium-201</td>
<td>²⁰¹Tl</td>
<td>81</td>
<td>3.04 d</td>
<td>eC</td>
<td>69–83 * (94%), 167 (10%)</td>
<td>-</td>
</tr>
<tr>
<td><strong>Therapy:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yttrium-90</td>
<td>⁹⁰Y</td>
<td>39</td>
<td>2.67 d</td>
<td>β⁻</td>
<td>-</td>
<td>2.280 (100%)</td>
</tr>
<tr>
<td>iodine-131</td>
<td>¹³¹I</td>
<td>53</td>
<td>8.02 d</td>
<td>β⁻</td>
<td>364 (81%)</td>
<td>0.807 (100%)</td>
</tr>
</tbody>
</table>

Z = atomic number, the number of protons; T<sub>1/2</sub> = half-life; decay = mode of decay; photons = principle photon energies in kilo-electron volts, keV, (abundance/decay); β = beta maximum energy in mega-electron volts, MeV, (abundance/decay); β⁺ = β⁺ decay; β⁻ = β⁻ decay; IT = isomeric transition; eC = electron capture; * X-rays from progeny, mercury, Hg.
# Therapeutic Beta Emitters

Table 1 - Physical and nuclear characteristics of bone-seeking therapeutic radionuclides\(^1\)

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Maximum energy (MeV)</th>
<th>Average energy (MeV)</th>
<th>Average Range (mm)</th>
<th>T(_{\text{half}}) (days)</th>
<th>yphoton (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strontium-89</td>
<td>1.46</td>
<td>0.58</td>
<td>2.4</td>
<td>50.5</td>
<td>None</td>
</tr>
<tr>
<td>Phosphorus-32</td>
<td>1.71</td>
<td>0.70</td>
<td>3.0</td>
<td>14.3</td>
<td>None</td>
</tr>
<tr>
<td>Tin-117m</td>
<td>0.13(^2) 0.15(^2)</td>
<td>---</td>
<td>0.22</td>
<td>14.0</td>
<td>0.159 (86%)</td>
</tr>
<tr>
<td>Erbium-169</td>
<td>0.34</td>
<td>0.11</td>
<td>0.30</td>
<td>9.3</td>
<td>None</td>
</tr>
<tr>
<td>Lutetium-177</td>
<td>0.50</td>
<td>0.14</td>
<td>0.35</td>
<td>6.7</td>
<td>0.208 (11%)</td>
</tr>
<tr>
<td>Rhenium-186</td>
<td>1.08</td>
<td>0.33</td>
<td>1.05</td>
<td>3.7</td>
<td>0.137 (9%)</td>
</tr>
<tr>
<td>Samarium-153</td>
<td>0.81</td>
<td>0.22</td>
<td>0.55</td>
<td>1.9</td>
<td>0.103 (29%)</td>
</tr>
<tr>
<td>Holmium-166</td>
<td>1.84</td>
<td>0.67</td>
<td>3.3</td>
<td>1.1</td>
<td>0.081 (6%)</td>
</tr>
<tr>
<td>Rhenium-188</td>
<td>2.12</td>
<td>0.64</td>
<td>3.8</td>
<td>0.71</td>
<td>0.155 (10%)</td>
</tr>
</tbody>
</table>

\(^1\)Arranged in order of decreasing half-life
\(^2\)Conversion electrons with discrete energies (and range).
Examples for Beta Decay

- Complicated decay schemes and the emission of other particles such as gamma rays.
- Pure beta emitters: $^3$H, $^{14}$C, $^{32}$P and $^{90}$Sr.

**Figure 4.7.** Iodine-131 transformation (decay) scheme.
Differentiation between malignancy and benign changes with SPECT/CT in a patient with thyroid cancer who underwent 131I whole-body imaging to assess for residual recurrent disease. Anterior (a) and posterior (b) 131I scans show focal activity in the right suprarenal region.

Coronal (c) and axial (d) SPECT/CT images show that the uptake is located in the renal collecting system (arrow), consistent with physiologic urinary activity, and not recurrence of disease.

www.lancastergeneralhealth.org/LGH/ECommerceSite/
Serial post-I 131 therapy scans (anterior whole body views). (A) Age 10, 3.01 GBq; (B) age 11, 2.95 GBq; (C) age 13, 4.3 GBq; (D) age 14, 5.1 GBq; (E) age 15, 9.9 GBq, 7 months before conception. Neck disease present at age 13 (C) was treated surgically. The final study (E) showed the presence of radioiodine avid bilateral pulmonary metastases (<5 mm maximum diameter on computed tomography) and very small, low-grade lower neck disease. The focal uptake in the left upper abdomen is colonic and is likely physiological in nature.

225Ac-PSMA-617 for PSMA-Targeted α-Radiation Therapy of Metastatic Castration-Resistant Prostate Cancer

Clemens Kratochwil1, Frank Bruchertseifer2, Frederik L. Giesel1, Mirjam Weis2, Frederik A. Verburg3, Felix Mottaghy3, Klaus Kopka4, Christos Apostolidis2, Uwe Haberkorn1, and Alfred Morgenstern2

1Department of Nuclear Medicine, University Hospital Heidelberg, Heidelberg, Germany; 2European Commission, Joint Research Centre, Institute for Transuranium Elements, Karlsruhe, Germany; 3Department of Nuclear Medicine, RWTH University Hospital Aachen, Aachen, Germany; and 4Division of Radiopharmaceutical Chemistry, German Cancer Research Center, Heidelberg, Germany

![Image of PET/CT scans]

**FIGURE 1.** 68Ga-PSMA-11 PET/CT scans of patient A. Pretherapeutic tumor spread (A), restaging 2 mo after third cycle of 225Ac-PSMA-617 (B), and restaging 2 mo after one additional consolidation therapy (C).
Estimated crosstalk of Th227/Ra223 imaging.

VOI quantification. Left: Th-227; Right: Ra-223.

H&E staining of VX2 tumor and surrounding liver section (A) and corresponding α-Camera image (C) 3 hours post-injection of $^{225}$Ac-DOTAGA-TDA. (B) Merged image showing segmented high-intensity regions overlayed on H&E stained slide. Segmented regions do not exactly match the tumor boundaries due to differences in tissue processing and because the comparison is made across two different 12-μm-thick sections.

Data presented in this slide is kindly provided by our collaborators, Dr. Eric Frey and Dr. Yong Du at Johns Hopkins Medical School
Atomic Emissions
Energy Release of Orbital Electron Capture

For Positron Decay to be possible, we need

\[ Q = M_p - M_d - M_e - M_{e^+} > 0, \]

so

\[ M_p > M_d + M_e + M_{e^+} = M_d + 2M_e \]

\( M_p \) and \( M_d \) are the atomic masses of the parent and daughter atoms.

For Electron Capture to occur,

\[ Q = M_p - M_d - \phi > 0 \]

so that

\[ M_p > M_d + \phi \]

where \( \phi \) is the binding energy of the orbital electron.
Internal Conversion

Begins with an excited nuclear state

De-excite through the emission of a gamma ray

\[ E_{\beta^-} = E_{ex} - E_b \]

The excitation energy is transferred directly to an orbital electron, causing it to be ejected from the atom

Conversion electron with an energy

IC Coefficient (or Branching Ratio) = \( \frac{N_{\gamma}}{N_e} \)
Auger Electrons and Characteristic X-rays

- The excitation energy of the atom may be transferred to one of the outer electrons, causing it to be ejected from the atom.
- Auger electrons are roughly the analogue of internal conversion electrons when the excitation energy originates in the atom rather than in the nucleus.

Figure 3.7 (A) The usual emission of a K characteristic X-ray, $h\nu$, energy equal to $E_K - E_L$, the difference in binding energy for the two orbital electrons, K and L. (B) $h\nu$ has been absorbed and a monoenergetic Auger electron is emitted, in the example shown, from the M shell, the energy of which is $E_K - E_L - E_M$. (C) In its final state the atom has vacancies in the L and M orbitals.

$$E_{\text{a.e.}} = (E_K - E_{L_1}) - E_{L_{23}}$$
Examples of Radioactivity from Medical Radioisotopes

2.2 Gamma Transitions and Internal Conversion Coefficients

<table>
<thead>
<tr>
<th>Energy keV</th>
<th>$P_{\text{Yee}} \times 100$</th>
<th>Multipolarity</th>
<th>$\alpha_K$</th>
<th>$\alpha_L$</th>
<th>$\alpha_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{10}(\text{Te})$</td>
<td>158.06 (5)</td>
<td>99.22 (30)</td>
<td>M1 + 1/2 E2</td>
<td>0.1684 (16)</td>
<td>0.02160 (22)</td>
</tr>
<tr>
<td>$\gamma_{13}(\text{Te})$</td>
<td>174.19 (3)</td>
<td>0.00999 (30)</td>
<td>M1 + 50% E2</td>
<td>0.150 (32)</td>
<td>0.020 (12)</td>
</tr>
<tr>
<td>$\gamma_{14}(\text{Te})$</td>
<td>192.61 (8)</td>
<td>0.021 (6)</td>
<td>M1 + 50% E2</td>
<td>0.138 (24)</td>
<td>0.024 (10)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>192.18 (9)</td>
<td>0.0227 (10)</td>
<td>M1 + 50% E2</td>
<td>0.118 (20)</td>
<td>0.020 (8)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>197.22 (11)</td>
<td>0.00837 (19)</td>
<td>M1 + 50% E2</td>
<td>0.139 (18)</td>
<td>0.018 (7)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>198.25 (12)</td>
<td>0.0040 (8)</td>
<td>M1 + 50% E2</td>
<td>0.107 (17)</td>
<td>0.018 (7)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>206.79 (10)</td>
<td>0.038 (7)</td>
<td>M1 + 50% E2</td>
<td>0.094 (14)</td>
<td>0.010 (5)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>207.32 (13)</td>
<td>0.00125 (30)</td>
<td>M1 + 50% E2</td>
<td>0.093 (14)</td>
<td>0.010 (5)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>247.36 (8)</td>
<td>0.073 (25)</td>
<td>M1 + 50% E2</td>
<td>0.054 (5)</td>
<td>0.0084 (21)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>257.52 (9)</td>
<td>0.0017 (2)</td>
<td>M1 + 50% E2</td>
<td>0.0528 (26)</td>
<td>0.0018 (2)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>278.26 (8)</td>
<td>0.022 (4)</td>
<td>M1 + 50% E2</td>
<td>0.0387 (22)</td>
<td>0.0052 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>281.73 (7)</td>
<td>0.025 (2)</td>
<td>M1 + 13.2% E2</td>
<td>0.0362 (20)</td>
<td>0.0048 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>295.17 (21)</td>
<td>0.00158 (2)</td>
<td>E2</td>
<td>0.0227 (7)</td>
<td>0.00576 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>329.38 (18)</td>
<td>0.0026 (6)</td>
<td>M1 + 50% E2</td>
<td>0.0387 (22)</td>
<td>0.0052 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>330.77 (11)</td>
<td>0.01197 (34)</td>
<td>M1 + 50% E2</td>
<td>0.0387 (22)</td>
<td>0.0052 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>343.73 (8)</td>
<td>0.0044 (3)</td>
<td>M1 + 50% E2</td>
<td>0.0387 (22)</td>
<td>0.0052 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>363.63 (7)</td>
<td>0.1297 (9)</td>
<td>M1 + 50% E2</td>
<td>0.0387 (22)</td>
<td>0.0052 (11)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>403.84 (14)</td>
<td>0.00298 (23)</td>
<td>E2</td>
<td>0.0227 (7)</td>
<td>0.00576 (11)</td>
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<td>$\gamma_{16}(\text{Te})$</td>
<td>437.53 (3)</td>
<td>0.00607 (7)</td>
<td>E2</td>
<td>0.0227 (7)</td>
<td>0.00576 (11)</td>
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<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>456.79 (15)</td>
<td>0.01412 (22)</td>
<td>M1 + 50% E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>528.06 (7)</td>
<td>1.28 (12)</td>
<td>M1 + 50% E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
</tr>
<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>539.54 (9)</td>
<td>0.309 (33)</td>
<td>M1 + 50% E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>556.94 (16)</td>
<td>0.0029 (3)</td>
<td>E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>582.84 (12)</td>
<td>0.00117 (7)</td>
<td>E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<tr>
<td>$\gamma_{16}(\text{Te})$</td>
<td>586.18 (20)</td>
<td>0.00128 (6)</td>
<td>E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<td>$\gamma_{16}(\text{Te})$</td>
<td>599.89 (16)</td>
<td>0.00269 (17)</td>
<td>E2</td>
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<td>0.00163 (27)</td>
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<td>$\gamma_{16}(\text{Te})$</td>
<td>610.27 (23)</td>
<td>0.0011 (3)</td>
<td>E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
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<td>$\gamma_{16}(\text{Te})$</td>
<td>624.64 (7)</td>
<td>0.0012 (3)</td>
<td>E2</td>
<td>0.0208 (17)</td>
<td>0.00163 (27)</td>
</tr>
</tbody>
</table>

3 Atomic Data

3.1 Te

| $\omega_K$ | 0.875 (4) |
| $\omega_L$ | 0.8682 (35) |
| $\pi_{KL}$ | 0.917 (4) |
3.1.1 X Radiations

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Relative Probability</th>
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<tr>
<td>$X_{K}$</td>
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<tr>
<td>$K_{\alpha_2}$</td>
<td>27.202</td>
</tr>
<tr>
<td>$K_{\alpha_1}$</td>
<td>27.4726</td>
</tr>
<tr>
<td>$K_{\beta_3}$</td>
<td>30.9416</td>
</tr>
<tr>
<td>$K_{\beta_1}$</td>
<td>30.996</td>
</tr>
<tr>
<td>$K_{\beta_2}$</td>
<td>31.236</td>
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<tr>
<td>$K_{\beta_0}$</td>
<td>31.241</td>
</tr>
<tr>
<td>$K_{\delta_2}$</td>
<td>31.7088</td>
</tr>
<tr>
<td>$K_{\delta_1}$</td>
<td>31.774</td>
</tr>
<tr>
<td>$K_{\delta_0}$</td>
<td>31.812</td>
</tr>
<tr>
<td>$X_{L}$</td>
<td></td>
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<tr>
<td>$L_{\gamma}$</td>
<td>3.336</td>
</tr>
<tr>
<td>$L_{\alpha}$</td>
<td>3.76 - 3.77</td>
</tr>
<tr>
<td>$L_{\eta}$</td>
<td>3.996</td>
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<tr>
<td>$L_{\beta}$</td>
<td>4.62 - 4.37</td>
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<td>$L_{\gamma}$</td>
<td>4.44 - 4.82</td>
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3.1.2 Auger Electrons

<table>
<thead>
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<th>Energy (keV)</th>
<th>Relative Probability</th>
</tr>
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<tbody>
<tr>
<td>Auger $K$</td>
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</tr>
<tr>
<td>KLL</td>
<td>21.804 - 22.989</td>
</tr>
<tr>
<td>KLLX</td>
<td>25.814 - 27.470</td>
</tr>
<tr>
<td>KXY</td>
<td>29.80 - 31.81</td>
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<tr>
<td>Auger $L$</td>
<td>2.3 - 4.8</td>
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4 Electron Emissions

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Electrons per 100 disint.</th>
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</thead>
<tbody>
<tr>
<td>$\epsilon_{\alpha_2}$ (Te)</td>
<td>2.3 - 4.8</td>
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<tr>
<td>$\epsilon_{\alpha_1}$ (Te)</td>
<td>21.804 - 22.989</td>
</tr>
<tr>
<td>$\epsilon_{\alpha_0}$ (Te)</td>
<td>25.814 - 27.470</td>
</tr>
<tr>
<td>$\epsilon_{\alpha_0}$ (KXY)</td>
<td>29.80 - 31.81</td>
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<tr>
<td>$\epsilon_{\alpha_2}$ (Te)</td>
<td>127.176 (5)</td>
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<td>$\epsilon_{\alpha_0}$ (Te)</td>
<td>154.651 - 154.669</td>
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5 Photon Emissions

5.1 X-Ray Emissions

<table>
<thead>
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<th>Energy (keV)</th>
<th>Photons per 100 disint.</th>
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<tbody>
<tr>
<td>XL (Te)</td>
<td>3.336 - 4.82</td>
</tr>
<tr>
<td>XK$\alpha_2$ (Te)</td>
<td>27.292</td>
</tr>
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<td>XK$\beta_3$ (Te)</td>
<td>30.9446</td>
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<td>30.996</td>
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<tr>
<td>XK$\beta_0$ (Te)</td>
<td>31.241</td>
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<td>31.774</td>
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<td>XK$\delta_0$ (Te)</td>
<td>31.812</td>
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5.2 Gamma Emissions

<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Photons per 100 disint.</th>
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<tbody>
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<td>$\gamma_{2,6}(\text{Te})$</td>
<td>158.97 (5)</td>
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<td>$\gamma_{3,10}(\text{Te})$</td>
<td>174.2 (3)</td>
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### Table de Radionucléides

<table>
<thead>
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<th>Energy (keV)</th>
<th>Photons per 100 disint.</th>
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<tr>
<td>$\gamma_{4.5}$ (Te)</td>
<td>182.01 (6)</td>
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<td>$\gamma_{4.0}$ (Te)</td>
<td>192.17 (7)</td>
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<tr>
<td>$\gamma_{0.7}$ (Te)</td>
<td>197.22</td>
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<tr>
<td>$\gamma_{0.6}$ (Te)</td>
<td>198.23</td>
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<tr>
<td>$\gamma_{0.5}$ (Te)</td>
<td>200.79</td>
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<td>207.8</td>
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<td>$\gamma_{0.5}$ (Te)</td>
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<td>$\gamma_{0.3}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>629.26 (22)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
<td>909.12 (12)</td>
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<td>$\gamma_{0.0}$ (Te)</td>
<td>1036.63 (17)</td>
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<tr>
<td>$\gamma_{0.0}$ (Te)</td>
<td>1068.12 (15)</td>
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### Main Production Modes

- Sb – $121(\alpha,2n)I – 123$
  - Possible impurities: $I – 121$, $I – 124$, $I – 125$, $I – 126$
- I – $127(p,5\alpha)Xe – 123$
- Te – $124(p,2n)I – 123$
  - Possible impurities: $I – 124$, $I – 125$, $I – 126$
- Te – $123(p,n)I – 123$
  - Possible impurities: $I – 121$, $I – 124$, $I – 125$, $I – 126$
- Te – $122(\alpha,3n)Xe – 123$
  - Possible impurities: $I – 125$
- Te – $121(He – 3,3n)Xe – 123$
  - Possible impurities: $I – 125$
- Te – $121(He – 3,4n)Xe – 123$
  - Possible impurities: $I – 125$
- Te – $122(\alpha,ln)I – 123$
Alpha Decay
Alpha Emission

- An alpha particle is a highly energetic helium nucleus consisting of two neutrons and 2 protons.
- It is normally emitted from isotopes when the neutron-to-proton ratio is too low – through the so-called alpha decay.
- Atomic number and atomic mass number are conserved in alpha decays.
Alpha Emission

- In heavy elements, a potential barrier of ~25MeV has to be overcome for an alpha particle to escape from the potential well.
- It would require a minimum kinetic energy of ~3.8MeV for the alpha particle to “tunneling through” the potential well is

---

**Fig. 4.1.** Potential inside and in the vicinity of a nucleus.
Energy Spectra of Alpha Particles

Alpha decays are sometimes accompanied by the excited daughter products which complicates the resultant alpha particle spectra.

The kinetic energy of alpha particles generated is given by

\[ E_\alpha = Q \cdot (A - 4) / A \]
Energy Release in Alpha Emission

- The required kinetic energy has to come from the decrease in mass following the decay process.
- The relationship between mass and energy associated with an alpha emission is given as

\[ M_p = M_d + M_\alpha + 2M_e + Q, \]  \hspace{1cm} (4.1)

where \( M_p, M_d, M_\alpha, \) and \( M_e \) are respectively equal to the masses of the parent, the daughter, the emitted alpha particle, and the two orbital electrons that are lost during the transition to the lower atomic numbered daughter, while \( Q \) is the total energy release associated with the radioactive transformation.
Energy Loss Mechanisms

**FIGURE 5.1.** (Top) Alpha-particle autoradiograph of rat bone after inhalation of $^{241}$Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The $^{14}$C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Witten-dorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)
Single Collision Energy-Loss Spectrum

![Graph showing single collision energy-loss spectra for 50-eV, 150-eV electrons, and 1-MeV protons in liquid water.](image)

**Fig. 5.3** Single-collision energy-loss spectra for 50-eV and 150-eV electrons and 1-MeV protons in liquid water. (Courtesy Oak Ridge National Laboratory, operated by Martin Marietta Energy Systems, Inc., for the Department of Energy.)
Targeted Therapy – Basic Principles

Alpha emitting isotopes for therapeutic applications in nuclear medicine

<table>
<thead>
<tr>
<th>Radionuclide</th>
<th>Half-Life</th>
<th>Max. Particle Energy</th>
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</thead>
<tbody>
<tr>
<td>At-211</td>
<td>7.2 hrs</td>
<td>6.0 MeV</td>
</tr>
<tr>
<td>Bi-213</td>
<td>46 min</td>
<td>6.0 MeV</td>
</tr>
<tr>
<td>Ra-223</td>
<td>11.4 days</td>
<td>5.8 MeV</td>
</tr>
<tr>
<td>Ac-225</td>
<td>10.0 days</td>
<td>5.9 MeV</td>
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</table>

# Decay of Ra-223

<table>
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<tr>
<th>Decay Chain</th>
<th>Branching Ratio</th>
<th>Half-life</th>
<th>Alpha and Recoil Nuclei (MeV)</th>
<th>Beta and Auger Electrons (MeV)</th>
<th>Gamma and X-Rays (MeV)</th>
</tr>
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<tbody>
<tr>
<td>Ra-223</td>
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<td>11.43 days</td>
<td>5.77</td>
<td>0.078</td>
<td>0.141</td>
</tr>
<tr>
<td>Rn-219</td>
<td>100%</td>
<td>3.96 s</td>
<td>6.88</td>
<td>0.007</td>
<td>0.059</td>
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<tr>
<td>Po-215</td>
<td>100%</td>
<td>1.78 ms</td>
<td>7.49</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Pb-211</td>
<td>100%</td>
<td>36.10 min</td>
<td>-</td>
<td>0.454</td>
<td>0.064</td>
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<tr>
<td>Bi-211</td>
<td>100%</td>
<td>2.14 min</td>
<td>6.66</td>
<td>0.010</td>
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<tr>
<td>Tl-207</td>
<td>99.7%</td>
<td>4.77 min</td>
<td>-</td>
<td>0.494</td>
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<tr>
<td>Po-211</td>
<td>0.3%</td>
<td>0.52 s</td>
<td>7.61</td>
<td>0.000</td>
<td>0.008</td>
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</table>

Biodistribution of Ra-223

Original article

Quantitative imaging of $^{223}$Ra-chloride (Alpharadin) for targeted alpha-emitting radionuclide therapy of bone metastases

Cecilia Hindorf, Sarah Chittenden, Anne-Kirsti Aksnes, Chris Parker, and Glenn D. Flux

Counts

Energy (keV)

4h Anterior

48h Posterior

144h

Dosimetry of bone metastases in targeted radionuclide therapy with alpha-emitting $^{223}\text{Ra}$-dichloride

Authors

Massimiliano Pacilio, Guido Ventroni, Giuseppe De Vincentis, Bartolomeo Cassano, Rosanna Pellegrini, Elisabetta Di Castro, Viviana Frantellizzi, Giulia Anna Follacchio, Tatiana Garkavya, Leda Lorenzon, Pasquale Ialongo, Roberto Pani, Lucio Mango
Therapeutic Radiometals Beyond $^{177}$Lu and $^{90}$Y: Production and Application of Promising $\alpha$-Particle, $\beta^-$-Particle, and Auger Electron Emitters

Cristina Müller$^{1,2}$, Nicholas P. van der Meulen$^{1,3}$, Martina Benešová$^{1,2}$, and Roger Schibli$^{1,2}$

$^1$Center for Radiopharmaceutical Sciences ETH-PSI-USZ, Paul Scherrer Institut, Villigen-PSA, Switzerland; $^2$Department of Chemistry and Applied Biosciences, ETH Zurich, Zurich, Switzerland; and $^3$Laboratory of Radiochemistry, Paul Scherrer Institut, Villigen-PSA, Switzerland

**FIGURE 1.** (A) Decay of $^{149}$Tb to stable $^{149}$Sm, $^{145}$Nd, and $^{141}$Pr. EC = electron capture. (B) Maximum-intensity projection (MIP) of PET/CT image of AR42J tumor-bearing mouse 2 h after injection of $^{149}$Tb-DOTANOC (7 MBq). Bl = urinary bladder; Ki = kidney; Tu = tumor. (Adapted with permission of (45).)
Therapeutic Radiometals Beyond $^{177}$Lu and $^{90}$Y: Production and Application of Promising $\alpha$-Particle, $\beta^-$-Particle, and Auger Electron Emitters

Cristina Müller$^{1,2}$, Nicholas P. van der Meulen$^{1,3}$, Martina Benešová$^{1,2}$, and Roger Schibli$^{1,2}$

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**FIGURE 4.** (A) $^{68}$Ga-PSMA-11 PET/CT scan of patient with pretherapeutic tumor spread. PSA = prostate-specific antigen. (B) Restaging 2 mo after third cycle of $^{225}$Ac-PSMA-617 (9–10 MBq). (C) Restaging 2 mo after single additional consolidation therapy (6 MBq). $^{177}$Lu-PSMA-617 was contraindicated because of diffuse red marrow infiltration. (Reproduced from (68).)

**FIGURE 3.** Decay of $^{225}$Ac to $^{213}$Bi and stable $^{209}$Bi.
X-Rays Sources
What Can We Learn from X-rays?

“The radiograph of Bera Rontgen’s hand”,
taken 22 Dec. 1895

Wilhelm Conrad Röntgen,
The Nobel Prize in Physics 1901.

A brain image from the first prototype clinical CT imager, 1972.

Godfrey N. Hounsfield, The Nobel Prize in Physiology or Medicine 1979
X-ray Imaging Examples

Double mandibular fracture with strong displacement to the left.

Solitary humeral bone cyst known as "fallen leaf sign"
Sources of Electromagnetic Radiation
– X-ray Emission Through Bremsstrahlung

- When fast electrons decelerated in the Coulomb field of a nucleus, part of its energy is converted into electromagnetic radiation in the form of Bremsstrahlung (a German word for “braking radiation”).

Covered in lecture
X-ray Generation – Bremmstrahlung

- Target nucleus positive charge \((Z \cdot p^+)\) attracts incident \(e^-\)
- Deceleration of an incident \(e^-\) occurs in the proximity of the target atom nucleus
- \(E\) lost by \(e^-\) is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the \(e^-\) determines the amount of \(E\) loss
  - The Coulomb force of attraction varies strongly with distance \((\propto 1/r^2)\); ↓ distance → ↑ deceleration and \(E\) loss → ↑ photon \(E\)
  - Direct impact on the nucleus determines the maximum x-ray \(E\) \((E_{\text{max}})\)
Interestingly, this process creates a relatively uniform spectrum.

\[ \text{Intensity} = n h \nu \]

**Bremmstrahlung X-ray Energy Spectrum**

**Photon energy spectrum**

\( \varepsilon_0 \)
The Unfiltered Bremsstrahlung Spectrum
Thick Target X-ray Formation

We can model target as a series of thin targets. Electrons successively loses energy as they moves deeper into the target.

Each layer produces a flat energy spectrum with decreasing peak energy level.
X-ray Generation – Characteristic X-rays

Electron binding energy

Hydrogen $Z = 1$

Tungsten $Z = 74$

Covered in lecture
X-ray Generation – Characteristic X-rays

- $e^-$ of the target atom have a binding energy (BE) that depends on atomic Z (rem: $BE_K \propto Z^2$) and the shell ($BE_K > BE_L > BE_M > \ldots$)

- When $e^-(KE)$ incident on the target exceeds the target atom $e^-(BE)$, it’s energetically possible for a collisional interaction to eject the bound electron and ionize the atom.

- What would happen then?
X-ray Generation – Characteristic X-rays

- Within each shell (other than K) there are discrete E orbitals ($\ell = 0, 1, ... , n-1$) → characteristic x-ray fine E splitting
- Characteristic x-rays other than those generated through K-shell transitions are unimportant

### TABLE 5-2. K-SHELL CHARACTERISTIC X-RAY ENERGIES (keV) OF COMMON X-RAY TUBE TARGET MATERIALS

<table>
<thead>
<tr>
<th>Shell Transition</th>
<th>Tungsten</th>
<th>Molybdenum</th>
<th>Rhodium</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{\alpha 1}$</td>
<td>59.32</td>
<td>17.48</td>
<td>20.22</td>
</tr>
<tr>
<td>$K_{\alpha 2}$</td>
<td>57.98</td>
<td>17.37</td>
<td>20.07</td>
</tr>
<tr>
<td>$K_{\beta 1}$</td>
<td>67.24</td>
<td>19.61</td>
<td>22.72</td>
</tr>
</tbody>
</table>

*Note: Only prominent transitions are listed.*
Physical Designs Considerations

Typical micro-focus X-ray source

Rotating anode X-ray source

(Left) Liquid metal jet anode
http://www.excillum.com/technology/metal-jet-technology.html
X-ray Generation – X-ray Tube

Motor, Why?

Rotating target

Electron beam? How are electrons generated?

High voltage

Stator

Anode assembly

Ground

Filament circuit

Cathode assembly

X-rays

Figure 5.3
An x-ray tube.

Figure 5.4
Schematic diagram of an x-ray tube.
**X-ray Generation – Characteristic X-rays**

**Figure 5.5**
Relative intensity of x-ray photons. (Adapted from Webster, 1998. This material is used by permission of John Wiley & Sons, Inc.)
X-ray Computed Tomography (CT)

0.2 % attenuation change detectable in CT Images !!
Example of Potential Problems Caused by the Imperfection of X-ray Sources – Beam Hardening

![Beam hardening with cupping]

![Normal CT scan]

- Bremstrahlung (x-rays within anode)
- Leaving anode
- Leaving tube
- Characteristic radiation
- After filter
- Leaving body

Relative intensity vs. Photon energy, keV

0 20 40 60 80 100 120

Chapter 3: Radioactivity

NP4E 435, Radiological Imaging, Fall 2021
Example of Potential Problems Caused by the Imperfection of X-ray Sources – Beam Hardening

Beam hardening artifact occurs because the algorithm that reconstructs the image fails when faced with an interface between a high-density structure such as a metallic prosthesis or thick bone and surrounding soft tissue. The 'spray' or 'streak' artifact results. It can be seen in the posterior fossa in most CT brains.
Sources of Electromagnetic Radiation
– Bremsstrahlung

- The bremsstrahlung spectrum can be altered by filtration.
- This is the underlying process for standard X-ray tubes.
Synchrotron Radiation
Sources of Electromagnetic Radiation
– Synchrotron Radiation

- Emitted when charged particles are accelerated (or decelerated) in a curved path or orbit.
- From visible light (a few eV) to X-ray energies (~10^4 eV).
- Very intense !!!
Synchrotron Radiation

Bending magnet

Undulator magnet


Covered in lecture
X-ray Generation – Bremmstrahlung

- Target nucleus positive charge \((Z \cdot p^+)\) attracts incident \(e^-\)
- Deceleration of an incident \(e^-\) occurs in the proximity of the target atom nucleus
- \(E\) lost by \(e^-\) is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the \(e^-\) determines the amount of \(E\) loss
  - The Coulomb force of attraction varies strongly with distance \((\propto 1/r^2)\); ↓ distance → ↑ deceleration and \(E\) loss → ↑ photon \(E\)
  - Direct impact on the nucleus determines the maximum x-ray \(E\) \((E_{\text{max}})\)
Sources of Electromagnetic Radiation
– Selection of X-rays of a Single Energy

A typical monochromator setup

Covered in lecture
Sources of Electromagnetic Radiation
– Synchrotron Radiation

European Synchrotron Radiation Facility

Circumference
844 m

“booster” synchrotron
6 GeV

linear accelerator
200 MeV

storage ring

ID 18

Advanced Photon Source (APS)

From http://www-hasylab.desy.de/science/groups/schneider_group/misc/RR-Lecture-1.pdf
• Stunning imaging resolution at the order down to a few tens of nm

• Reveal interesting physical properties of the object, X-ray fluorescence, diffraction–enhanced imaging
Interactions of Ionizing Radiation with Matter

Reading Material:
Interactions of X and Gamma Rays
X-ray and Gamma Ray Interactions

![Graph showing energy dependence of various gamma-ray interaction processes in sodium iodide.](image)

**Figure 2.18** Energy dependence of the various gamma-ray interaction processes in sodium iodide. (From *The Atomic Nucleus* by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

Photoelectric Effect

Photoelectric interaction is with the atom in a whole and can not take place with free electrons.

Photoelectric effect leaves a vacancy in one of the electron shells, which leaves the atom at an excited state.

\[ E_{e^-} = h\nu - E_b \]

- \( h \) is the Planck's constant
- \( \nu \) is the frequency of the photon

Covered in lecture
Photoelectric Effect Cross Section

Probability of photoelectric absorption per atom is

\[ \tau \propto \begin{cases} 
\frac{Z^4}{(hv)^{3.5}} & \text{low energy} \\
\frac{Z^5}{(hv)^{3.5}} & \text{high energy}
\end{cases} \]

- The interaction cross section depends strongly on Z.
- Photoelectric effect is favored at lower photon energies.

Photoelectric Effect (2) – Absorption Edges

Figure 2: Total and partial atomic photoeffect of Ag.
Photoelectric Effect (2) – Absorption Edges

Contrast enhanced CT image of the lung
www.healthcare.siemens.com
**Functional CT imaging techniques for the assessment of angiogenesis in lung cancer**

Thomas Henzler¹, Jingyun Shi², Hashim Jafarov³, Stefan O. Schoenberg¹, Christian Manegold³, Christian Fink¹, Gerald Schmid-Blindert¹

¹Institute of Clinical Radiology and Nuclear Medicine, University Medical Center Mannheim, Medical Faculty Mannheim-Heidelberg University, Germany; ²Department of Radiology, Shanghai Pulmonary Hospital, Tongji University School of Medicine, China; ³Interdisciplinary Thoracic Oncology, University Medical Center Mannheim, Medical Faculty Mannheim - Heidelberg University, Germany

Corresponding to: Dr. Jingyun Shi, Department of Radiology, Shanghai Pulmonary Hospital, Tongji University School of Medicine, 507 Zhang Min Road, Shanghai, 200433, China. Email: shijingyun89179@126.com.

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Figure 1 Contrast enhanced calculated "virtual 120 kV" dual energy CT image of a 56-year-old male patient with an adenocarcinoma of the left upper lobe (A). Fused (B) and isolated selective iodine perfusion maps (D) revealed hyper-perfusion of the peripheral tumor margins with less iodine uptake in the central areas. The corresponding virtual non-contrast CT image (C) shows hypo-attenuation of the hypo-perfused central tumor area indicating less tumor vitality.
What is Compton Scattering?

The differential scattering cross section \((d\sigma)\) – the probability of a photon scattered into a unit solid angle around the scattering angle \(\theta\), when passing normally through a layer of material containing one electron per unit area.

**Fig. 5.15.** Compton scattering diagram to illustrate differential scattering cross section. \(S\) is a sphere of unit radius whose center is the scattering electron.
Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

\[ h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))} \]

where \( h = 6.757704 \text{ meter} \cdot \text{kilogram/second} \) (Planck’s constant), \( \nu \) is the photon frequency, \( m_0c^2 \) is the mass of the electron, and \( \theta \) is the scattering angle.

and the photon transfers part of its energy to the electron (assumed to be at rest), which is known as a recoil electron. Its energy is simply

\[ E_{\text{recoil}} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))} \]

The one-to-one relationship between scattering angle and energy loss!!
Basic Kinematics in Compton Scattering

The energy transfer in Compton scattering may be derived as the following:

Assuming that the electron binding energy is small compared with the energy of the incident photon – elastic scattering.

Write out the conservation of energy and momentum:

Conservation of energy

\[ h\nu + mc^2 = h\nu' + E' \]

Conservation of momentum

\[ \frac{h\nu}{c} = \frac{h\nu'}{c} \cos \theta + P' \cos \varphi \]
\[ \frac{h\nu'}{c} \sin \theta = P' \sin \varphi \]
Compton Scattering with Non-stationary Electrons – Doppler Broadening

\[
E_{\text{recoil}}(\theta) = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos \theta)}
\]

\[
h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))} \pm \sigma(h\nu')
\]
Energy Transfer in Compton Scattering

If we assume that the electron is free and at rest, the scattered gamma ray has an energy

\[ h\nu' = \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos \theta)}, \]

and the photon transfers part of its energy to the electron (assumed to be at rest before the collision), which is known as the recoil electron. Its energy is simply

\[ E_{recoil} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))}. \]

In the simplified elastic scattering case, there is an one-to-one relationship between scattering angle and energy loss!!

Derivation of the Relationship Between Scattering Angle and Energy Loss

The relation between energy the scattering angle and energy transfer are derived based on the conservation of energy and momentum:

\[
\vec{P}_{hv} + \vec{P}_e = \vec{P}_{hv'} + \vec{P}_e'
\]

\[
E_{hv} + E_e = E_{hv'} + E_{e'}
\]

Are those terms truly zero?
Compton Scattering with Non-stationary Electrons – Doppler Broadening

- It is so far assumed that (a) the electron is free and stationary and (b) the incident photon is unpolarized.

- When an incident photon is reflected by a non-stationary electron, for example an bond electron, an extra uncertainty is added to the energy of the scattered photon. This extra uncertainty is called Doppler broadening.

\[
\frac{h\nu'}{\frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos(\theta))}} = \pm \sigma(h\nu')
\]

The one-to-one relationship between scattering angle and energy loss holds only when incident photon energy is far greater than the bonding energy of the electron...
Compton Scattering with Non-stationary Electrons – Doppler Broadening

Comparison of the energy spectra for the photons scattered by C and Cu samples. $E_{hv}=40$keV, $\theta=90$ degrees

The Doppler broadening is stronger in Cu than in C because of the Cu electrons have greater bonding energy.
Energy Transfer in Compton Scattering

The maximum energy carried by the recoil electron is obtained by setting $\theta$ to $180^\circ$,

$$E_{\text{max}} = \frac{2hv}{2 + mc^2/hv}$$

The maximum energy transfer is exemplified by the Compton edge in measured gamma ray energy spectra.

Figure from Atoms, Radiation, and Radiation Protection, James E. Turner, p180.

**FIGURE 8.5.** Relative number of Compton recoil electrons as a function of their energy for 1-MeV photons.
Angular Distribution of the Scattered Gamma Rays

Klein-Nishina formula:

\[
\frac{d\sigma}{d\Omega} (\theta) = Zr^2_e \left( \frac{1}{1 + \alpha(1 - \cos(\theta))} \right)^2 \left[ 1 + \frac{1 + \cos^2(\theta)}{2} \right] \left[ 1 + \frac{1 + \cos^2(\theta)}{(1 + \cos^2(\theta))[1 + \alpha(1 - \cos(\theta))]} \right], \quad \alpha = \frac{hv}{m_0c^2}
\]

Probability of Compton scattering within a unit solid angle around a scattering angle \( \theta \).

Incident photons with higher energy tend to scatter with smaller angle (forward scattering)

Incident photons with lower energy (a few hundred keV) have greater chances of undergoing large angle scattering (back scattering)

Radial distance represents the differential cross section.
Klein-Nishina formula can be used to calculate the expected energy spectrum of recoil electrons as the following:

$$\frac{d\sigma}{dE_{\text{recoil}}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} d\theta$$

and the probability that a recoil electron possesses an energy between $E_{\text{recoil}} - \Delta E/2$ and $E_{\text{recoil}} + \Delta E/2$ is

$$\propto \Delta E \cdot \frac{d\sigma}{dE_{\text{recoil}}}$$
Application of the Klein-Nishina Formula (1)

\[
\frac{d\sigma}{dE_{\text{recoil}}} = \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} \frac{d\theta}{dE_{\text{recoil}}}
\]

The terms on the right hand side of the equation can be derived from the following relationships,

\[
\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left( \frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right) \left( 1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right)
\]

\[
E_{\text{recoil}} = h\nu - h\nu' = h\nu - \frac{h\nu}{1 + \frac{h\nu}{m_0c^2}(1 - \cos \theta)} \quad \Rightarrow \quad \frac{d\theta}{dE_{\text{recoil}}} = -\frac{m_0c^2}{E_{\text{recoil}}^2 \sin \theta}
\]

\[
d\Omega = 2\pi \sin \theta d\theta \quad \Rightarrow \quad \frac{d\Omega}{d\theta} = 2\pi \sin \theta
\]

The expected energy spectrum of recoil electrons can be evaluated numerically using these relationships.
Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – the probability of a photon scattered into a unit solid angle around the scattering angle \( \theta \), when passing normally through a very thin layer of scattering material that contains one electron per unit area.

\[
\frac{d\sigma}{d\Omega} (\theta) = r_e^2 \left( \frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right) \left( 1 + \frac{\alpha^2 (1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right) \left( m^2 sr^{-1} \right)
\]

**Fig. 5.15.** Compton scattering diagram to illustrate differential scattering cross section. \( S \) is a sphere of unit radius whose center is the scattering electron.
Angular Distribution of the Scattered Gamma Rays

Incident photons with higher energies tend to scatter with smaller angles (forward scattering).

Incident photons with lower energy (a few hundred keV) have greater chance of undergoing large angle scattering (back scattering).

The higher the energy carried by an incident gamma ray, the more likely that the gamma ray undergoes forward scattering ...
Total Compton Collision Cross Section

Compton Collision Cross Section is defined as the total cross section per electron by Compton scattering. It can be derived by integrating the differential cross section over the $4\pi$ solid angle.

Since

$$d\Omega = 2\pi \sin \theta d\theta$$

The total scattering cross section is

$$\sigma = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta \, d\theta \, \text{m}^2$$

Note that the Compton collision cross section is given in unit of $\text{m}^2$. 

![Diagram showing the scattering of a Compton collision with incident and scattered photons, angle $\theta$, radius $r$, and solid angle $d\Omega$ calculation.]}
Application of the Klein-Nishina Formula (1)

The energy distribution of the recoil electrons derived using the Klein-Nishina formula is closely related to the energy spectrum measured with “small” detectors.

Figure 10.1 Shape of the Compton continuum for various gamma-ray energies. (From S. M. Shafroth (ed.), Scintillation Spectroscopy of Gamma Radiation. Copyright 1964 by Gordon & Breach, Inc. By permission of the publisher.)
Total Compton Collision Cross Section

Compton Collision Cross Section is defined as the total cross section per electron by Compton scattering. It can be derived by integrating the differential cross section over the $4\pi$ solid angle.

Since

$$d\Omega = 2\pi \sin \theta d\theta$$

The total scattering cross section is

$$\sigma = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta \, d\theta \text{ m}^2$$

Note that the Compton collision cross section is given in unit of m$^2$. 
Angular Distribution of the Scattered Gamma Rays

The differential scattering cross section per electron – the probability of a photon scattered into a unit solid angle around the scattering angle $\theta$, when passing normally through a very thin layer of scattering material that contains one electron per unit area.

\[
\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left(\frac{1}{1 + \alpha(1 - \cos \theta)}\right)^2 \left(\frac{1 + \cos^2 \theta}{2}\right) \left(1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]}\right) \left(m^2 sr^{-1}\right)
\]

Fig. 5.15. Compton scattering diagram to illustrate differential scattering cross section. $S$ is a sphere of unit radius whose center is the scattering electron.
Question 2:

If we know the total Compton scattering cross section

\[ \sigma = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta \ d\theta \ m^2 \]

How do we drive the linear attenuation coefficient of gamma rays through Compton scattering?

The linear attenuation coefficient is the probability of a gamma rays Compton scattered in the material while traveling through a unit distance.
Application of the Klein-Nishina Formula (3)

The differential Compton cross section given by the Klein-Nishina Formula can also be related to another important parameter for gamma ray dosimetry – the linear attenuation coefficient.

Suppose that the total number of atoms per m$^3$ in the absorber is N and the atomic number is Z, the electron density in the absorber is NZ.

Therefore the product

$$\sigma_{linear} = NZ\sigma_{compton} \quad (cm^{-1})$$

and

$$\sigma_{compton} = \int \frac{d\sigma}{d\Omega} d\Omega = \int \frac{d\sigma}{d\Omega} \frac{d\Omega}{d\theta} d\theta = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta d\theta \quad (cm^2)$$

$$\frac{d\sigma}{d\Omega}(\theta) = r_e^2 \left( \frac{1}{1 + \alpha(1 - \cos \theta)} \right)^2 \left( \frac{1 + \cos^2 \theta}{2} \right) \left( 1 + \frac{\alpha^2(1 - \cos \theta)^2}{(1 + \cos^2 \theta)[1 + \alpha(1 - \cos \theta)]} \right) \left( m^2 sr^{-1} \right)$$
Application of the Klein-Nishina Formula (3)

\[ \sigma_{\text{linear}} = NZ \sigma_{\text{compton}} \]

has the unit of $m^{-1}$, is

- The total cross section of all electrons “seen” by a photon while traversing a unit distance in the absorber, or
- The total probability that the photon will interact with the absorber through Compton scattering while traversing a unit distance in the absorber.
Compton Scattering for Imaging?

The CGRO Mission

The Compton Gamma Ray Observatory was the second of NASA's Great Observatories. Compton, at 17 tons, was the heaviest astrophysical payload ever flown at the time of its launch on April 5, 1991 aboard the space shuttle Atlantis. Compton was safely deorbited and re-entered the Earth's atmosphere on June 4, 2000.

Compton had four instruments that covered an unprecedented six decades of the electromagnetic spectrum, from 30 keV to 30 GeV. In order of increasing spectral energy coverage, these instruments were the Burst And Transient Source Experiment (BATSE), the Oriented Scintillation Spectrometer Experiment (OSSE), the Imaging Compton Telescope (COMPTEL), and the Energetic Gamma Ray Experiment Telescope (EGRET). For each of the instruments, an improvement in sensitivity of better than a factor of ten was realized over previous missions.

The Observatory was named in honor of Dr. Arthur Holly Compton, who won the Nobel prize in physics for work on scattering of high-energy photons by electrons - a process which is central to the gamma-ray detection techniques of all four instruments.

CGRO Observation Timelines

If you have a question about CGRO, please contact us via the Feedback form.

This page was last modified on Wednesday, 30-Nov-2005 11:44:27 EST.
Compton Scattering for Imaging?
Principle of Compton Imaging

\[ \cos \theta = 1 + \frac{m_e c^2}{E_0} - \frac{m_e c^2}{E_0 - E_1} \]
Principle of Compton Imaging

- No need for mechanical collimation – much higher detection efficiency.
- Requires the first detector to have a very good energy resolution.

Covered in lecture
Experimental Compton Camera

Developed by the Detector Physics Group, University of Michigan, lead by Neal Clinthorne and Les. Rogers.

Covered in lecture
Detector system

- DAQ card
- 68 pole cable with SCSI II connectors
- Interface
- 50 pole cable with ERNI and SCSI II connectors
- +5V, -5V power supply
- Detector + ASIC

[Diagram of the detector system with labeled components]
Imaging Spectroscopy

[Graphs and data visualizations related to imaging spectroscopy are shown.]

Covered in lecture
Pair Production

• Pair production refers to the creation of an electron-positron pair by an incident gamma ray in the vicinity of a nucleus.

• The minimum energy required is

$$E_{\gamma} \geq 2m_e c^2 + \frac{2m_e^2 c^2}{m_{\text{nucleus}}} \approx 2m_e c^2 = 1.022\text{MeV}$$

• The process is more probable with a heavy nucleus and incident gamma rays with higher energies.

• The positron will soon annihilates with ordinary electrons near by and produces two 511keV gamma rays.
The Relative Importance of the Three Major Type of X and Gamma Ray Interactions

---


NPRE 435, Radiological Imaging, Fall 2021

X-ray and Gamma Ray Interactions
X-ray and Gamma Ray Interactions

Figure 2.18 Energy dependence of the various gamma–ray interaction processes in sodium iodide. (From The Atomic Nucleus by R. D. Evans. Copyright 1955 by the McGraw-Hill Book Company. Used with permission.)

Case Study I – Dual Energy CT

What are the challenges for CT imaging?

• Compton scattering is the predominant interaction for x-ray attenuation.

• Compton scattering depends on Z (or electron density)

• There are many materials of interest that have similar electron density.

If we solely rely on Compton scattering as the contrast mechanism for X-ray imaging, we are measuring the attenuation factor that is a linear function of the density ... This may not be sufficient for many cases ...
# Classification of Photon Interactions

## Table 1: Physical Properties of Several Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Chemical Compo.</th>
<th>Density (g/cm³)</th>
<th>Effective Z</th>
<th>Linear Attenuation Coefficients (cm⁻¹)</th>
<th>( \mu_{30keV} : \mu_{80keV} : \mu_{300keV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30keV</td>
<td>80keV</td>
</tr>
<tr>
<td>Iron</td>
<td>Fe</td>
<td>7.6</td>
<td>26</td>
<td>3.14</td>
<td>59</td>
</tr>
<tr>
<td>Aluminium</td>
<td>Al</td>
<td>2.7</td>
<td>13</td>
<td>0.69</td>
<td>2.35</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>(CH₂)n</td>
<td>~1.2</td>
<td>3.75</td>
<td>0.266</td>
<td>0.059</td>
</tr>
<tr>
<td>Wood</td>
<td>C₂₉H₄₂N</td>
<td>~0.9</td>
<td>4.05</td>
<td>0.195</td>
<td>0.047</td>
</tr>
<tr>
<td>Cocaine</td>
<td>C₁₇H₂₁O₄N</td>
<td>~1.2</td>
<td>4.77</td>
<td>0.256</td>
<td>0.093</td>
</tr>
<tr>
<td>TNT</td>
<td>C₃H₅O₆N₅</td>
<td>1.64</td>
<td>6.14</td>
<td>0.344</td>
<td>0.179</td>
</tr>
<tr>
<td>RDX</td>
<td>C₆H₈O₇N₆</td>
<td>~1.50</td>
<td>6.39</td>
<td>0.319</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Airport X-ray Inspection

Covered in lecture
Case Study I – Dual Energy CT

Organic explosive material is easily visible in Orange, metallic components in Blue and soft inorganic materials in Green

http://www.vidisco.com/node/338
# Classification of Photon Interactions

<table>
<thead>
<tr>
<th>Type of interaction</th>
<th>Absorption</th>
<th>Scattering</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interaction with:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atomic electrons</td>
<td>Photoelectric effect (\sigma_{pe}) (\sim Z^4 (L.E.)) (\sim Z^5 (H.E.))</td>
<td>Rayleigh scattering (\sigma_R \sim Z^2 (L.E.))</td>
</tr>
<tr>
<td>Nucleus</td>
<td>Photonuclear reactions ((\gamma,n),(\gamma,p)), photofission, etc. (\sigma_{ph.n.} \sim Z (h\nu \geq 10\text{MeV}))</td>
<td>Elastic nuclear scattering ((\gamma,\gamma) \sim Z^2)</td>
</tr>
<tr>
<td>Electric field surrounded charged particles</td>
<td>Electron-positron pair production in field of nucleus, (\sigma_{pair} \sim Z^2 (h\nu \geq 1.02\text{MeV}))</td>
<td></td>
</tr>
</tbody>
</table>
Energy Loss Mechanisms for Fast Electrons
Tracks of Beta Particles in Absorbing Medium

- Since beta particles have the same mass as the orbital electrons, they are easily scattered during collision and therefore follow tortuous paths in absorbing medium.
- The electrons are “wondering” more significantly near the end of their tracks.
- Energy-loss interactions are more sparsely distributed at the beginning of the track.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p150
Specific Energy Loss of Fast Electrons

- The specific energy loss of electrons by excitation and ionization is

\[-\left(\frac{dE}{dx}\right)_c = \frac{2\pi e^4 NZ}{m_e v^2} \left[ \ln \frac{m_e v^2 E}{2 I^2 (1 - \beta^2)} - (\ln 2)(2\sqrt{1 - \beta^2} - 1 + \beta^2) \right] \]

\[
+ (1 - \beta^2) + \frac{1}{8} \left(1 - \sqrt{1 - \beta^2}\right)^2
\]

\[
\beta \equiv \frac{v}{c}
\]

No. of atoms per unit volume

Electron velocity

Effective atomic number of the absorbing material
Specific Energy Loss of Fast Electrons

- The linear specific energy loss through Bremsstrahlung is

\[
-\left( \frac{dE}{dx} \right)_r = \frac{NEZ(Z+1)e^4}{137m_e^2c^4} \left( 4 \ln \frac{2E}{m_ec^2} - \frac{4}{3} \right)
\]

more important for higher-Z materials, such as lead and tungsten.
Energy Loss by Bremsstrahlung

- For beta particles to stop completely, the fraction of energy loss by Bremsstrahlung process is approximately given by

\[ f_\beta = 3.5 \times 10^{-4} ZE_m, \]

where

- \( f_\beta \) = the fraction of the incident beta energy converted into photons,
- \( Z \) = atomic number of the absorber,
- \( E_m \) = maximum energy of the beta particle, MeV.
Specific Energy Loss of Fast Electrons

• The total specific energy loss of electrons is

$$\left( \frac{dE}{dx} \right)_{tot} = \left( \frac{dE}{dx} \right)_{\text{coulomb}} + \left( \frac{dE}{dx} \right)_{\text{radiation}}$$

• The ratio of specific energy loss is

$$\frac{(dE/dx)_r}{(dE/dx)_c} \approx \frac{EZ}{700}$$

Bremsstrahlung is the favored process at higher electron energies and for high-Z materials.

Minimum occurs when $v > 0.96c$

The critical energy

MIP (Minimum Ionizing Particle)
Specific Energy Loss of Fast Electrons

- The energy for which the two terms become equal, is called the critical energy.

\[ E_C \approx \frac{700}{Z} \text{(MeV)} \]

<table>
<thead>
<tr>
<th>Material</th>
<th>Radiation Length (g/cm²)</th>
<th>Critical Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂</td>
<td>63</td>
<td>340</td>
</tr>
<tr>
<td>Al</td>
<td>24</td>
<td>47</td>
</tr>
<tr>
<td>Ar</td>
<td>20</td>
<td>35</td>
</tr>
<tr>
<td>Fe</td>
<td>13.8</td>
<td>24</td>
</tr>
<tr>
<td>Pb</td>
<td>6.3</td>
<td>6.9</td>
</tr>
<tr>
<td>H₂O</td>
<td>36</td>
<td>93</td>
</tr>
<tr>
<td>NaI(Tl)</td>
<td>9.5</td>
<td>12.5</td>
</tr>
<tr>
<td>BGO</td>
<td>8.0</td>
<td>10.5</td>
</tr>
</tbody>
</table>

- At keV-MeV energy range, ionization is the dominant mechanism for energy loss.
Range of Fast Electrons
Range of Fast Electrons in Medium

- As a very crude estimate, electron ranges tend to be about 2mm per MeV in low-density materials and 1mm per MeV in materials of moderate density.

- To a fair degree of approximation, the product of the range times the density of the absorber is a constant for different materials for electrons of equal initial energy.

Absorption of Beta Particles

- The number of beta particles emerging from a absorber of a given thickness tends to follow a exponential behavior,

\[ I = I_0 e^{-nt} \]

- Note that there is no fundamental basis for interpreting this exponential behavior, as does in gamma ray attenuation.

Interaction of Positrons

- Positron shares the major mechanism of energy loss with their negative counterparts (electrons).
- However, positrons differ significantly in the annihilation radiation process that results in 0.511MeV gamma rays.
A Useful Website

http://physics.nist.gov/PhysRefData/contents.html

By NIST, Physics Laboratory,
Interaction of Heavy Charged Particles
Energy Loss Mechanisms

- Heavy charged particles loss energy primarily though the ionization and excitation of atoms.
- Heavy charged particles can transfer only a small fraction of its energy in a single collision. Its deflection in collision is almost negligible. Therefore heavy charged particles travel in a almost straight paths in matter, losing energy continuously through a large number of collisions with atomic electrons.
- At low velocity, a heavy charged particle may losses a negligible amount of energy in nuclear collisions. It may also pick up free electrons along its path, which reduces its net charge.
- Energetic heavy charged particle can also induce nuclear reactions.
Energy Loss Mechanisms

**FIGURE 5.1.** (Top) Alpha-particle autoradiograph of rat bone after inhalation of $^{241}$Am. Biological preparation by R. Masse and N. Parmentier. (Bottom) Beta-particle autoradiograph of isolated rat-brain nucleus. The $^{14}$C-thymidine incorporated in the nucleolus is located at the track origin of the electron emitted by the tracer element. Biological preparation by M. Wintzerith and P. Mandel. (Courtesy R. Rechenmann and E. Witten-dorp-Rechenmann, Laboratoire de Biophysique des Rayonnements et de Methodologie INSERM U.220, Strasbourg, France.)
Energy Loss Mechanisms

For heavy charged particles, the maximum energy that can be transferred in a single collision is given by the conservation of energy and momentum:

\[ \frac{1}{2} MV^2 = \frac{1}{2} MV_1^2 + \frac{1}{2} mv_1^2 \]

\[ MV = MV_1 + mv_1. \]

where M and m are the mass of the heavy charged particle and the electron. V is the initial velocity of the charged particle. \( V_1 \) and \( v_1 \) are the velocities of both particles after the collision.

The maximum energy transfer is therefore given by

\[ Q_{\text{max}} = \frac{1}{2} MV^2 - \frac{1}{2} MV_1^2 = \frac{4mME}{(M + m)^2} \]
Energy Loss Mechanisms

For a more general case, which includes the relativistic effect, the maximum energy transferred by a single collision is

\[
Q_{\text{max}} = \frac{2 \gamma^2 mV^2}{1 + 2 \gamma m/M + m^2/M^2}
\]

where \(\gamma = 1/\sqrt{1 - \beta^2}\), \(\beta = V/c\), and \(c\) is the speed of light.

In extreme relativistic case, \(\gamma m/M \ll 1\). So the above equation reduces to

\[
Q_{\text{max}} = 2 \gamma^2 mV^2 = 2 \gamma^2 mc^2 \beta^2
\]

<table>
<thead>
<tr>
<th>Proton Kinetic Energy (E) (MeV)</th>
<th>(Q_{\text{max}}) (MeV)</th>
<th>Maximum Percentage Energy Transfer (100Q_{\text{max}}/E)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.00022</td>
<td>0.22</td>
</tr>
<tr>
<td>1</td>
<td>0.0022</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>0.0219</td>
<td>0.22</td>
</tr>
<tr>
<td>100</td>
<td>0.229</td>
<td>0.23</td>
</tr>
<tr>
<td>(10^3)</td>
<td>3.33</td>
<td>0.33</td>
</tr>
<tr>
<td>(10^4)</td>
<td>136.</td>
<td>1.4</td>
</tr>
<tr>
<td>(10^5)</td>
<td>(1.06 \times 10^4)</td>
<td>10.6</td>
</tr>
<tr>
<td>(10^6)</td>
<td>(5.38 \times 10^6)</td>
<td>53.8</td>
</tr>
<tr>
<td>(10^7)</td>
<td>(9.21 \times 10^6)</td>
<td>92.1</td>
</tr>
</tbody>
</table>

m: mass of a electron at rest
M: mass of the heavy charged particle
Linear Stopping Power of a Medium for Heavy Charged Particles

The linear stopping power of a medium is given by the Bethe formula,

\[- \frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[ \ln \frac{2mc^2 \beta^2}{I(1 - \beta^2)} - \beta^2 \right].\]

In this relation

\[k_0 = 8.99 \times 10^9 \text{ N m}^2 \text{ C}^{-2}\]

\[z = \text{atomic number of the heavy particle},\]

\[e = \text{magnitude of the electron charge},\]

\[n = \text{number of electrons per unit volume in the medium},\]

\[m = \text{electron rest mass},\]

\[c = \text{speed of light in vacuum},\]

\[\beta = V/c = \text{speed of the particle relative to } c,\]

\[I = \text{mean excitation energy of the medium}.\]
Linear Stopping Power of a Medium for Heavy Charged Particles

The Bethe formula can be further simplified by substituting known constants, which gives

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} \left[ \ln \frac{1.02 \times 10^6 \beta^2}{I (1 - \beta^2)} - \beta^2 \right] \text{MeV cm}^{-1}$$

It is further simplified to emphasize some important quantities related to the stopping power, the “speed” of the particle $\beta$, atomic mass of the charged particle $z$, the number of electron per cm$^3$ $n$ and the mean excitation-ionization potential $I$:

$$-\frac{dE}{dx} = \frac{5.08 \times 10^{-31} z^2 n}{\beta^2} [F(\beta) - \ln I] \text{ MeV cm}^{-1}$$

where

$$F(\beta) = \ln \frac{1.02 \times 10^6 \beta^2}{1 - \beta^2} - \beta^2$$
Linear Stopping Power of a Medium for Heavy Charged Particles (revisited)

The linear stopping power of a medium is given by the Bethe formula,

\[-\frac{dE}{dx} = \frac{4\pi k_0^2 z^2 e^4 n}{mc^2 \beta^2} \left[ \ln \frac{2mc^2 \beta^2}{I(1 - \beta^2)} - \beta^2 \right].\]

In this relation

\( k_0 = 8.99 \times 10^9 \text{ N } \text{ m}^2 \text{ C}^{-2} \)
\( z = \) atomic number of the heavy particle,
\( e = \) magnitude of the electron charge,
\( n = \) number of electrons per unit volume in the medium,
\( m = \) electron rest mass,
\( c = \) speed of light in vacuum,
\( \beta = V/c = \) speed of the particle relative to \( c \),
\( I = \) mean excitation energy of the medium.
Range for Heavy Charged Particles

There are two related definitions of the range of heavy charged particles:

1. Mean range: the absorber thickness that reduces the alpha particle count to exactly one-half of its value in the absence of the absorber.

2. Extrapolated range: extrapolating the linear portion of the end of the transmission curve to zero.

**Figure 2.5** An alpha particle transmission experiment. $I$ is the detected number of alpha particles through an absorber thickness $t$, whereas $I_0$ is the number detected without the absorber. The mean range $R_m$ and extrapolated range $R_e$ are indicated.
Range for Alpha Particles

The range of alpha particles in air (15°C, 1atm) can be approximately given by

\[ R = \begin{cases} 
0.56E, & E < 4; \\
\end{cases} \]

where \( E \) is given in MeV and \( R \) is given in cm.

The range of alpha particles in any other medium with a similar atomic composition can be computed from the following relationship:

\[ R_m, \text{mg/cm}^2 = 0.56A^{1/3} R, \]

where \( A = \) atomic mass number of the medium, 
\( R = \) range of the alpha particle in air, cm.

Because the effective atomic composition of tissue is not very much different from that of air, the following relationship may be used to calculate the range of alpha particles in tissue:

\[ R_a \times \rho_a = R_t \times \rho_t \]
Interactions of Neutrons
Physics Behind Neutron Radiography – Attenuation of Neutrons by Different Materials
Neutron Radiography

Neutron Radiography is an imaging technique which provides images similar to X-ray radiography. The difference between neutron and X-ray interaction mechanisms produce significantly different and often complementary information. While X-ray attenuation is directly dependent on atomic number, neutrons are efficiently attenuated by only a few specific elements.

For example, organic materials or water are clearly visible in neutron radiographs because of their high hydrogen content, while many structural materials such as aluminium or steel are nearly transparent. The next table shows how most materials behave when placed in the path of a neutron beam.
Neutron Imaging

- Aluminum optical table
- Monochromator crystal
- Collimating guide
- CCD camera
- Neutron converter screen
- Sample
- Mirror
- Lens
- Neutron beam
- 1 m
Neutron Radiography

This neutron image of the injection nozzle for diesel engines demonstrates the advantage of the method: metallic parts become very transparent, whereas small amounts of hydrogenous liquids (diesel fuel) can be detected, visualized and quantified with high precision.
Neutron Radiography

Organic materials deliver very high contrasts for neutrons in the transmission mode. This dried fish (Piranha) can be studied in very detail because a high resolution was obtained in the image (50 μm are possible nowadays). There are many interesting aspects in Biology to be investigated with the help of neutrons: root growing, moisture distribution in soil and plants, the detection of pollutions and poisons.
Imaging With Neutrons

The 1994 Nobel Prize in Physics – Shull & Brockhouse

Neutrons show where the atoms are….

…and what the atoms do.

From http://www.dep.anl.gov/nx/lectrnotes.pdf

NPRE 435, Radiological Imaging, Fall 2021
Classification of Neutrons

Thermal neutrons:
- Thermal neutrons are in approximate thermal equilibrium with their surroundings. Their energy distribution is given by the Maxwell-Boltzmann formula.
- The most probably energy at room temperature (20°C) is 0.025eV. The average energy is 0.038eV.

Fast neutrons: neutrons with energies above 0.1MeV.
Slow neutrons: neutrons with energies between those for thermal and fast neutrons.
Interactions of Neutron with Matter

Attenuation of neutrons:

- Neutrons are uncharged and can travel appreciable distance in matter without interaction.
- Under conditions of “good geometry”, a narrow beam of monoenergetic neutrons is attenuated exponentially.

Neutrons can interact with an atomic nucleus through

- Elastic scattering: the total kinetic energy is conserved – the energy loss by the neutron is equal to the kinetic energy of the recoil nucleus.
- Inelastic scattering: the nucleus absorbs some energy internally and is left to an excited state.
- (Thermal) neutron capture: the neutron is captured or absorbed by a nucleus, leading to a reaction such as \((n,p)\), \((n,2n)\), \((n,\alpha)\) or \((n,\gamma)\). The reaction changes the atomic number and/or atomic mass number of the struck nucleus.
Elastic Scattering of Fast Neutrons

Elastic scattering is the most important process for slowing down fast neutrons. Due to the rapid increase in the probability of neutron capture with decreasing energy, the neutrons will eventually be captured by target nuclei.

Here we discuss a few aspects of nuclear scattering in matter ... 

- Energy transfer through elastic scattering.
- Angular distribution of scattered neutrons.
- Energy distribution of scattered neutrons.
- Average logarithmic energy decrement of neutrons.
Elastic Scattering of Neutrons

Energy transfer of neutron through elastic scattering.

For the geometry shown, we can write the conservation of energy and momentum as

\[
\begin{align*}
\frac{1}{2} M V_i^2 &= \frac{1}{2} M V_f^2 + \frac{1}{2} m v_{f1}^2 & <1> \\
M V &= M V_i \cos \theta + m v_{f1} \cos \theta & <2> \\
M V_i \sin \theta + m v_{f1} \sin \theta &= 0 & <3>
\end{align*}
\]

From <2> and <3>, one can write

\[
\frac{1}{2} M V_i^2 = \frac{1}{2} (M V - m v_{f1} \cos \theta)^2 + \frac{1}{2} (m v_{f1} \sin \theta)^2
\]

Substitute <4> into <1>, we have

\[
m V_i \cos \theta = \frac{1}{2} \frac{m}{M} V_i^2 + \frac{1}{2} m v_{f1}^2
\]

and

\[
V_i \cos \theta = \frac{1}{2} \frac{M+m}{M} V_i \quad \text{and} \quad V_i = \frac{2M}{M+m} \cdot \cos \theta \cdot V.
\]

So

\[
E_{\text{recoil}} = \frac{1}{2} m v_{f1}^2 = \frac{m}{2} \left( \frac{2M}{M+m} V \cos \theta \right)^2 = \frac{4Mm}{(M+m)^2} \cdot \frac{1}{2} M V^2 \cos^2 \theta
\]

and

\[
E_{\text{recoil}} \approx \frac{1}{2} M V^2 \cdot \cos^2 \theta = E_n \cdot \cos^2 \theta,
\]

when \( m = M \).
Elastic Scattering of Neutrons

The maximum energy that a neutron of mass \( M \) and kinetic energy \( E_n \) can transfer to a nucleus of mass \( m \) in a single elastic collision is given by

\[
E_{\text{max}} = E_n \frac{4Mm}{(M + m)^2}
\]

**TABLE 9.4. Maximum Fraction of Energy Lost, \( Q_{\text{max}}/E_n \) from Eq. (9.3), by Neutron in Single Elastic Collision with Various Nuclei**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( Q_{\text{max}}/E_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>({}_1^1\text{H})</td>
<td>1.000</td>
</tr>
<tr>
<td>({}_1^2\text{H})</td>
<td>0.889</td>
</tr>
<tr>
<td>({}_2^4\text{He})</td>
<td>0.640</td>
</tr>
<tr>
<td>({}_2^8\text{Be})</td>
<td>0.360</td>
</tr>
<tr>
<td>({}_6^{12}\text{C})</td>
<td>0.284</td>
</tr>
<tr>
<td>({}_8^{16}\text{O})</td>
<td>0.221</td>
</tr>
<tr>
<td>({}_{26}^{56}\text{Fe})</td>
<td>0.069</td>
</tr>
<tr>
<td>({}_{50}^{118}\text{Sn})</td>
<td>0.033</td>
</tr>
<tr>
<td>({}_{92}^{238}\text{U})</td>
<td>0.017</td>
</tr>
</tbody>
</table>
Elastic Scattering of Neutrons

The elastic scattering plays an important role in neutron energy measurements. For example, a proton-neutron telescope illustrated below can be used to accurately measure the spectrum of neutrons in a collimated beam.

\[ E_{\text{proton}} = E_{\text{neutron}} \cos \theta \]

**FIGURE 10.36.** Arrangement of proton-recoil telescope for measuring spectrum neutron beam.

Figure from Atoms, Radiation, and Radiation Protection, James E Turner, p281
Angular Distributions of Scattered Neutrons and Recoil Nuclei

Experimentally, for neutrons energies up to 10MeV, it is observed that the scattering of neutrons is isotropic in the center-of-mass coordinate system. The neutron and the recoil nuclei are scattered with equal probability in any direction in this 3-D coordinate system.

\[
\text{d} \Omega = \frac{2\pi r \sin \theta \, r \, d\theta}{r^2}
\]
Energy Distributions of Scattered Neutrons

The speed of the scattered neutron can be derived as the following.

\[
\begin{align*}
\text{Before collision in lab system} & \\
\vec{v} &= \vec{v}_0 \\
\text{Before collision in CM system} & \\
\vec{v}' &= \vec{v}_0 - \vec{v}_e \\
\text{After collision in lab system} & \\
\vec{v}'' &= \vec{v}_0 + \vec{v}_e' \\
\text{After collision in CM system} & \\
\vec{v}' &= \frac{m}{m+m'} \vec{v}_0 \\
\end{align*}
\]

Therefore, the speed of the scattered neutron is given by

\[
\vec{v}' = \vec{v}_0 + \vec{v}_e'
\]

and

\[
\vec{v}'^2 = v_0^2 + v_e^2 - 2 v_0 v_e \cos \theta
\]

So the kinetic energy of the neutron after the collision is

\[
E' = \frac{1}{2} m v'^2
\]

and

\[
\frac{E'}{E_0} = \left[ \frac{m}{m+m'} \right]^2 + \left( \frac{m}{m+m'} \right)^2 + 2 \left( \frac{m}{m+m'} \right) v_0 \cos \theta
\]

\[
= \frac{m^2 + 2 m' v_0 \cos \theta}{(m+m')^2}
\]

Since the scattering in CM system is isotropic, the prob. of scatter into angular interval d\Omega can be written as

\[
P(0) : d\Omega = \left[ 2 \pi \sin \theta \cdot d\Omega \right] / 4\pi = \frac{1}{2} \sin \theta \cdot d\Omega
\]

Therefore, the probability of the emerging neutron carries an energy between \( E' \) and \( dE' \) is

\[
P(E') \cdot dE' = \frac{P(0) \cdot d\Omega}{\sin \theta}
\]

Since both \( E_0 \) and \( \left( \frac{m}{m+m'} \right)^2 \) are constants, the above equation implies that the energy of the scattered neutron follows a uniform distribution.

\[
P(E')
\]

And the average scattered neutron energy is

\[
E_{avg} = \left( \frac{3}{8} \right) E_0
\]
Energy Distributions of Scattered Neutrons

\[ P(E') : \text{kinetic energy carried by the scattered neutron} \]

\[ \alpha = \left[ \frac{(M - m)}{(M + m)} \right]^2 \]

The fraction of energy carried by the scattered neutron is

\[ \frac{E}{E_0} = \frac{M + m}{(M + m)^2} \phi \frac{(M - m) \cos \theta}{(M + m)^2} \]

The distribution of the energy of the scattered neutrons is given by

\[ p(E') = \frac{1}{1 - \alpha} \frac{1}{E_0}, \quad E \subset [\alpha E_0, E_0]. \]
Average Logarithm Energy Decrement

For fast neutrons undergo successive collisions in the absorber. The average decrease per collision in the logarithm of the neutron energy (the average logarithmic energy decrement) remains constant:

\[ \xi = \Delta \ln E = \ln E_0 - \ln E = \ln \frac{E_0}{E} = -\ln \frac{E}{E_0} \]

and

\[ \xi = 1 + \frac{\alpha \ln \alpha}{1 - \alpha} \]

where

\[ \alpha = \left(\frac{M - m}{M + m}\right)^2 \]

The average logarithmic energy decrement is independent of the neutron energy and is a function only of the mass of the scattering nuclei.
Average Logarithmic Energy Decrement

The average logarithmic energy decrement per collision is defined as

\[ \bar{\xi} = \Delta \ln E = \ln E_0 - \ln E = -\ln \frac{E}{E_0} \]

\[ = \int_{\alpha E_0}^{E_0} -\ln \left( \frac{E}{E_0} \right) \cdot p(E) \cdot dE \]

\[ = -p(E) \cdot \left( E \ln \frac{E}{E_0} - E \right) \bigg|_{\alpha E_0}^{E_0} \]

\[ = -\frac{(M+m)^2}{4mm} \frac{1}{E_0} \left[ (E_0 \ln \frac{E_0}{E_0} - E_0) - (\alpha E_0 \ln \alpha - \alpha E_0) \right] \]

\[ = \frac{1}{1-\alpha} \left( 1 + \alpha \ln \alpha - \alpha \right) \]

\[ = \frac{1}{1-\alpha} \left( 1 - \alpha + \alpha \ln \alpha \right) = 1 + \frac{\alpha \ln \alpha}{1-\alpha} \]

where

\[ \alpha = \left( \frac{M-m}{M+m} \right)^2 \]
Interaction of Slow Neutrons (E<0.5eV)

- Significant interactions include elastic scattering and neutron induced nuclear reactions.
- Due to the low neutron energy, very little energy can be transferred by elastic scattering.
- The more significant effect of elastic scattering is to slow down slow neutrons and turn them into thermal neutrons (average E<0.025eV at room temperature).
Interaction of Slow Neutrons (E<0.5eV)

- The most important interactions between slow neutrons and absorbing materials are neutron-induced reactions, such as (n,γ), (n,α), (n,p) and (n, fission) etc. These interactions lead to more prominent signatures for neutron detection.

\[
\text{neutron + target nucleus} \Rightarrow \begin{cases} 
\text{recoil nucleus} \\
\text{protons} \\
\text{alpha particles} \\
\text{fission fragments}
\end{cases}
\]
Neutron Induced Reactions

\[ ^1_0 n + ^1_1 H \rightarrow ^2_1 H + ^0_0 \gamma \]

- Neutron absorption followed by the immediate emission of a gamma ray photon.
- Since the thermal neutron has negligible energy by comparison, the gamma photon has the energy \( Q = 2.22 \text{MeV} \) released by the reaction, which represents the binding energy of the deuteron.
- The capture cross section per atom is 0.33 barn.
- When tissue is exposed to thermal neutrons, this reaction provides a source of gamma rays that delivers dose to the tissue.
Neutron Induced Reactions

Capture cross sections for low energy neutrons generally decreases as the reciprocal of the velocity as the neutron energy increases (the 1/v law).

So if the capture cross section $\sigma_0$ is known for a given velocity $v_0$, then the cross section at some other velocity $v$ can be estimated from the relations

$$\frac{\sigma}{\sigma_0} = \frac{v_0}{v} = \sqrt{\frac{E_0}{E}}$$

This equation can be used for neutrons of energies up to 100eV or 1keV, depending on the absorbing nucleus.
Energy Dependence of Thermal Neutron Absorption Cross section

**Figure 5.23.** Neutron absorption cross section for boron, showing the validity of the $1/\nu$ law for neutrons from 0.02 to 1000 eV in energy. The equation of the curve is $\sigma = 116\sqrt{\text{eV}}$ barns.
Neutron Induced Reactions

\[ {}_0^1n + {}_2^3\text{He} \rightarrow {}_1^3\text{H} + {}_1^1\text{p} \]

- Cross section for thermal neutron is 5330 barns.
- Q=765keV.
- Commonly used in proportional counters for fast neutron monitoring.

**FIGURE 10.35.** Pulse-height spectrum from \(^3\text{He}\) proportional counter for monoenergetic neutrons of energy T.
Neutron Induced Reactions

\[ ^1_0 n + ^6_3 Li \rightarrow ^4_2 He + ^3_1 H \]

- Cross section for thermal neutron is 940 barns.
- Q=4.78MeV.
- Widely used for thermal neutron detection.
  
  Neutron sensitive LiI scintillator cam be made or Li can be added to other scintillator to register neutrons.

\(^6\text{Li}\) is 7.42% abundant and Li enriched in the isotope \(^6\text{Li}\) is available.
Neutron Induced Reactions

\[ {}^1_0n + {}^{10}_5B \rightarrow {}^7_3Li + {}^4_2He \]

- Cross section for thermal neutron is 3840 barns.
- \( Q = 2.31 \text{MeV} \) when the daughter nucleus is in an excited state (93%) and 
  \( 2.79 \text{MeV} \) when the Li nucleus is in ground state (7%).
- Widely used for thermal neutron detection.
  - BF\(_3\) is a gas that can be used directly in a neutron counter.
  - Boron is also used as a liner inside the tubes of proportional counters for 
    neutron detection.
Neutron Activation

Neutron activation is the production of a radioactive isotope by absorption of a neutron, such as the \((n,p)\) reaction.

(a) Materials irradiated by neutrons may become radioactive.
(b) Neutron activation provides a convenient way to measure neutron flux.
(c) By spectroscopic examination of the induced radiation, quantitative analysis of the unknown samples is also possible.
Neutron Activation

Neutron activated radioactivity of cement and aggregate
<table>
<thead>
<tr>
<th>Isotope</th>
<th>$E_{\text{peak}}$ [keV]</th>
<th>$I_\gamma$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{22}\text{Na}$</td>
<td>1274.53</td>
<td>99.944</td>
</tr>
<tr>
<td>$^{57}\text{Co}$</td>
<td>122.0614</td>
<td>85.6</td>
</tr>
<tr>
<td>$^{57}\text{Co}$</td>
<td>136.4743</td>
<td>10.688</td>
</tr>
<tr>
<td>$^{60}\text{Co}$</td>
<td>1173.237</td>
<td>99.9736</td>
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Table A.1: overview of the peak energies and gamma emission probabilities of the radioactive sources used for the efficiency callibration.