Sources of Ionizing Radiation and Radiation Interactions

Reading Material:
Chapter 1, Radiation Detection and Measurements,
Forth Edition, by G. F. Knoll
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   • Gamma Rays Sources
   • X-Rays Sources
   • Neutron Sources

II. Radiation Interactions
   • Interactions of X ray and Gamma Rays
   • Interactions of Fast Electrons and Heavy Charged Particles.
   • Interactions of Neutrons
Functional Imaging with Tremendous Sensitivity Under In Vivo Settings

PET studies of glucose metabolism to map human brain’s response in performing different tasks.


PET-labeled probes for biological imaging: (from S. Cherry, UC Davis)

hemodynamic parameters ($H_2^{15}O$, $^{15}O$-butanol, $^{11}CO$, $^{13}NH_3$...), substrate metabolism($^{18}F$-FDG, $^{15}O_2$, $^{11}C$-palmitic acid...), protein synthesis ($^{11}C$-leucine, $^{11}C$-methionine, $^{11}C$-tyrosine), enzyme activity ($^{11}C$-deprenyl, $^{18}F$-deoxyuracil...), drugs ($^{11}C$-cocaine, $^{13}N$-cisplatin, $^{18}F$-fluorouracil...), receptor affinity ($^{11}C$-raclopride, $^{11}C$-carfentanil, $^{11}C$-scopolamine), neurotransmitter biochemistry ($^{18}F$-fluorodopa, $^{11}C$-ephedrine...), gene expression ($^{18}F$-penciclovir, $^{18}F$-antisense oligonucleotides) ........

L. J. Meng, University of Illinois Technology Showcase, April 5th, 2017
Special SPECT Imaging Applications:
Single Photon Emission Microscopy

Imaging of dopamine transporter in mouse brain. Data from Center for Gamma Ray Imaging, University of Arizona

In vivo imaging of (~1500) radiolabeled T cells in mouse brain using SPECT (Meng, 2009).
The Promise of Targeted Particle Therapy

By Deborah A. Mulford, MD; David A. Scheinberg, MD, PhD and Joseph G. Jurcic, MD

**TABLE 1**
Characteristics of Selected α-Emitting Radioisotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Particle(s) emitted</th>
<th>Half-life</th>
<th>Energy of α-particle (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>^{211}\text{At}</td>
<td>1 α</td>
<td>7.2 h</td>
<td>6</td>
</tr>
<tr>
<td>^{225}\text{Ac}</td>
<td>4 α, 2 β</td>
<td>10 d</td>
<td>6–8</td>
</tr>
<tr>
<td>^{212}\text{Bi}</td>
<td>1 α, 1 β</td>
<td>60.6 min</td>
<td>6</td>
</tr>
<tr>
<td>^{213}\text{Bi}</td>
<td>1 α, 2 β</td>
<td>46 min</td>
<td>6</td>
</tr>
<tr>
<td>^{223}\text{Ra}</td>
<td>4 α, 2 β</td>
<td>11.4 d</td>
<td>6–7</td>
</tr>
<tr>
<td>^{212}\text{Pb}</td>
<td>1 α, 2 β</td>
<td>10.6 h</td>
<td>7.8</td>
</tr>
<tr>
<td>^{149}\text{Tb}</td>
<td>1 α</td>
<td>4.2 h</td>
<td>4</td>
</tr>
</tbody>
</table>

**FIGURE 1.** ^{229}\text{Th} decay scheme. ^{225}\text{Ac} is isolated from ^{229}\text{Th} sources and decays by α-emission through ^{211}\text{Fr}, ^{217}\text{At}, and ^{213}\text{Bi}, each of which also emits an α-particle.
Alpha Emission

• An alpha particle is a highly energetic helium nucleus consisting of two neutrons and 2 protons.
• It is normally emitted from isotopes when the neutron-to-proton ratio is too low – through the so-called alpha decay.
• Atomic number and atomic mass number are conserved in alpha decays
Alpha Emission

- In heavy elements, a **potential barrier of ~25 MeV** has to be overcome for an alpha particle to escape from the potential well.
- It would require a **minimum kinetic energy of ~3.8 MeV** for the alpha particle to “tunneling through” the potential well is

Fig. 4.1. Potential inside and in the vicinity of a nucleus.
Energy Spectra of Alpha Particles

Alpha decays are sometimes accompanied by the excited daughter products which complicates the resultant alpha particle spectra.

The kinetic energy of alpha particles generated is given by

$$E_\alpha = Q \cdot \left( A - 4 \right) / A$$
Energy Release in Alpha Emission

- **The required kinetic energy** has to come from the decrease in mass following the decay process.
- The **relationship between mass and energy** associated with an alpha emission is given as

\[
M_p = M_d + M_a + 2M_e + Q, \tag{4.1}
\]

where \(M_p\), \(M_d\), \(M_a\), and \(M_e\) are respectively equal to the masses of the parent, the daughter, the emitted alpha particle, and the two orbital electrons that are lost during the transition to the lower atomic numbered daughter, while \(Q\) is the total energy release associated with the radioactive transformation.
A Few Remarks on Alpha Decay

- **Q value positive** for alpha decay.
- Energies of the alpha particles generally increase with the atomic number of parent. For example, 1.8 MeV for $^{144}$Nd to 11.6 MeV for $^{212m}$Po.
- All nuclei with mass numbers greater than $A$ of 150 are thermodynamically unstable against alpha emission ($Q$ is positive). However, alpha emission is a dominant decay process only for heaviest nuclei, $A \geq 210$. 
Beta Decay
# Common isotopes used in nuclear medicine

<table>
<thead>
<tr>
<th>Isotope</th>
<th>Symbol</th>
<th>Z</th>
<th>T(_{1/2})</th>
<th>Decay</th>
<th>Gamma (keV)</th>
<th>Positron (keV)</th>
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<tbody>
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<tr>
<td>fluorine-18</td>
<td>(^{18}\text{F})</td>
<td>9</td>
<td>109.77 m</td>
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<td>511 (193%)</td>
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<td>gallium-67</td>
<td>(^{67}\text{Ga})</td>
<td>31</td>
<td>3.26 d</td>
<td>ec</td>
<td>93 (39%), 185 (21%), 300 (17%)</td>
<td>-</td>
</tr>
<tr>
<td>krypton-81m</td>
<td>(^{81m}\text{Kr})</td>
<td>36</td>
<td>13.1 s</td>
<td>IT</td>
<td>190 (68%)</td>
<td>-</td>
</tr>
<tr>
<td>rubidium-82</td>
<td>(^{82}\text{Rb})</td>
<td>37</td>
<td>1.27 m</td>
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<td>511 (191%)</td>
<td>3.379 (95%)</td>
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<tr>
<td>nitrogen-13</td>
<td>(^{13}\text{N})</td>
<td>7</td>
<td>9.97 m</td>
<td>(\beta^+)</td>
<td>511 (200%)</td>
<td>1190 (100%)</td>
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<td>technetium-99m</td>
<td>(^{99m}\text{Tc})</td>
<td>43</td>
<td>6.01 h</td>
<td>IT</td>
<td>140 (89%)</td>
<td>-</td>
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<td>indium-111</td>
<td>(^{111}\text{In})</td>
<td>49</td>
<td>2.80 d</td>
<td>ec</td>
<td>171 (90%), 245 (94%)</td>
<td>-</td>
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<tr>
<td>iodine-123</td>
<td>(^{123}\text{I})</td>
<td>53</td>
<td>13.3 h</td>
<td>ec</td>
<td>159 (83%)</td>
<td>-</td>
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<tr>
<td>xenon-133</td>
<td>(^{133}\text{Xe})</td>
<td>54</td>
<td>5.24 d</td>
<td>(\beta^-)</td>
<td>81 (31%)</td>
<td>0.364 (99%)</td>
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<tr>
<td>thallium-201</td>
<td>(^{201}\text{Tl})</td>
<td>81</td>
<td>3.04 d</td>
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<td>69–83(^*) (94%), 167 (10%)</td>
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<td>(^{90}\text{Y})</td>
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<td>-</td>
<td>2.280 (100%)</td>
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<td>(^{131}\text{I})</td>
<td>53</td>
<td>8.02 d</td>
<td>(\beta^-)</td>
<td>364 (81%)</td>
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\(Z = \) atomic number, the number of protons; \(T_{1/2} = \) half-life; decay = mode of decay

Photons = principle photon energies in kilo-electron volts, keV, (abundance/decay)

\(\beta = \) beta maximum energy in mega-electron volts, MeV, (abundance/decay)

\(\beta^+ = \beta^+ \text{ decay}; \beta^- = \beta^- \text{ decay}; IT = \text{ isomeric transition}; ec = \text{ electron capture}\)

\(^*\) X-rays from progeny, mercury, Hg
An Example of Cancer Therapy with Radiopharmaceuticals

Applying the same tracer for both diagnostic as treatment applications is called “Theranostics”.
The Figure illustrates this concept in the diagnosis and treatment of patients with metastatic neuroendocrine tumors resistant to all standard treatments.

On the left the diagnostic imaging using whole body PET performed one hour after intravenous administration of 3 mCi 68Ga-octreotate showing an intense expression of the somatostatine receptor in all tumor sites in liver, bone and abdomen; and, on the right is the same patient imaged by whole body SPECT at 24 hours after administration of a radiotherapeutic dose of 177Lu-octreotate for selective targeted radiotherapy.
Three Types of Beta Decay

Beta decay

Positron decay

Orbital Electron Capture
Beta Decay

• **Beta particle** is an ordinary electron. Many atomic and nuclear processes result in the emission of beta particles.

• One of the most common source of beta particles is the **beta decay** of nuclides, in which

\[ A \ _Z \ X \rightarrow A \ _{Z+1} \ Y + \ _{-1}^0 \beta + \ _0 \bar{\nu} \]

For example

\[ ^{60}_{27}\text{Co} \rightarrow ^{60}_{28}\text{Ni} + _{-1}^0 \beta + _0 \bar{\nu} \]
Energy Release of Beta Decay

The energy release in a beta decay is given as

\[ Q = M_p - (M_d + M_e) \]

- The energy release is once again given by the conversion of a fraction of the mass into energy. Note that atomic electron bonding energy is neglected.
- For a beta decay to be possible, the energy release has to be positive.
The energy release is shared by all three daughter products. Due to the relatively large mass of the daughter nucleus, it attains only a small fraction of the energy. Therefore, the kinetic energy of the beta particle is

\[ E_{\beta^-} \approx Q - E_{\bar{\nu}} \]
Gamma Ray Emission

- Gamma rays are emitted from nuclei following the transition between different nuclear states.

- Gamma rays are emitted with discrete energies. A gamma ray spectrum is characteristic to the particular radionuclide that are present.

**FIGURE 3.7**. Detailed decay scheme for $^{226}\text{Ra}$, showing origin of photons found in its gamma spectrum (position of initial $^{226}\text{Ra}$ energy level not to scale).
Metastable Nuclear States and Gamma Ray Emission

The lifetimes of nuclear excited states vary, but $\sim 10^{-10}$ s can be regarded as typical. Thus, gamma rays are usually emitted quickly after radioactive decay to an excited daughter state.

In some cases, however, selection rules prevent photon emission for an extended period of time. The excited state of $^{137}_{56}$Ba following the decay of $^{137}_{55}$Cs has a half-life of 2.55 min. Such a long-lived nuclear state is termed metastable and is designated by the symbol m: $^{137m}_{56}$Ba.

Another example of a metastable nuclide is $^{99m}_{43}$Tc, which results from the beta decay of the molybdenum isotope $^{99}_{42}$Mo.
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<tr>
<th>Isotope</th>
<th>Symbol</th>
<th>Mass Number</th>
<th>Atomic Number</th>
<th>T_{1/2}</th>
<th>Mode of Decay</th>
<th>Gamma (keV)</th>
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\(\beta^+ = \text{beta decay} ; \beta^- = \text{beta decay} ; \text{IT} = \text{isomeric transition} ; \text{ec} = \text{electron capture}\)

\(\text{x-rays from progeny, mercury, Hg}\)
Examples for Beta Decay

- **Complicated decay schemes** and the emission of other particles such as gamma rays.
- **Pure beta emitters**: $^3$H, $^{14}$C, $^{32}$P and $^{90}$Sr.

*Figure 4.7. Iodine-131 transformation (decay) scheme.*
**Figure 4.7.** Iodine-131 transformation (decay) scheme.
Beta‐plus (or Positron) Decay
Positron Emission

- A **positron** is the anti-particle of electrons, which carries the same mass as an electron but is positively charged.
- Positrons are normally generated by those nuclides having a **relatively low neutron-to-proton ratio**.
- An typical example of positron emitter is

\[ ^{22}_{11}\text{Na} \rightarrow ^{22}_{10}\text{Ne} + ^0_1\beta + \nu \]

**FIGURE 3.11.** Decay scheme of \(^{22}_{11}\text{Na}\).
Three Types of Beta Decay

Positron decay
Energy Release

The energy release $Q$ from the positron emission process is given by

$$Q \approx M_p - M_d - M_e - M_{e^+} = M_p - (M_d + 2M_e)$$

where the atomic electron binding energy is ignored.
Positron Emission Tomography

Beta-plus decay or positron decay:

\[ {}_{z}^{A}X \rightarrow {}_{z-1}^{A}Y + {}_{1}^{0}\beta + \nu \]

Example of positron annihilation
### Commonly Used PET Isotopes

<table>
<thead>
<tr>
<th>Isotope</th>
<th>half-life (min)</th>
<th>Maximum positron energy (MeV)</th>
<th>Positron range in water (FWHM in mm)</th>
<th>Production method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{C}$</td>
<td>20.3</td>
<td>0.96</td>
<td>1.1</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{13}\text{N}$</td>
<td>9.97</td>
<td>1.19</td>
<td>1.4</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>2.03</td>
<td>1.70</td>
<td>1.5</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{18}\text{F}$</td>
<td>109.8</td>
<td>0.64</td>
<td>1.0</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{68}\text{Ga}$</td>
<td>67.8</td>
<td>1.89</td>
<td>1.7</td>
<td>generator</td>
</tr>
<tr>
<td>$^{82}\text{Rb}$</td>
<td>1.26</td>
<td>3.15</td>
<td>1.7</td>
<td>generator</td>
</tr>
</tbody>
</table>

Table 2. Properties of commonly used positron emitting radio-isotopes
Positron Emission Tomography (PET)

Collection of Line-integrals

Typical Detection Process
Orbital Electron Capture
Orbital Electron Capture

In electron capture (EC), one of the atomic electrons is captured by the nucleus and unites with an proton to form an neutron, with the emission of a neutrino.

\[
\frac{A}{Z} X + e^- \rightarrow \frac{A}{Z-1} Y + \nu
\]

\[
_0^1 e + _1^1 H \rightarrow _0^1 n + \nu
\]

- For neutron deficient atoms to attain stability through positron emission, it must exceed the weight of the daughter by at least two electron masses. If this condition can not be satisfied, the neutron deficiency can be overcome by the EC process.
- For example,

\[
^{103}_{46}\text{Pd} + _{-1}^0 e \rightarrow^{103m}_{45}\text{Rh} + _0^0 \nu.
\]
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<td>¹³¹I</td>
<td>53</td>
<td>8.02 d</td>
<td>β⁻</td>
<td>364 (81%)</td>
<td>0.807 (100%)</td>
</tr>
</tbody>
</table>

*Z = atomic number, the number of protons; T½ = half-life; decay = mode of decay

Photons = principal photon energies in kilo-electron volts, keV, (abundance/decay)

β = beta maximum energy in mega-electron volts, MeV, (abundance/decay)

β⁺ = β⁺ decay; β⁻ = β⁻ decay; IT = isomeric transition; ec = electron capture

* X-rays from progeny, mercury, Hg
Energy Release of Orbital Electron Capture

For **Positron Decay** to be possible, we need

\[ Q = M_p - M_d - M_e - M_{e^+} > 0, \]

so

\[ M_p > M_d + M_e + M_{e^+} = M_d + 2M_e \]

\[ M_p \] and \[ M_d \] are the atomic masses of the parent and daughter atoms.

For **Electron Capture** to occur,

\[ Q = M_p - M_d - \phi > 0 \]

so that

\[ M_p > M_d + \phi \]

where \( \phi \) is the binding energy of the orbital electron.
Orbital Electron Capture and Positron Decay

- Electron capture and positron decay are competing processes through which a neutron deficient nucleus may attain an increased stability.

- Both the emission of a positron and the capture of an electron, a neutrino is always emitted in order to conserve energy.

- In positron decay, the neutrino carries the difference between the energy release and the energy of the resultant positron. In electron capture, however, the neutrino must be mono-energetic.

\[
\begin{align*}
\text{Beta-plus decay} & \quad A_Z^X \rightarrow A_{Z-1}^Y + 0^1\beta + \nu \\
\text{Electron capture} & \quad A_Z^X + e^- \rightarrow A_{Z-1}^Y + \nu
\end{align*}
\]
Orbital Electron Capture

• **Energy release.** The energy released in EC is transferred to the **neutrino**, which hardly interact with matter.

• **X-ray emission by excited atoms.** Electron capture leaves an vacancy in one of the electron shells, normally in the k-shell. This vacancy will soon be filled by the transition of an electron from other shells or by capturing a free electron. This leads to the generation of **characteristic x-ray**.

• **Gamma-ray emission by excited nuclei.** Electron capture may also lead to daughter nucleus at an excited state. The subsequent nuclear de-excitation will also induce **gamma radiations** that should be considered.
An intrinsically unstable nucleus may attain an increased stability by \textit{alpha decay}, \textit{electron capture} and \textit{positron decay}.

What would happen to an excited nucleus?
Internal Conversion

Begins with an excited nuclear state

De-excite through the emission of a gamma ray

\[ E_{\beta^-} = E_{ex} - E_b \]

The excitation energy is transferred directly to an orbital electron, causing it to be ejected from the atom

Conversion electron with an energy

\[ \text{IC Coefficient (or Branching Ratio)} = \frac{N_\gamma}{N_e} \]
Internal Conversion

- Conversion electrons can originate from several different electron shells within the atom, a single excited state generally leads to several groups of electrons with different energies.
- The only practical laboratory scale source of mono-energetic electron groups in high keV to MeV energy range.
Auger Electrons

- The excitation energy of the atom may be transferred to one of the outer electrons, causing it to be ejected from the atom.
- Auger electrons are roughly the analogue of internal conversion electrons when the *excitation energy originates in the atom rather than in the nucleus.*

![Diagram of Auger electron emission](image)

*Figure 3.7* (A) The usual emission of a K characteristic X-ray, $h\nu$, energy equal to $E_K - E_L$, the difference in binding energy for the two orbital electrons, K and L. (B) $h\nu$ has been absorbed and a monoenergetic Auger electron is emitted, in the example shown, from the M shell, the energy of which is $E_K - E_L - E_M$. (C) In its final state the atom has vacancies in the L and M orbitals.
Energies of Auger Electrons

\[ E_{\text{a.e.}} = (E_K - E_{L_1}) - E_{L_{23}} \]
Gamma Ray Emission

- Gamma rays are emitted from nuclei following the transition between different nuclear states.

- Gamma rays are emitted with discrete energies. A gamma ray spectrum is characteristic to the particular radionuclide that are present.
Metastable Nuclear States and Gamma Ray Emission

The lifetimes of nuclear excited states vary, but $\sim 10^{-10}$ s can be regarded as typical. Thus, gamma rays are usually emitted quickly after radioactive decay to an excited daughter state.

In some cases, however, selection rules prevent photon emission for an extended period of time. The excited state of $^{137}\text{Ba}$ following the decay of $^{137}\text{Cs}$ has a half-life of 2.55 min. Such a long-lived nuclear state is termed \textit{metastable} and is designated by the symbol m: $^{137m}\text{Ba}$.

Another example of a metastable nuclide is $^{99m}\text{Tc}$, which results from the beta decay of the molybdenum isotope $^{99}\text{Mo}$.
What is the ideal gamma ray emitter for nuclear imaging?

- Reasonably penetrative...
- Half-life comparable to the biological process we are trying to visualize...
- Easy radiochemistry...
- Biologically safe...
- Clean ...
Important Ideas to Carry Away
Most of Gamma Ray Sources used in Nuclear Imaging Are the Produce of Isotopes Undergo Beta Decay

### Common isotopes used in nuclear medicine \(^{[9][10]}\)

<table>
<thead>
<tr>
<th>isotope</th>
<th>symbol</th>
<th>Z</th>
<th>T(_{1/2})</th>
<th>decay</th>
<th>gamma (keV)</th>
<th>positron (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Imaging:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>fluorine-18</td>
<td>(^{18})F</td>
<td>9</td>
<td>109.77 m</td>
<td>(\beta^+)</td>
<td>511 (193%)</td>
<td>249.8 (97%)</td>
</tr>
<tr>
<td>gallium-67</td>
<td>67Ga</td>
<td>31</td>
<td>3.26 d</td>
<td>ec</td>
<td>93 (39%), 185 (21%), 300 (17%)</td>
<td>-</td>
</tr>
<tr>
<td>krypton-81m</td>
<td>(^{81})Kr</td>
<td>36</td>
<td>13.1 s</td>
<td>IT</td>
<td>190 (68%)</td>
<td>-</td>
</tr>
<tr>
<td>rubidium-82</td>
<td>82Rb</td>
<td>37</td>
<td>1.27 m</td>
<td>(\beta^+)</td>
<td>511 (191%)</td>
<td>3.379 (95%)</td>
</tr>
<tr>
<td>nitrogen-13</td>
<td>13(^{13})N</td>
<td>7</td>
<td>9.97 m</td>
<td>(\beta^+)</td>
<td>511 (200%)</td>
<td>1190 (100%)</td>
</tr>
<tr>
<td>technetium-99m</td>
<td>(^{99m})Tc</td>
<td>43</td>
<td>6.01 h</td>
<td>IT</td>
<td>140 (89%)</td>
<td>-</td>
</tr>
<tr>
<td>indium-111</td>
<td>(^{111})In</td>
<td>49</td>
<td>2.80 d</td>
<td>ec</td>
<td>171 (90%), 245 (94%)</td>
<td>-</td>
</tr>
<tr>
<td>iodine-123</td>
<td>(^{123})I</td>
<td>53</td>
<td>13.3 h</td>
<td>ec</td>
<td>159 (83%)</td>
<td>-</td>
</tr>
<tr>
<td>xenon-133</td>
<td>(^{133})Xe</td>
<td>54</td>
<td>5.24 d</td>
<td>(\beta^-)</td>
<td>81 (31%)</td>
<td>0.364 (99%)</td>
</tr>
<tr>
<td>thallium-201</td>
<td>(^{201})Tl</td>
<td>81</td>
<td>3.04 d</td>
<td>ec</td>
<td>69–83* (94%), 167 (10%)</td>
<td>-</td>
</tr>
<tr>
<td>Therapy:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>yttrium-90</td>
<td>(^{90})Y</td>
<td>39</td>
<td>2.67 d</td>
<td>(\beta^-)</td>
<td>-</td>
<td>2.280 (100%)</td>
</tr>
<tr>
<td>iodine-131</td>
<td>(^{131})I</td>
<td>53</td>
<td>8.02 d</td>
<td>(\beta^-)</td>
<td>364 (81%)</td>
<td>0.807 (100%)</td>
</tr>
</tbody>
</table>

Z = atomic number, the number of protons; T\(_{1/2}\) = half-life; decay = mode of decay
photons = principle photon energies in kilo-electron volts, keV, (abundance/decay)
\(\beta\) = beta maximum energy in mega-electron volts, MeV, (abundance/decay)
\(\beta^+\) = \(\beta^+\) decay; \(\beta^-\) = \(\beta^-\) decay; IT = isomeric transition; ec = electron capture
* X-rays from progeny, mercury, Hg
There are Three Types of Beta Decay

Beta decay

Positron decay

Orbital Electron Capture
Beta Decays Typically Lead to the Emission of Energetic Beta Particles (Fast Electrons)

- The energy release is shared by all three daughter products. Due to the relatively large mass of the daughter nucleus, it attains only a small fraction of the energy. Therefore, the kinetic energy of the beta particle is

\[ E_{\beta^-} \approx Q - E_\nu \]
Gamma Rays Emission Following Beta Decay

Begins with an excited nuclear state

De-excite through the emission of a gamma ray

\[ E_{\beta^-} = E_{ex} - E_{b} \]

The excitation energy is transferred directly to an orbital electron, causing it to be ejected from the atom

Conversion electron with an energy

IC Coefficient (or Branching Ratio) = \( \frac{N_{\gamma}}{N_{e}} \)
Many Gamma Ray Sources Emit Photons at Multiple Discrete Energies – Potential Complications for Imaging?

- **Complicated decay schemes** and the emission of other particles such as gamma rays.
- **Pure beta emitters**: $^3$H, $^{14}$C, $^{32}$P and $^{90}$Sr.

![Iodine-131 transformation (decay) scheme.](image)

**Figure 4.7.** Iodine-131 transformation (decay) scheme.
Annihilation Radiation following Positron Decay

Beta - plus decay or positron decay :

\[ {}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{1}^{0}\beta + \nu \]

Two important details:
- Positron range
- Non-collinearity

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Radiation Sources and Interactions
A Few Details on Positron Decay and Their Implications to PET

<table>
<thead>
<tr>
<th>Isotope</th>
<th>half-life (min)</th>
<th>Maximum positron energy (MeV)</th>
<th>Positron range in water (FWHM in mm)</th>
<th>Production method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{11}\text{C}$</td>
<td>20.3</td>
<td>0.96</td>
<td>1.1</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{13}\text{N}$</td>
<td>9.97</td>
<td>1.19</td>
<td>1.4</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{15}\text{O}$</td>
<td>2.03</td>
<td>1.70</td>
<td>1.5</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{18}\text{F}$</td>
<td>109.8</td>
<td>0.64</td>
<td>1.0</td>
<td>cyclotron</td>
</tr>
<tr>
<td>$^{68}\text{Ga}$</td>
<td>67.8</td>
<td>1.89</td>
<td>1.7</td>
<td>generator</td>
</tr>
<tr>
<td>$^{82}\text{Rb}$</td>
<td>1.26</td>
<td>3.15</td>
<td>1.7</td>
<td>generator</td>
</tr>
</tbody>
</table>

Table 2. Properties of commonly used positron emitting radio-isotopes
What would be an Ideal Gamma Ray Source for Nuclear Imaging?

Another example of a metastable nuclide is $^{99m}\text{Tc}$, which results from the beta decay of the molybdenum isotope $^{99}\text{Mo}$. An isomeric transition (IT) to the ground

- $\text{Tc-99m}$ accounts for $>90\%$ of imaging studies in nuclear medicine and therefore subject to extensive dosimetry study.
- Half-life: $\sim 6\text{h}$; gamma energy: $140\text{keV}$, both ideal for imaging applications.
- $\text{Tc-99m}$ is obtained from the decay of the molybdenum isotope $^{99}\text{Mo}$. 
Single Photon Emission Computed Tomography (SPECT)

Endocrine tumor visualized with a commercial SPECT system.

X-Rays Sources
What Can We Learn from X-rays?

“The radiograph of Bera Rontgen’s hand”, taken 22 Dec. 1895

Wilhelm Conrad Röntgen, The Nobel Prize in Physics 1901.

A brain image from the first prototype clinical CT imager, 1972.

Godfrey N. Hounsfield, The Nobel Prize in Physiology or Medicine 1979
Double mandibular fracture with strong displacement to the left.

Solitary humeral bone cyst known as "fallen leaf sign"
Sources of Electromagnetic Radiation
– X-ray Emission Through Bremsstrahlung

• When fast electrons decelerated in the Coulomb field of a nucleus, part of its energy is converted into electromagnetic radiation in the form of Bremsstrahlung (a German word for “braking radiation”).
X-ray Generation – Bremmstrahlung

- Target nucleus positive charge \((Z\cdot p^+)\) attracts incident \(e^-\)
- Deceleration of an incident \(e^-\) occurs in the proximity of the target atom nucleus
- \(E\) lost by \(e^-\) is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the \(e^-\) determines the amount of \(E\) loss
  - The Coulomb force of attraction varies strongly with distance \((\propto 1/r^2)\); ↓ distance → ↑ deceleration and \(E\) loss → ↑ photon \(E\)
  - Direct impact on the nucleus determines the maximum x-ray \(E\) \((E_{max})\)
Interestingly, this process creates a relatively uniform spectrum.

Intensity

\[ \text{Intensity} = nh\nu \]

Photon energy spectrum

\( \varepsilon_0 \)
The Unfiltered Bremsstrahlung Spectrum
Thick Target X-ray Formation

We can model target as a series of thin targets. Electrons successively loses energy as they moves deeper into the target.

Each layer produces a flat energy spectrum with decreasing peak energy level.
X-ray Generation – Characteristic X-rays

Electron binding energy
X-ray Generation – Characteristic X-rays

• $e^-$ of the target atom have a binding energy (BE) that depends on atomic $Z$ (rem: $BE_K \propto Z^2$) and the shell ($BE_K > BE_L > BE_M > ...$)

• When $e^-(KE)$ incident on the target exceeds the target atom $e^-(BE)$, it’s energetically possible for a collisional interaction to eject the bound electron and ionize the atom.

• What would happen then?
X-ray Generation – Characteristic X-rays

• Within each shell (other than K) there are discrete E orbitals ($\ell = 0, 1, \ldots, n-1$) → characteristic x-ray fine E splitting

• Characteristic x-rays other than those generated through K-shell transitions are unimportant
Physical Designs Considerations

Typical micro-focus X-ray source

Rotating anode X-ray source

(Left) Liquid metal jet anode
http://www.excillum.com/technology/metal-jet-technology.html
X-ray Generation – X-ray Tube

Figure 5.3
An x-ray tube.

Figure 5.4
Schematic diagram of an x-ray tube.

Motor, Why?

Anode assembly

High voltage

Stator

Cathode assembly

Ground

Filament circuit

Electron beam? How are electrons generated?

Rotating target

Filament

X-rays
X-ray Generation – Characteristic X-rays

Superimposed multiple flat spectrum with decreasing cutoff energy

Low energy X-rays suffer attenuation inside the anode

Further attenuation by the source package.

External filtering to reduce low E photons \(\rightarrow\) lower does

Beam hardening

**Figure 5.5**
Relative intensity of x-ray photons. (Adapted from Webster, 1998. This material is used by permission of John Wiley & Sons, Inc.)
Example of Potential Problems Caused by the Imperfection of X-ray Sources – Beam Hardening

![CT scan images with and without beam hardening](image)

![Graph showing beam hardening effects](graph)

- **Bremstrahlung** (x-rays within anode)
- **Leaving anode**
- **Leaving tube**
- **After filter**
- **Leaving body**

Photon energy, keV | Relative intensity
--- | ---
0 | 1
20 | 0.8
40 | 0.6
60 | 0.4
80 | 0.2
100 | 0
120 | 0

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Example of Potential Problems Caused by the Imperfection of X-ray Sources – Beam Hardening

Beam hardening artifact occurs because the algorithm that reconstructs the image fails when faced with an interface between a high density structure such as a metallic prosthesis or thick bone and surrounding soft tissue. The 'spray' or 'streak' artifact results. It can be seen in the posterior fossa in most CT brains.
Sources of Electromagnetic Radiation
– Bremsstrahlung

- The bremsstrahlung spectrum can be altered by filtration.
- This is the underlying process for standard X-ray tubes.
Synchrotron Radiation
Sources of Electromagnetic Radiation
– Synchrotron Radiation

• Emitted when charged particles are accelerated (or decelerated) in a curved path or orbit.
• From visible light (a few eV) to X-ray energies (~$10^4$eV).
• Very intense !!!
Synchrotron Radiation

Bending magnet

Undulator magnet

X-ray Generation – Bremmstrahlung

- Target nucleus positive charge \((Z\cdot p^+)\) attracts incident \(e^-\)
- Deceleration of an incident \(e^-\) occurs in the proximity of the target atom nucleus
- \(E\) lost by \(e^-\) is gained by the EM photon (x-ray) generated
  - The impact parameter distance, the closest approach to the nucleus by the \(e^-\) determines the amount of \(E\) loss
  - The Coulomb force of attraction varies strongly with distance \((\propto 1/r^2); \downarrow\) distance \(\rightarrow\) \(\uparrow\) deceleration and \(E\) loss \(\rightarrow\) \(\uparrow\) photon \(E\)
  - Direct impact on the nucleus determines the maximum x-ray \(E\) (\(E_{\text{max}}\))
Sources of Electromagnetic Radiation
– Selection of X-rays of a Single Energy

A typical monochromator setup
Sources of Electromagnetic Radiation
– Synchrotron Radiation

European Synchrotron Radiation Facility

Advanced Photon Source (APS)

From http://www-hasylab.desy.de/science/groups/schneider_group/misc/RR-Lecture-1.pdf
• Stunning imaging resolution at the order down to a few tens of nm
• Reveal interesting physical properties of the object, X-ray fluorescence, diffraction–enhanced imaging