Materials selection for mechanical design

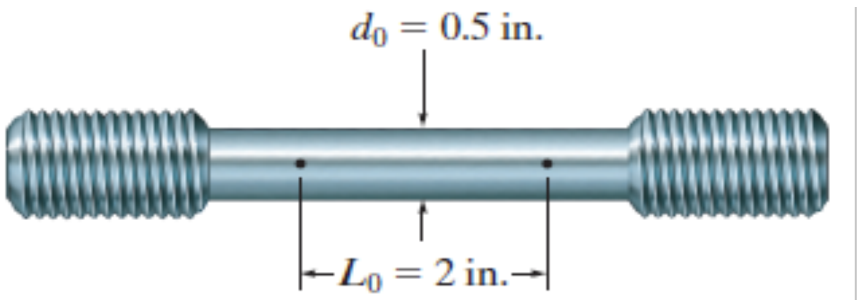
- Breaking down a design problem
- Identifying function, objectives, constraints
- Optimizing performance
- Ashby plots

Mechanical response of materials

- Mechanical response: how does a material respond to loads?
- **Elastic deformation**
  - strain: unitless change in dimension
  - stress: force per area
  - *elastic* deformation is *reversible* upon release of applied forces
- **Plastic deformation**
  - *permanent* deformation is what’s left over
  - response depends on *material history* (microstructure)
- **Fracture**
  - preexisting cracks/flaws in material, propagate under applied stress
  - catastrophic failure of material
- **Creep**
  - time-dependent *permanent* deformation to time-independent load
  - thermally activated response
- **Fatigue**
  - fracture response to time-dependent cyclic loading
  - cycling promotes crack nucleation and growth; catastrophic failure

Stress-strain diagram

- Uniaxial tension test:
  - Stress: what we do to the material
  - Strain: how the material responds
- Quantify:
  - Initial gauge length L_0 and area A_0; instantaneous length L and area A

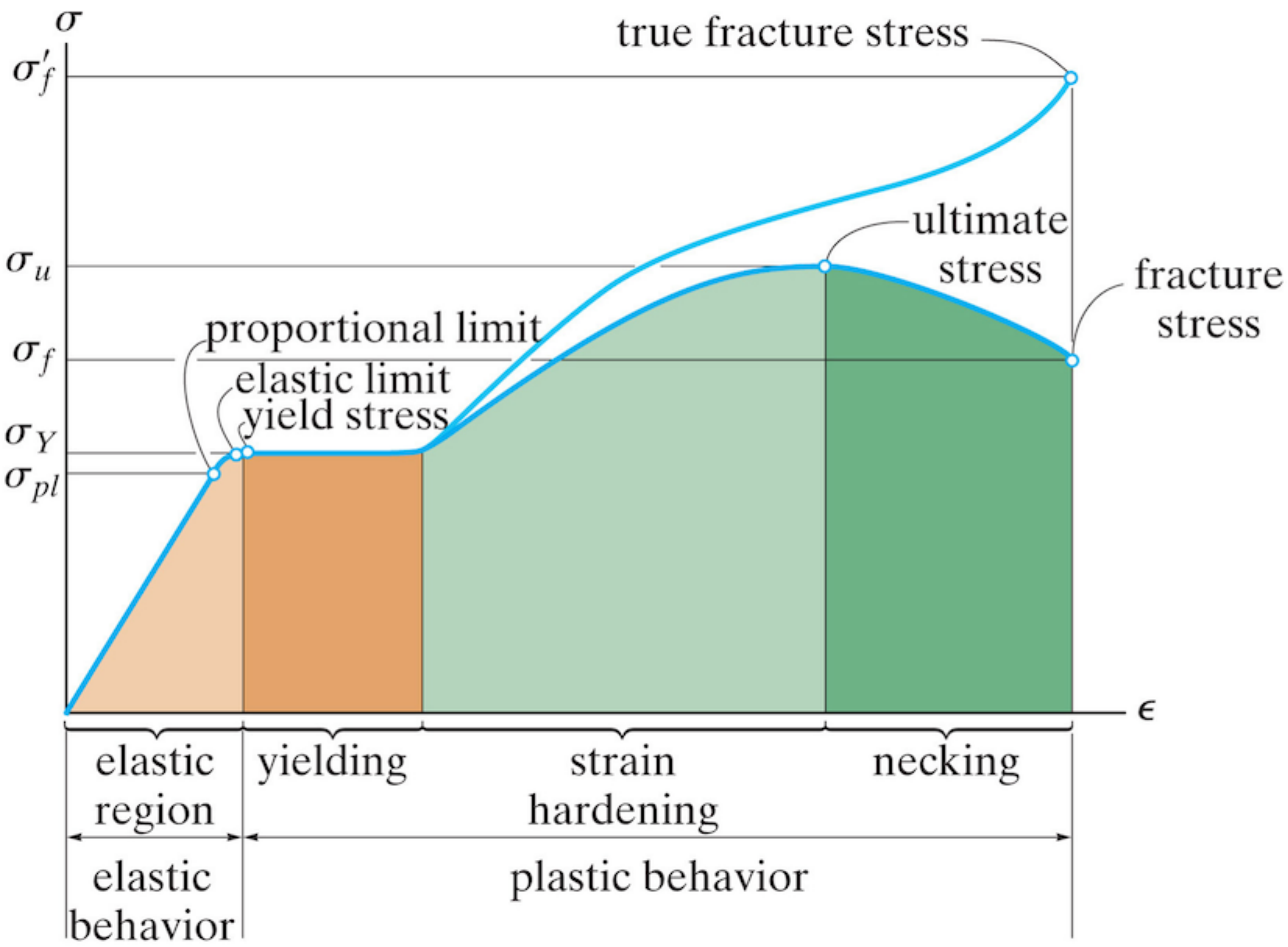


Engineering stress

$$\sigma_{\mathrm{E}} = \frac{P}{A_0}$$

True stress

$$\sigma_{\mathrm{T}} = \frac{P}{A}$$



schematic steel stress-strain curve
(not to scale)

Engineering strain

$$\epsilon_{\mathrm{E}} = \frac{\delta}{L_0} = \frac{L - L_0}{L_0}$$

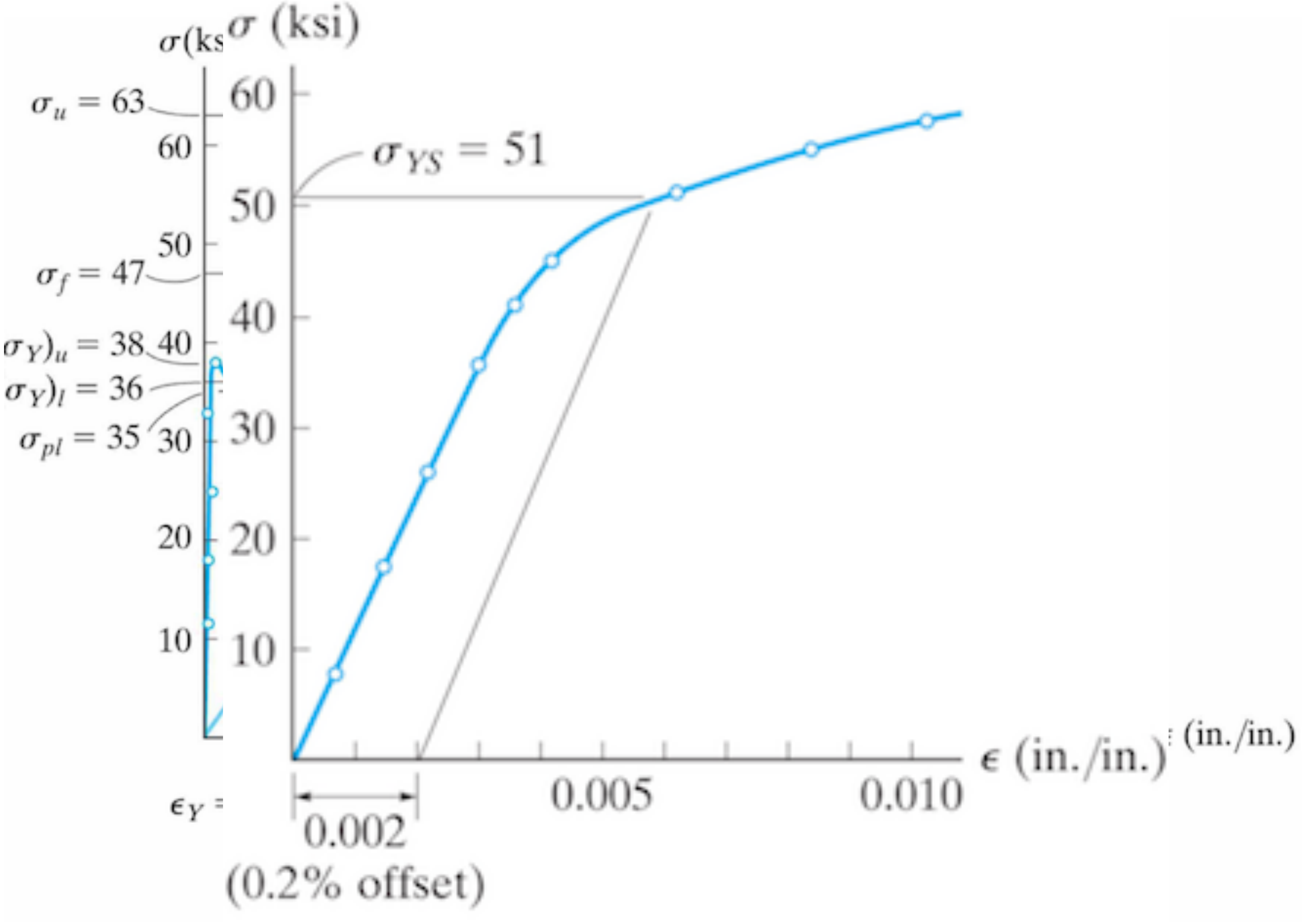
True strain

$$\epsilon_{\mathrm{T}} = \ln \frac{L}{L_0} = \ln \frac{A_0}{A} = \ln(1 + \epsilon_{\mathrm{E}})$$

Stress-strain diagram: features

- Engineering vs. true quantities
  - **Engineering:** loads and deformation from *initial geometry*
  - **True:** what the material experiences (stress), accumulated/additive dimension change (strain: 10% true + 10% true = 20% true)
- **Yield strength:** highest stress that the material can withstand without undergoing significant plastic (irreversible) deformation
  - May be defined by a **yield point** (rapid drop in stress at yield)
  - May be defined as 0.2% offset (stress to get 0.2% plastic strain)
- **Ultimate strength:** is the maximum value of stress (engineering stress) that the material can withstand
- **Fracture stress:** the value of stress at fracture
- **Stiffness:** ratio of stress to strain, primarily of interest in the elastic region. (elastic moduli)
- **Ductility:** Materials that undergo large strain before fracture are classified as ductile materials. Necks before failure
- Percent elongation: $100(L_f - L_0)/L_0$
- Percent reduction in area: $100(A_0 - A_f)/A_0$

Stress-strain diagram: ductile materials



- Rupture occurs along a cone-shaped surface that forms an angle of approximately 45° with the original surface of the specimen ("cup-cone" shape)

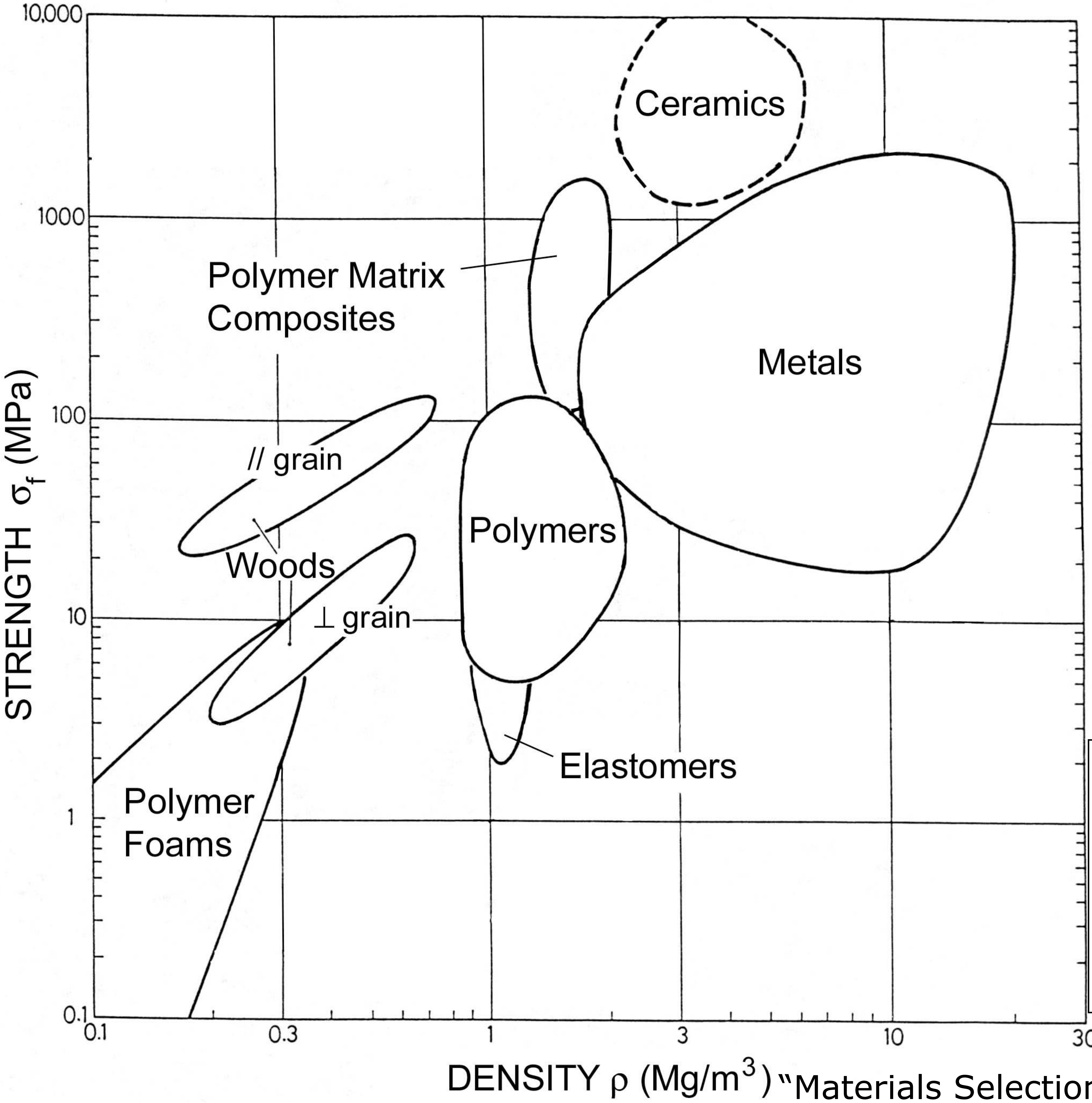
<u>Shear is primarily responsible for failure in ductile materials</u>

Axial loading: maximum shear stress occurs at 45°

Necking

Rupture

Ashby plots



Materials selection for mechanical design:

choose best material among competing properties

Our goal: understanding mechanisms responsible for behavior

M. Ashby
"Materials Selection in Mechanical Design"

Materials selection

Design concerns

- **function** - what a component does
- **constraints** - what must/must not occur
- **objective** - what is maximized/minimized

Example: tie-rod stretches to carry load, must not yield, and be lightweight

function **constraint** **objective**

Rank different designs **performance** as a function P(F, G, M):

- **functional needs** (F)
- **geometry** (G)
- **material properties** (M)

We *assume* a separable form: P(F, G, M) = $P_F(F)\,P_G(G)\,P_M(M)$ so that material choice can be optimized independent of design specifics, with flexibility

Ex: maximum pressure in cylindrical vessel to leak, but not fracture:

$$p_{\max} = \frac{2}{\alpha\pi r} \cdot \left(\frac{K_{\mathrm{Ic}}^2}{\sigma_{\mathrm{YS}}}\right)$$

$P_G(G)$ $P_M(M)$

The goal: **optimize performance**

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Materials properties

Material properties

- **mechanical:** modulus, yield stress, fracture toughness, ...
- **transport/thermal:** heat capacity, thermal expansion, resistivity
- **economic:** density, cost/mass
  - price includes: cost to extract, cost/energy to process, cost/energy to form, cost of disposal, regulation cost

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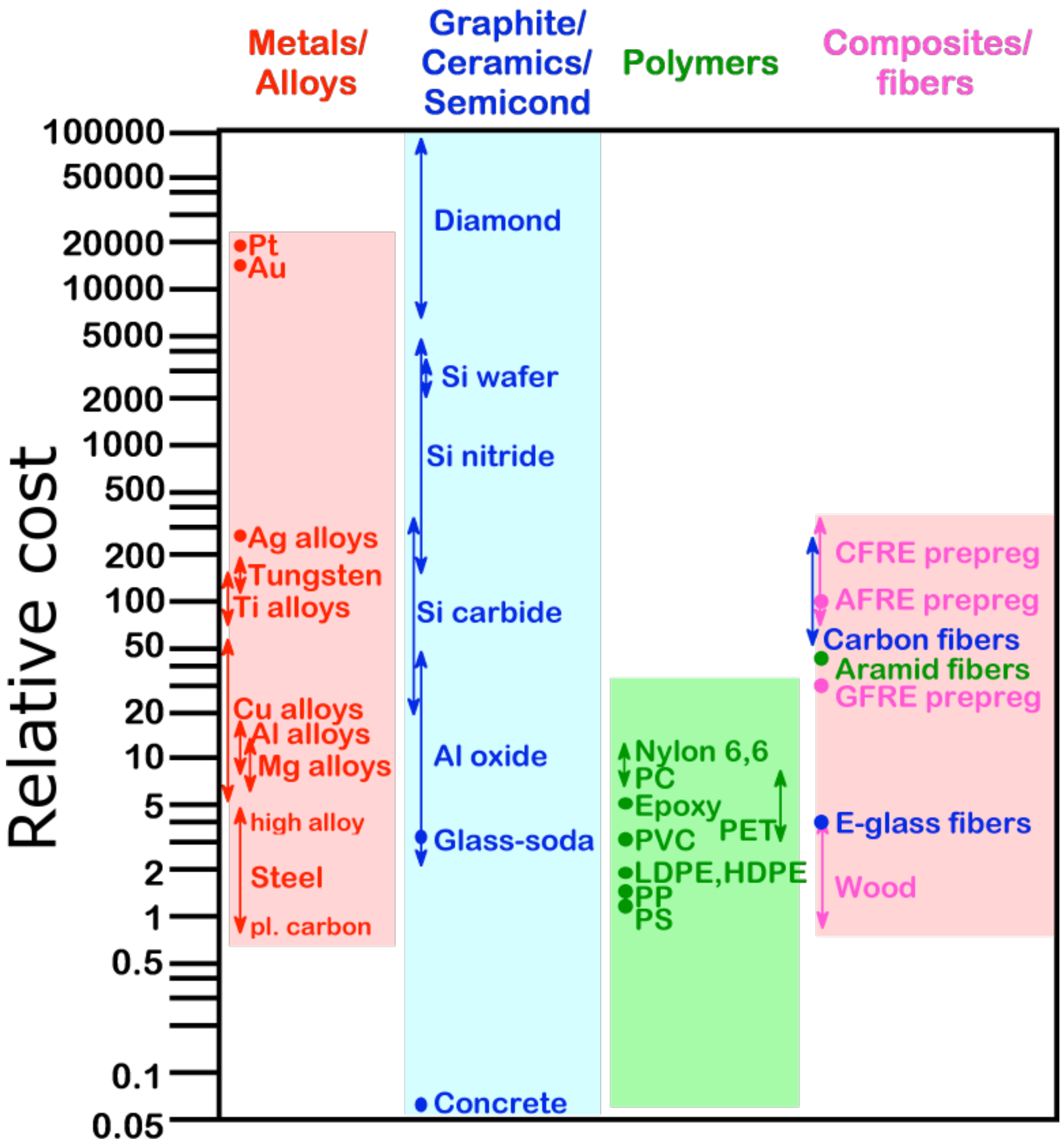
Relative material cost/mass



Reference material: rolled A36 carbon steel

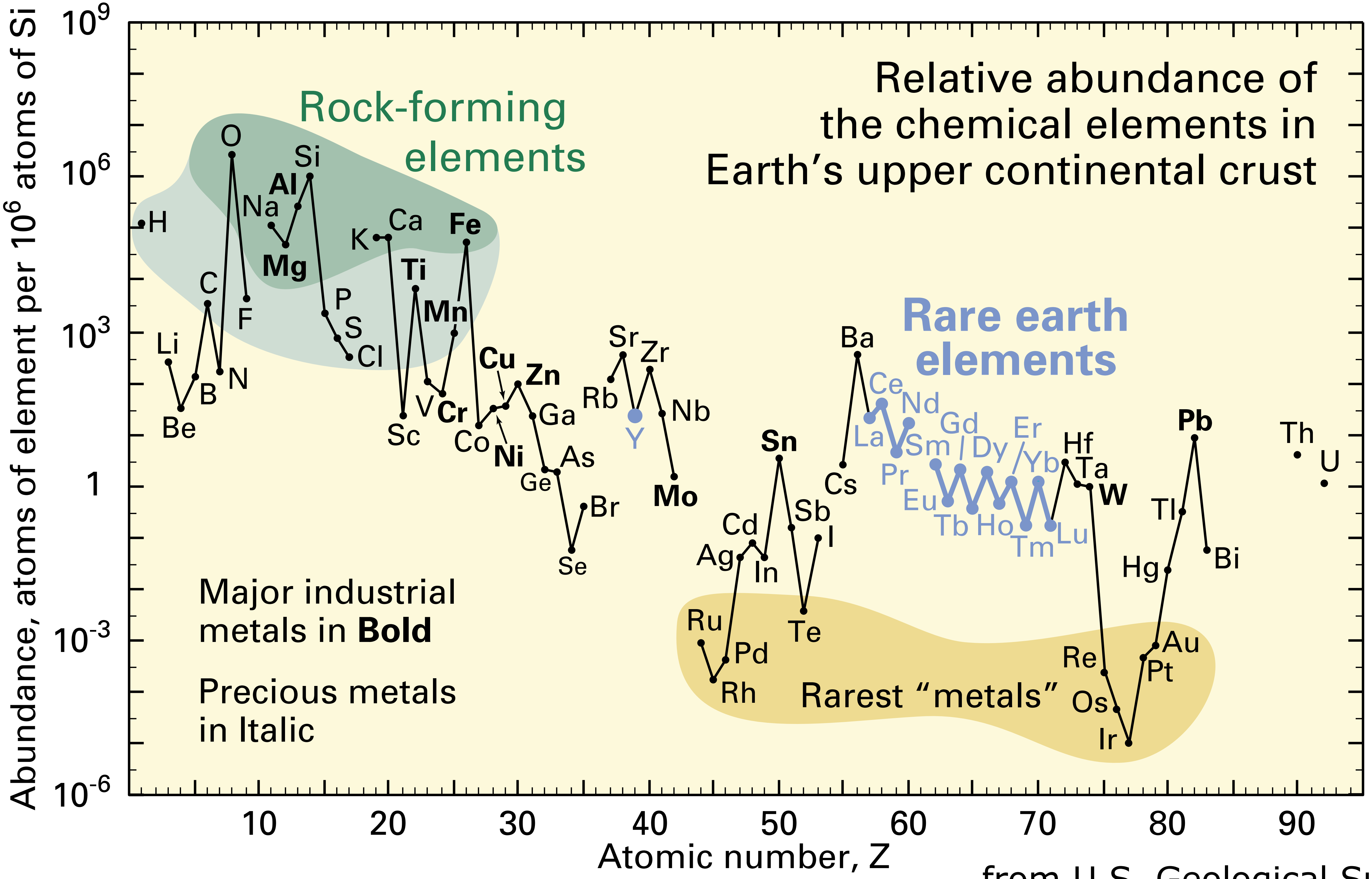
Relative costs fluctuate less than actual costs over time.

Cost includes both extraction and processing, but not production.

Based on Callister, 6th ed.

Relative (raw) abundances

Nucleosynthesis (stellar fusion processes) determines what "raw" materials we have available.



from U.S. Geological Survey

Materials properties and selection

Material properties

mechanical: modulus, yield stress, fracture toughness, ...

transport/thermal: heat capacity, thermal expansion, resistivity

economic: density, cost/mass

price includes: cost to extract, cost/energy to process, cost/energy to form, cost of disposal, regulation cost

Materials selection involves

1. determining combination of properties to maximize (function, constraint, and objective)
2. selecting material/material class to fill that need

We do selection via an “Ashby plot”: log-log plot of two material properties

Why log-log?

$$P_{\mathrm{M}}(M) = M_1^{\alpha_1} \cdot M_2^{\alpha_2} \cdots$$

$$\log P_{\mathrm{M}}(M) = \alpha_1 \log M_1 + \alpha_2 \log M_2 + \cdots$$

Constant (equal) performance is a **straight line** on an Ashby plot

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Light, strong tie-rod

A **tie-rod** carries load along its length. We want it to carry the load without yielding, and if it's in a vehicle, we want a low mass.

First: what is the *function*?

A. low mass
B. low area
C. carry load
D. long length
E. not yield

Light, strong tie-rod

A **tie-rod** carries load along its length. We want it to carry the load without yielding, and if it’s in a vehicle, we want a low mass.

Next: what is the *objective*?

A. low mass
B. low area
C. carry load
D. long length
E. not yield

Light, strong tie-rod

A **tie-rod** carries load along its length. We want it to carry the load without yielding, and if it's in a vehicle, we want a low mass.

Finally: what is the *constraint*?

A. low mass
B. low area
C. carry load
D. long length
E. not yield

Light, strong tie-rod

A **tie-rod** carries load along its length:

- Functional needs: carry load **F**
- Geometry: length L, area A
- Constants? **F**, L
- Variables? area A, material
- Constraint? stress below yield stress
- Performance? mass m

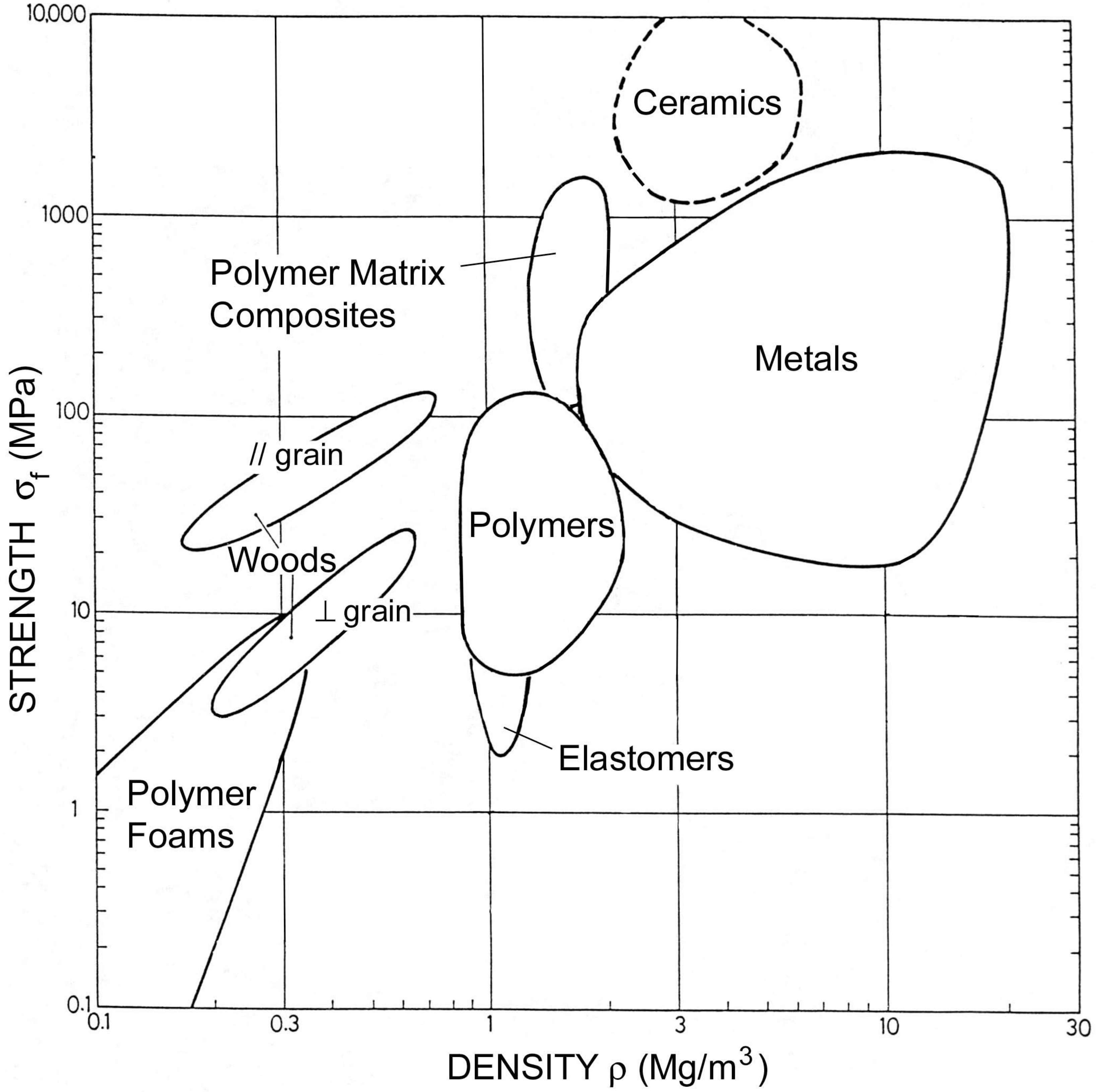
Light, strong tie-rod

A **tie-rod** carries load along its length:

- Functional needs: carry load **F**
- Geometry: length *L*, area *A*
- Constants? **F**, *L*
- Variables? area *A*, material
- Constraint? stress below yield stress
- Performance? mass *m*

$$m = SLF\frac{\rho}{\sigma_{\mathrm{YS}}}$$

Yield strength vs. density



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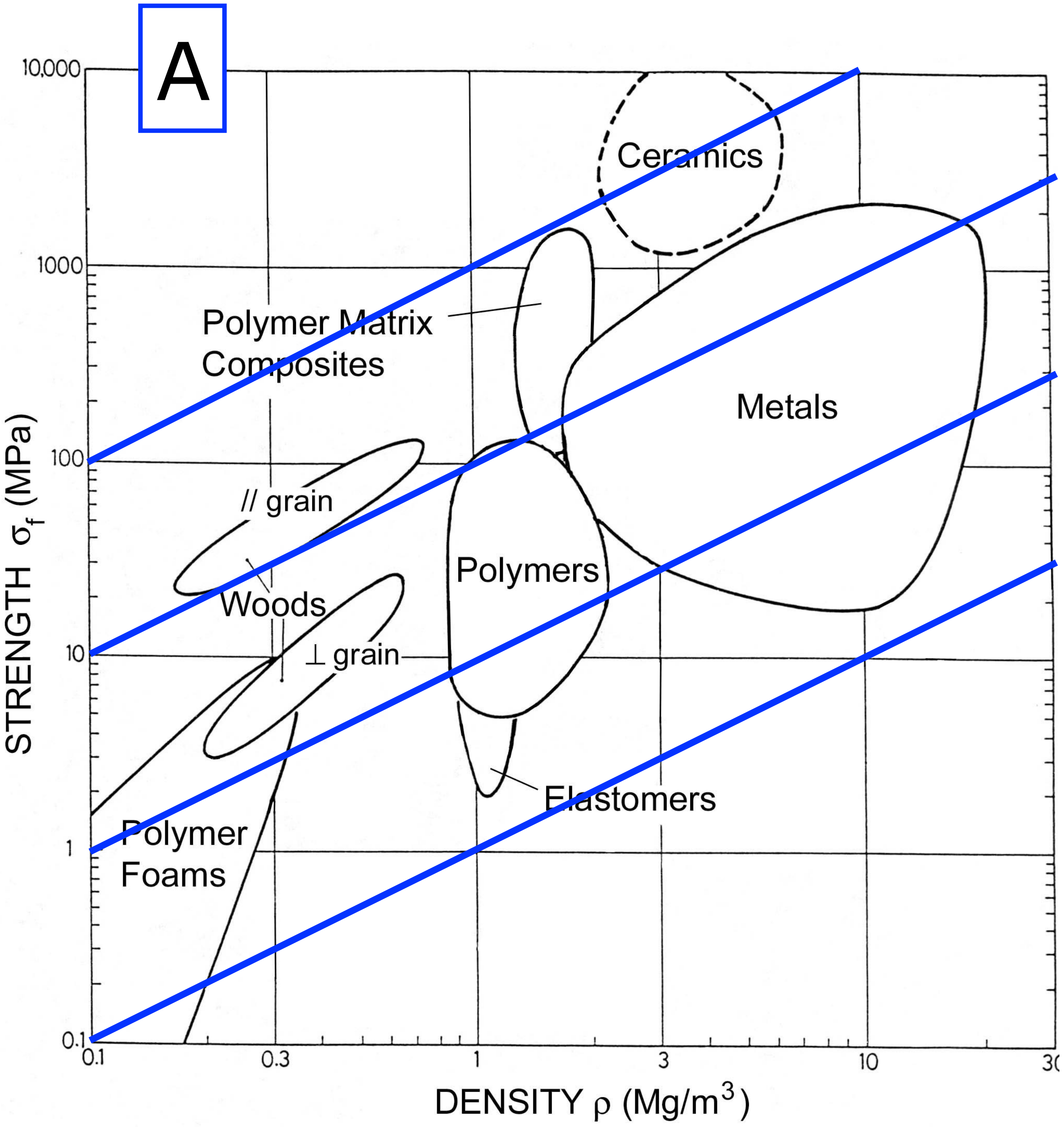
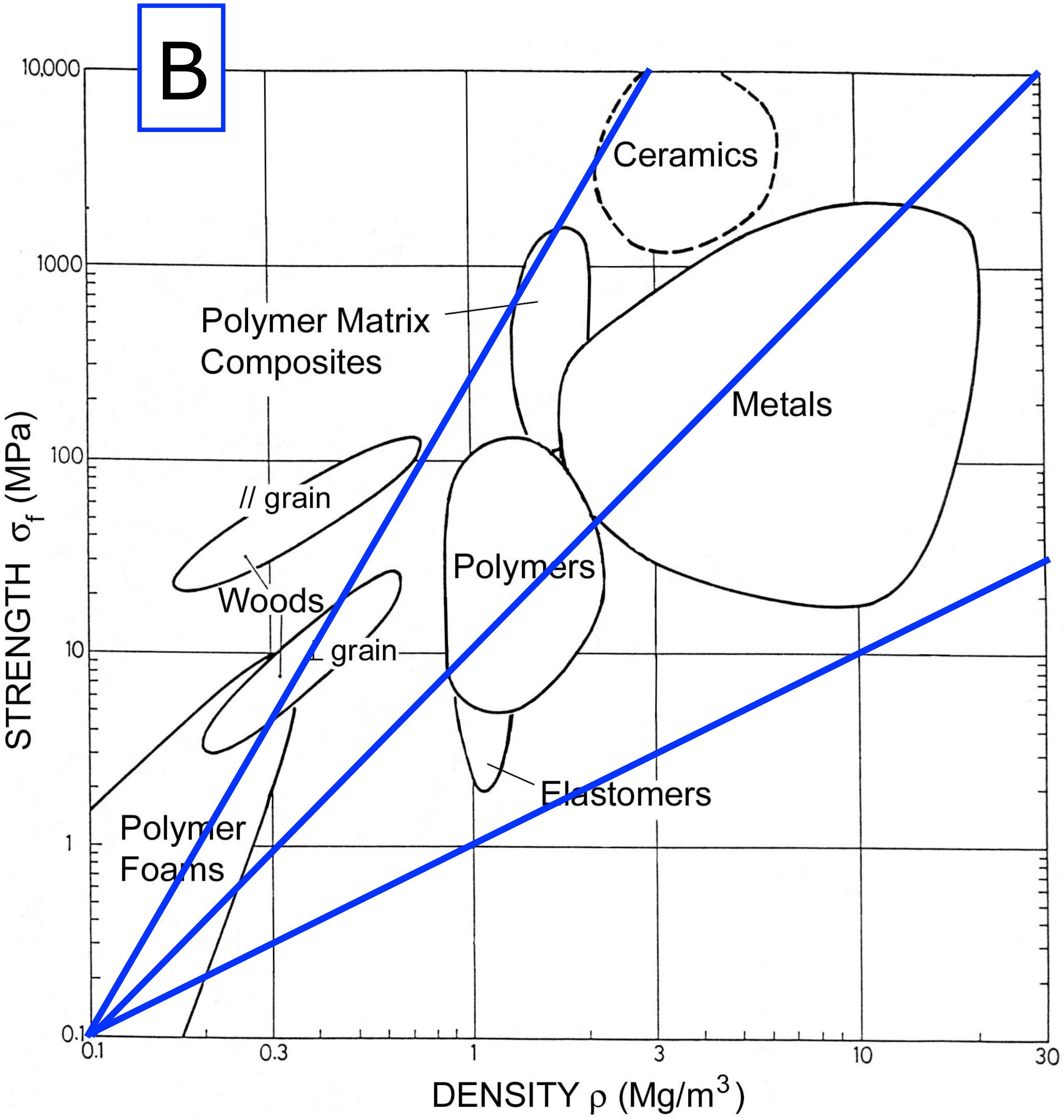
Yield strength vs. density

lines of constant performance?





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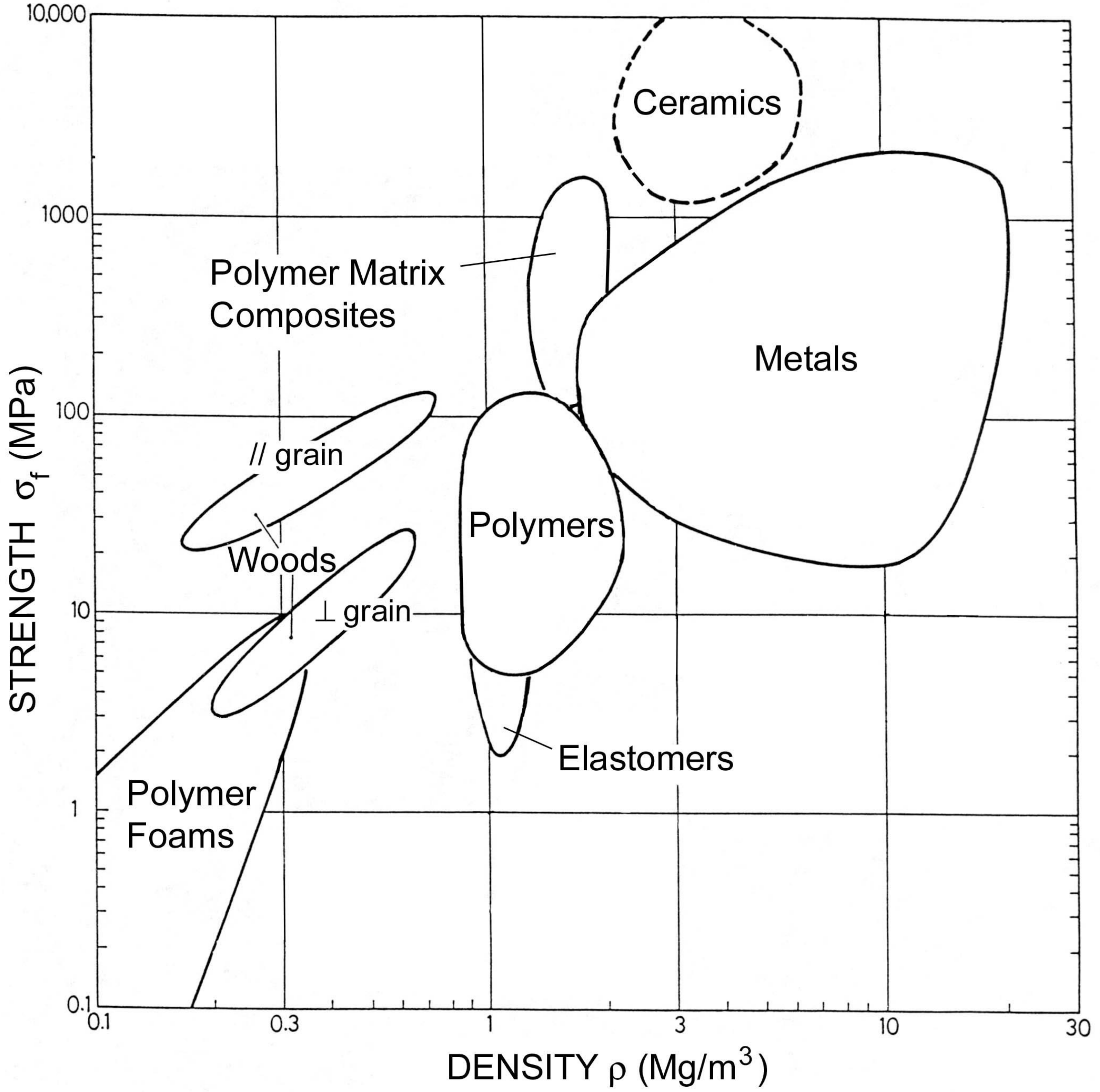
Yield strength vs. density



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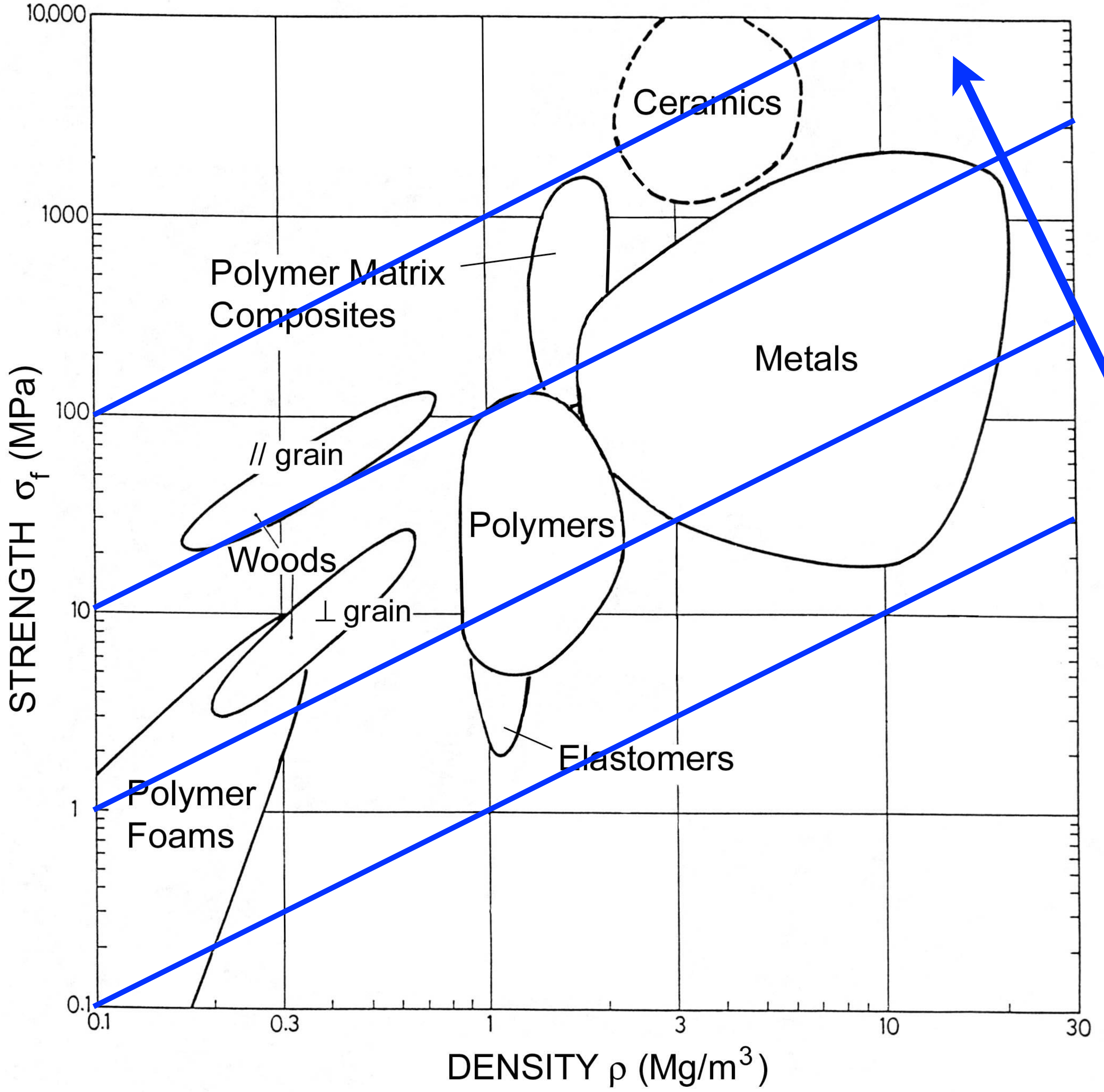
Yield strength vs. density



increasing YS/density

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Light, stiff tie-rod

A **tie-rod** carries load along its length. We want it to carry the load without extending more than length δ, and if it’s in a vehicle, we want a low mass.

- Functional needs: carry load **F**
- Geometry: length L, area A
- Constants? **F**, L
- Variables? area A, material
- Constraint? extension below δ
- Performance? mass m

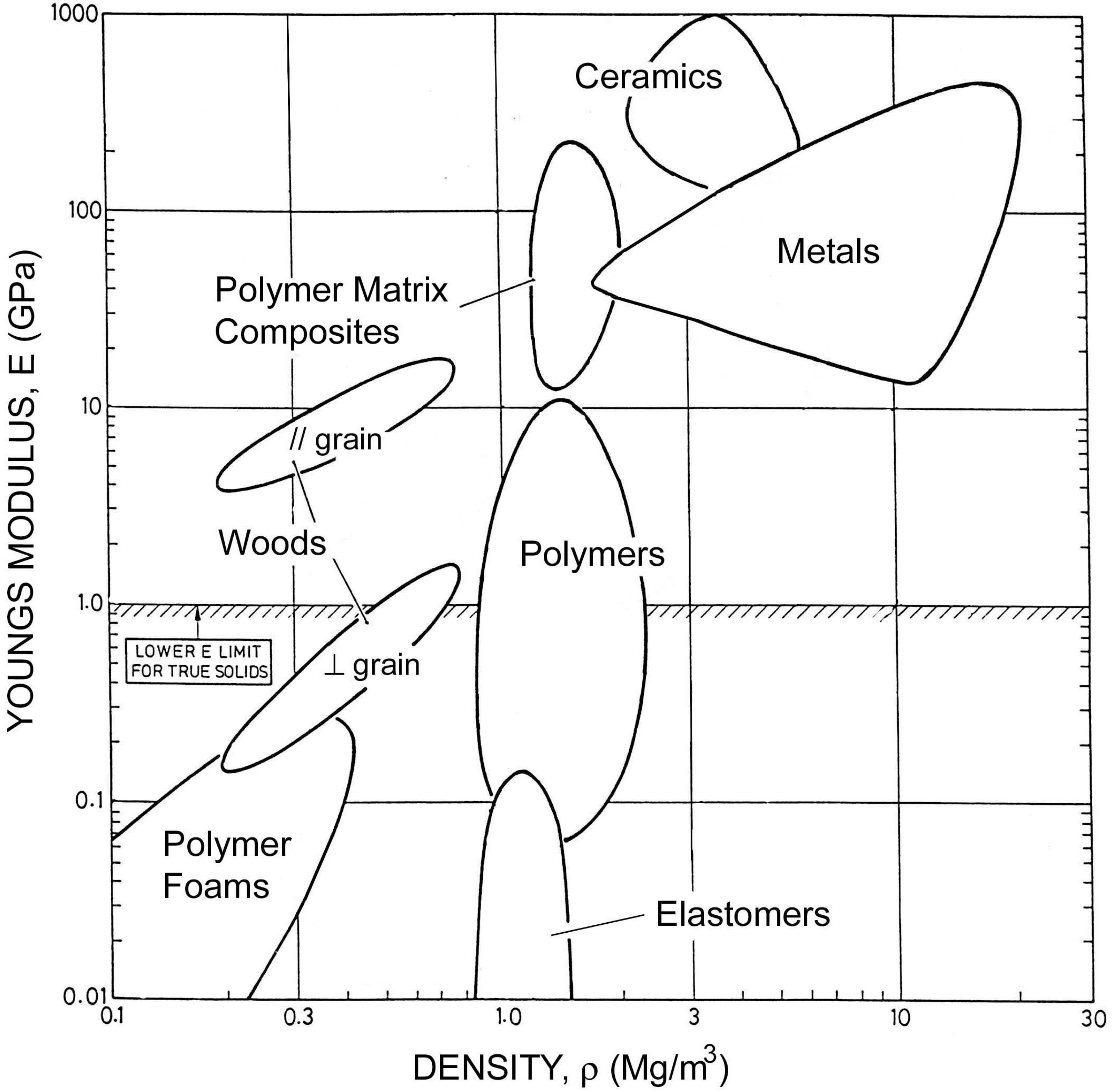
Light, stiff tie-rod

A **tie-rod** carries load along its length. We want it to carry the load without extending more than length δ, and if it's in a vehicle, we want a low mass.

- Functional needs: carry load **F**
- Geometry: length L, area A
- Constants? **F**, L
- Variables? area A, material
- Constraint? extension below δ
- Performance? mass m

$$m = \frac{SFL^2}{\delta}\frac{\rho}{E}$$

Young’s modulus vs. density



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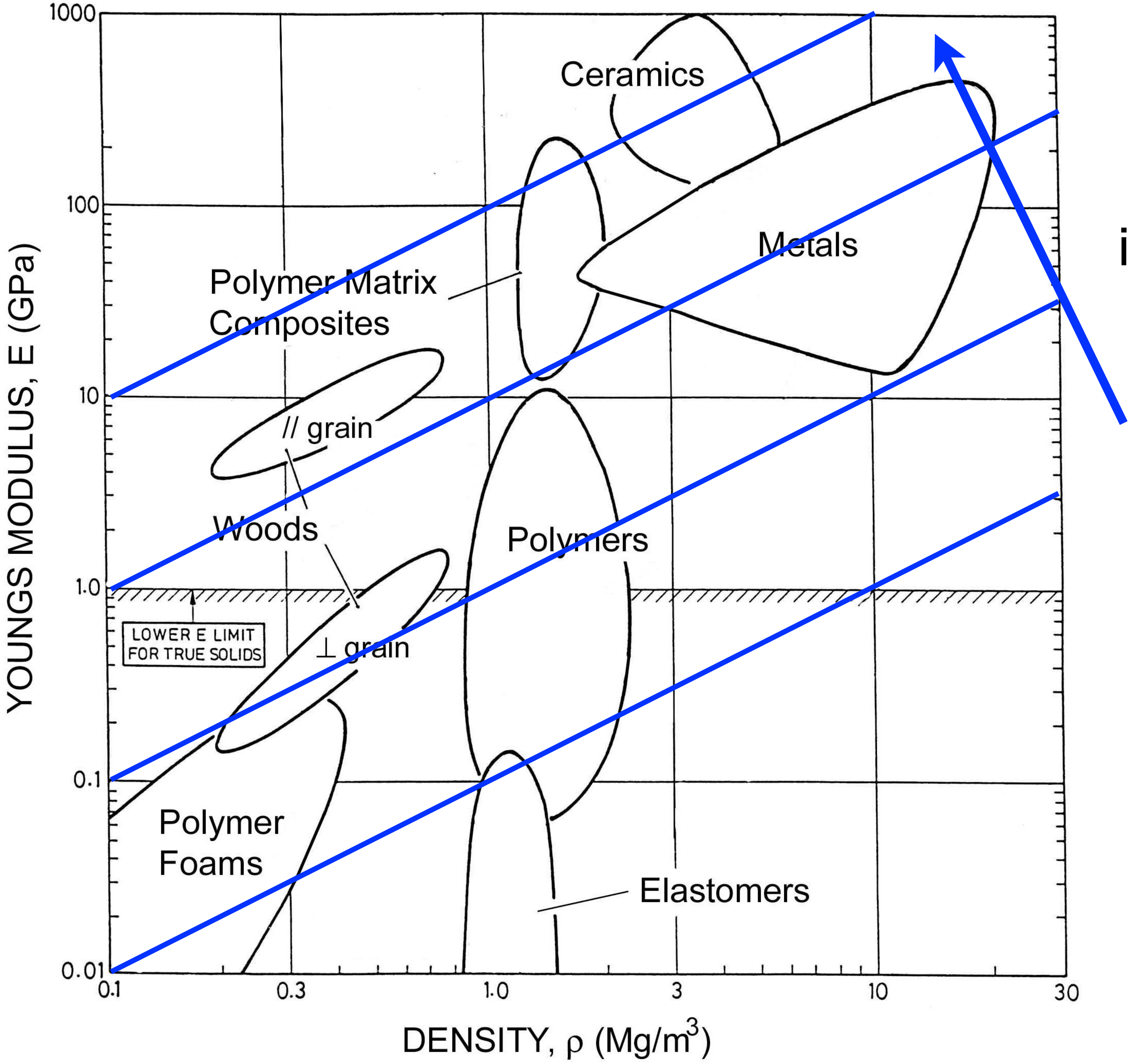
Young’s modulus vs. density



increasing E/density

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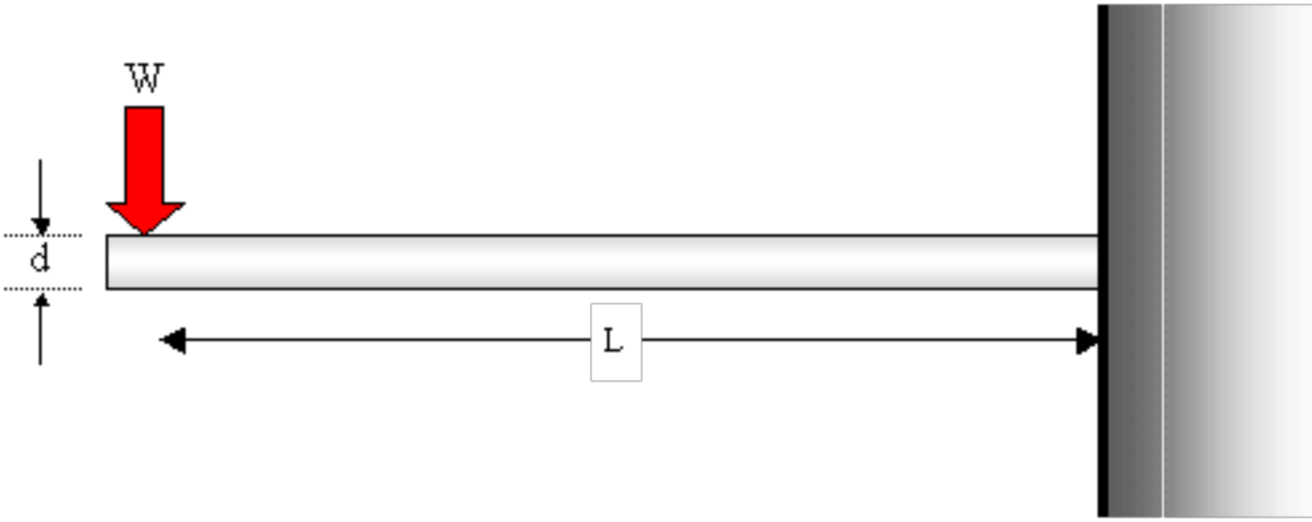
Light, strong cantilever

A **cantilever** is fixed at one end, and carries load perpendicular to its length. We want it to carry the load without yielding, we want a low mass.

- Functional needs: carry load **W**
- Geometry: length *L*, diameter *d*
- Constants? **W**, *L*
- Variables? diameter *d*, material
- Constraint? stress below yield
- Performance? mass *m*



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- Functional needs: carry load **W**
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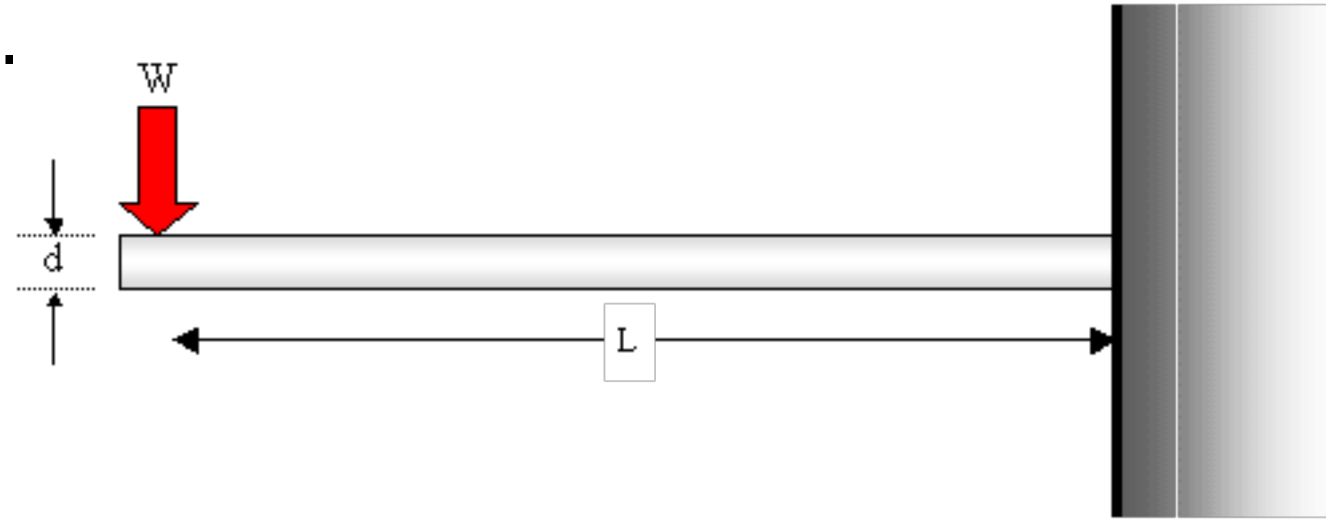
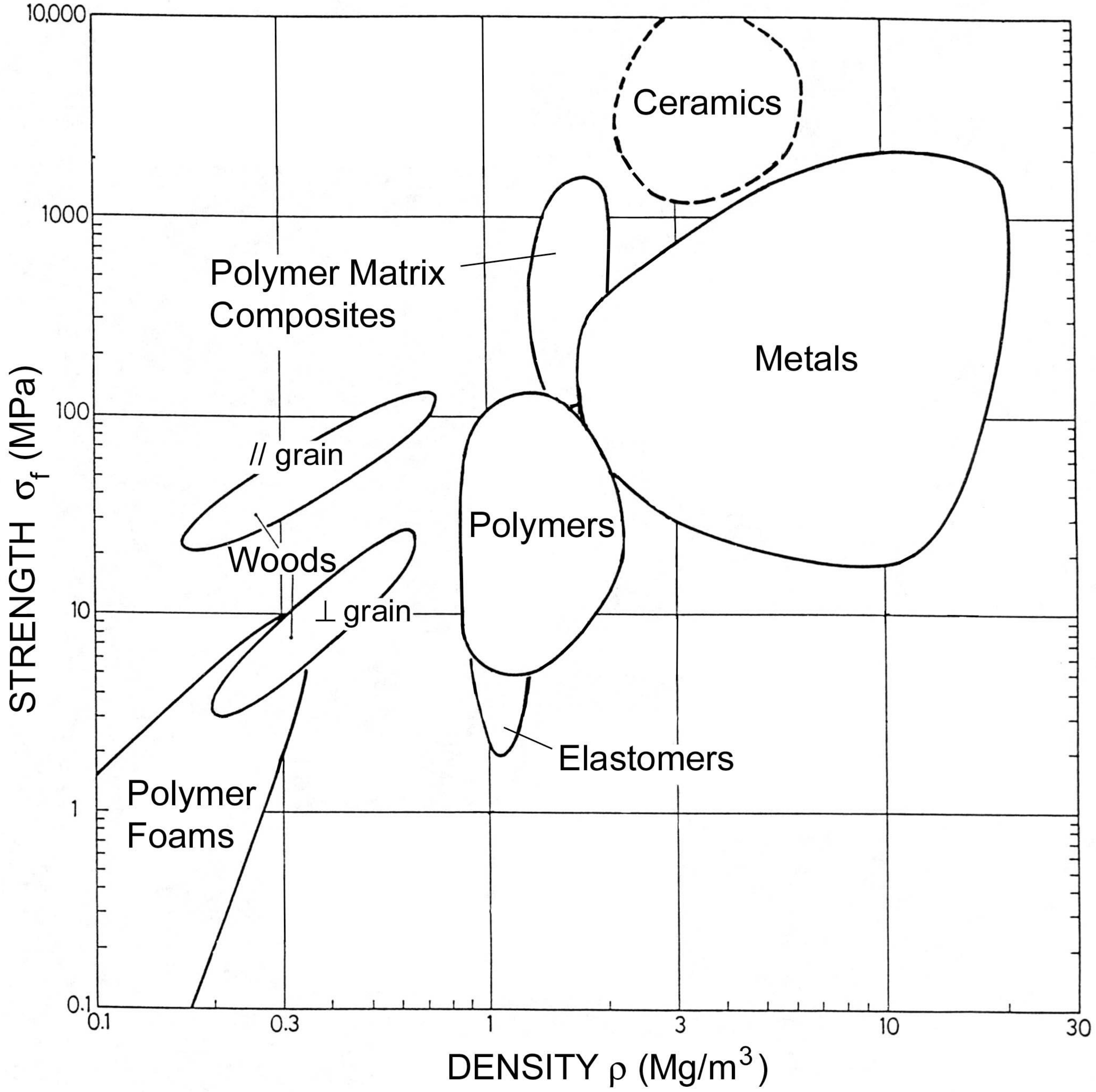


$$m \propto \frac{\rho}{\sigma_{\mathrm{YS}}^{2/3}}$$

Yield strength vs. density



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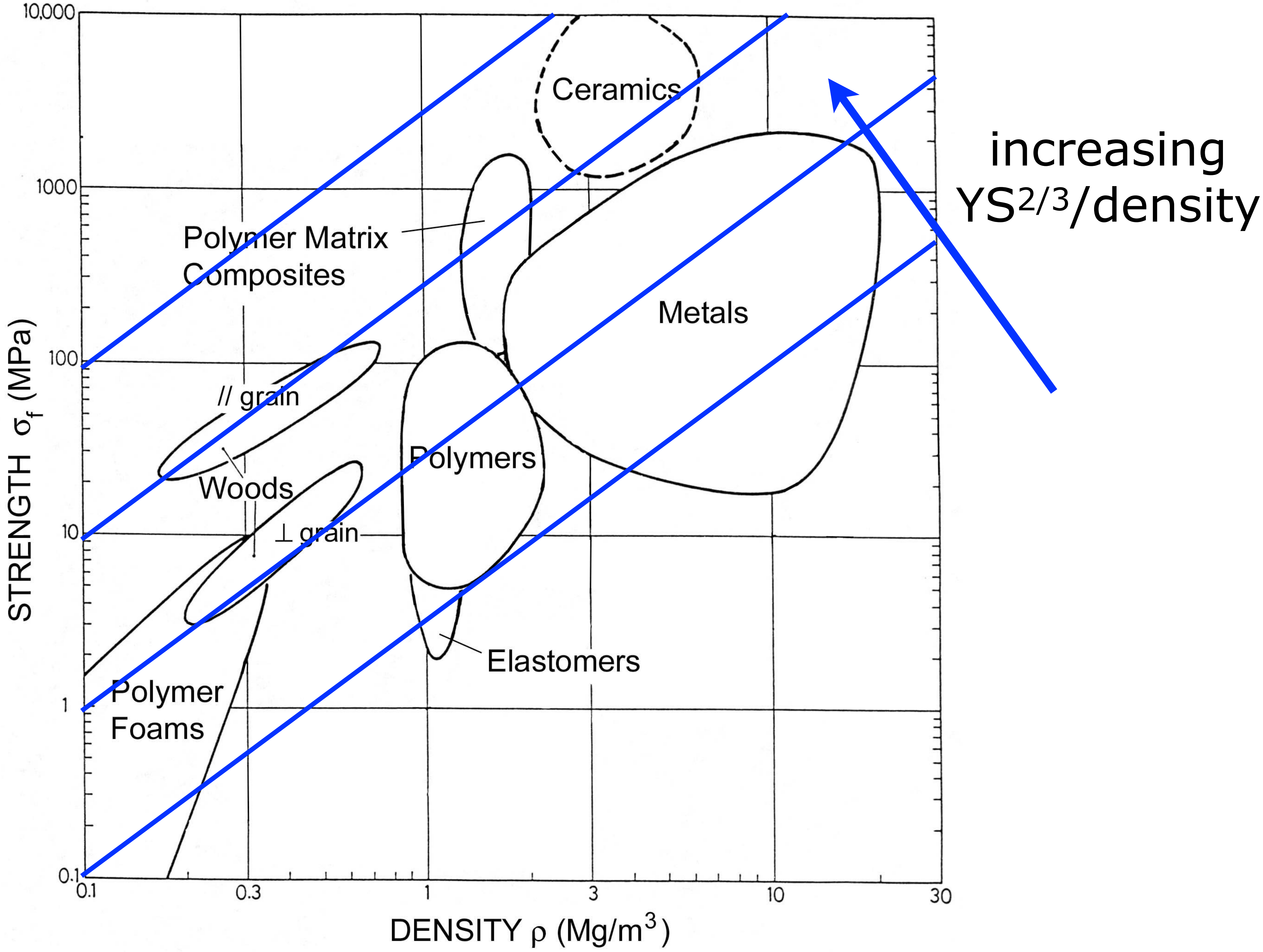
Yield strength vs. density



increasing YS$^{2/3}$/density

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What about cost?

Material properties

- **mechanical:** modulus, yield stress, fracture toughness, …
- **transport/thermal:** heat capacity, thermal expansion, resistivity
- **economic:** density, cost/mass
  - price includes: cost to extract, cost/energy to process, cost/energy to form, cost of disposal, regulation cost

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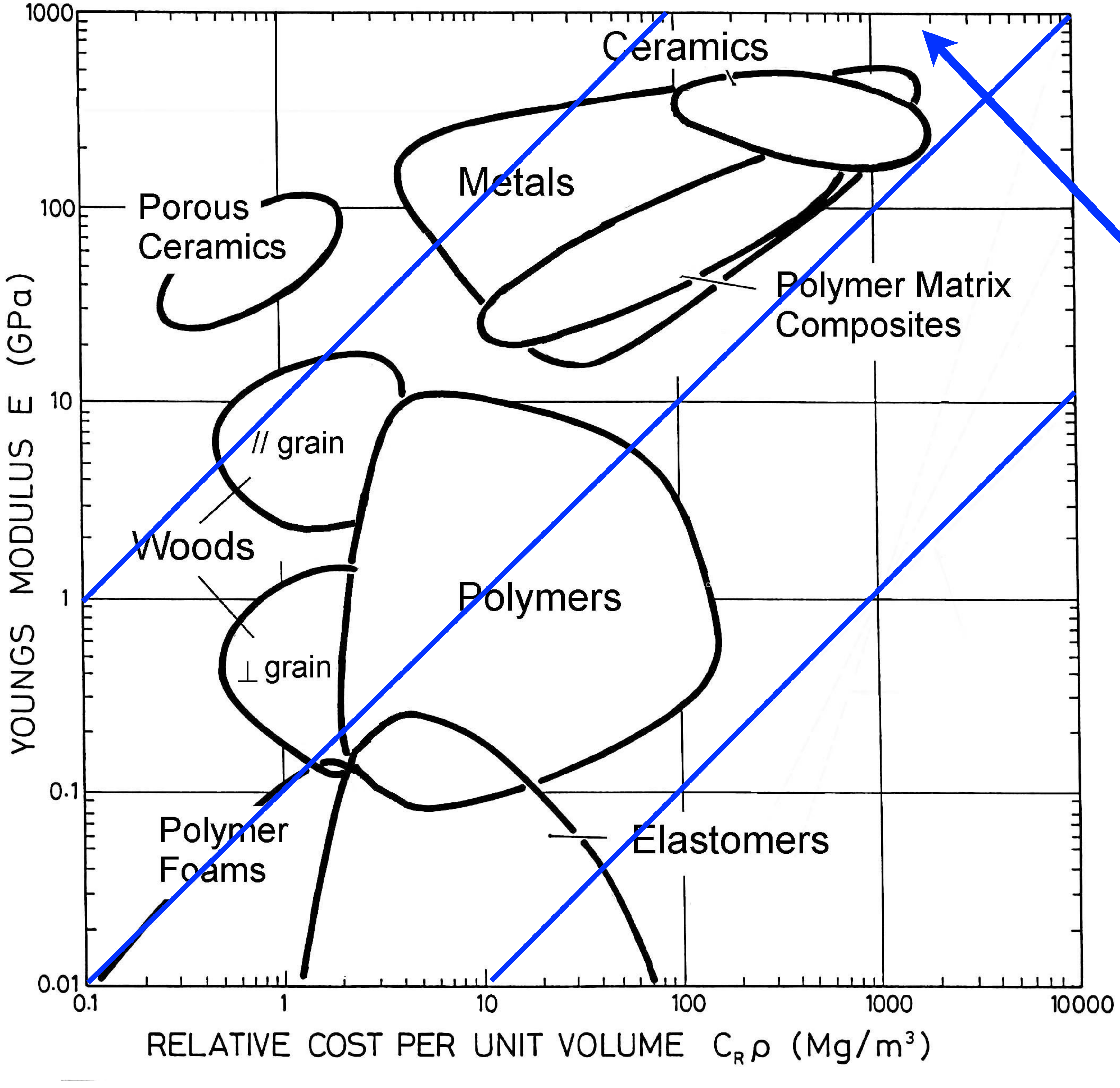
Young’s modulus vs. cost



increasing E/cost

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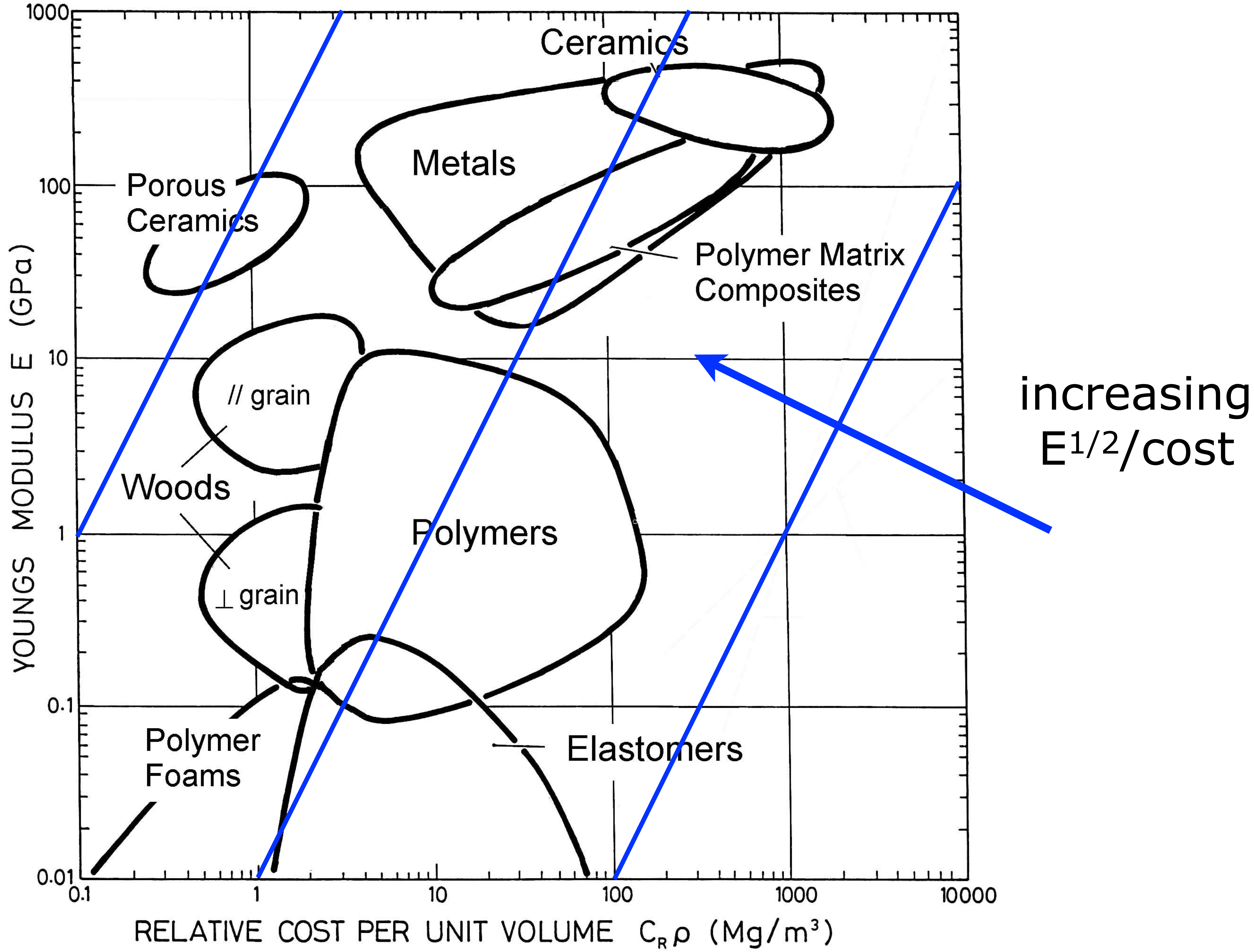
Young's modulus vs. cost



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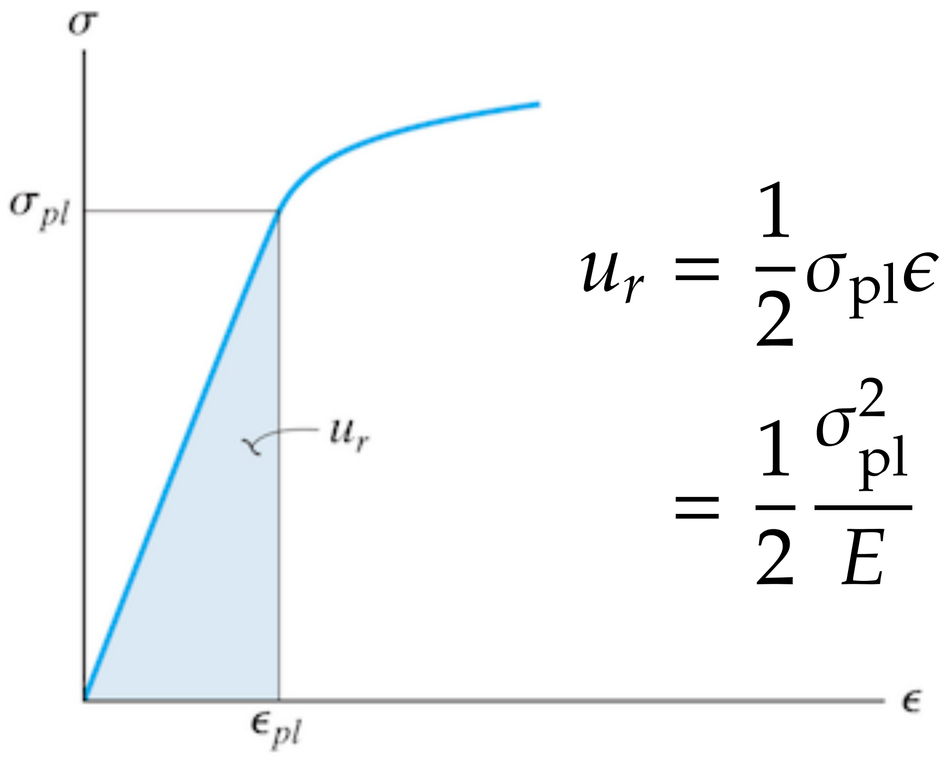
Strain energy density

- External load does **work** on a body: change in internal energy
- Work = integral of force × distance
  - Force = (stress) × (area)
  - Distance = (strain) × (length)
- stress × strain → energy/volume
- If the deformation is **recoverable** then so is the energy.

$$W = \int \mathbf{F} \cdot \mathbf{dr}$$

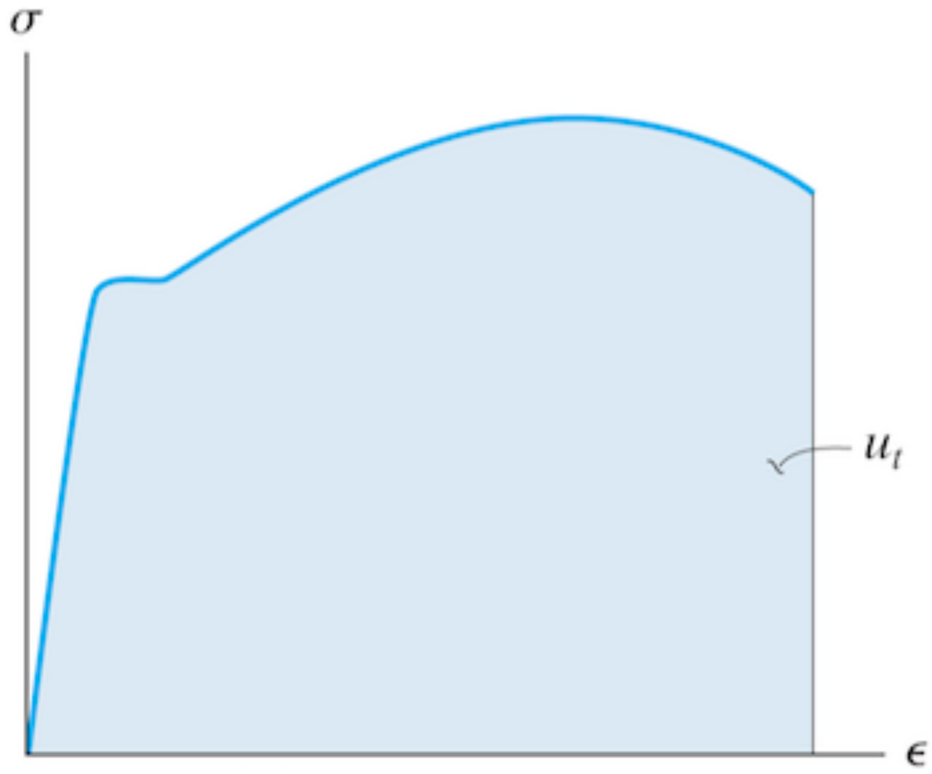
$$= \int (\sigma A_0)(L_0 \, d\epsilon)$$

$$= V_0 \int \sigma \, d\epsilon$$



$$u_r = \frac{1}{2}\sigma_{\rm pl}\epsilon_{\rm pl}$$

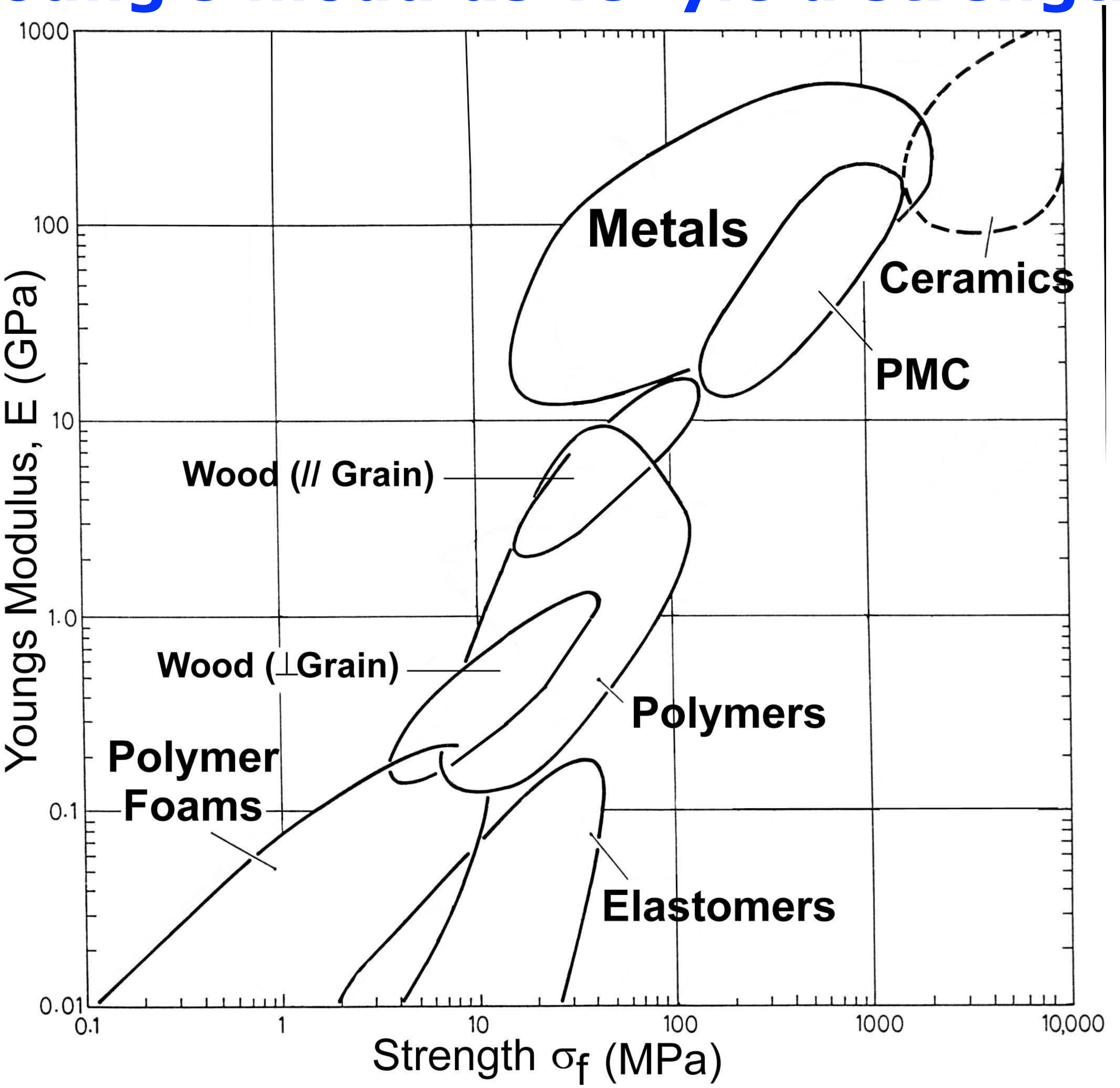
$$= \frac{1}{2}\frac{\sigma_{\rm pl}^2}{E}$$



Elastic energy storage density: **modulus of resilience**

Total strain energy density from fracture: **modulus of toughness**

Young’s modulus vs. yield strength



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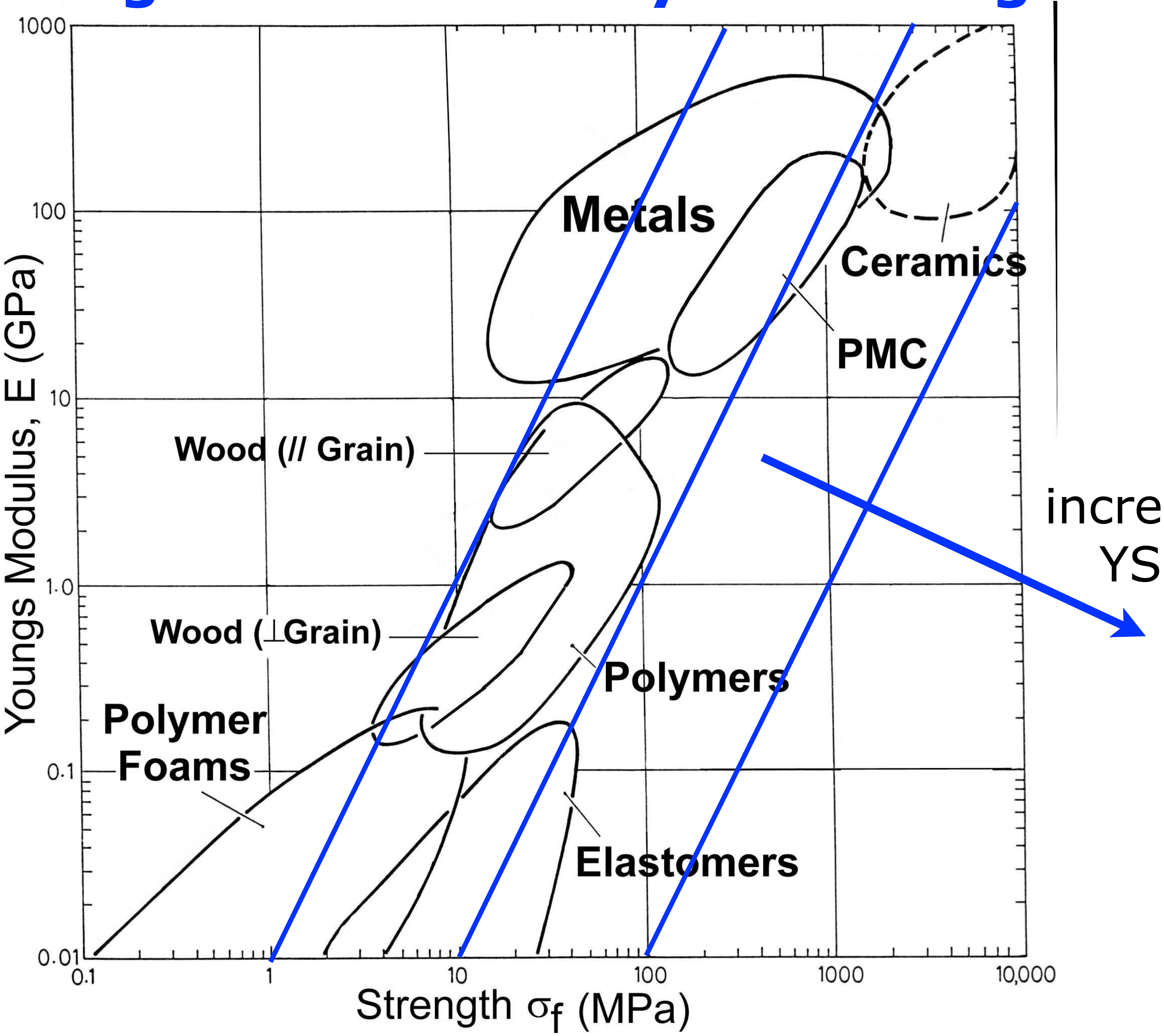
Young’s modulus vs. yield strength



increasing YS²/E

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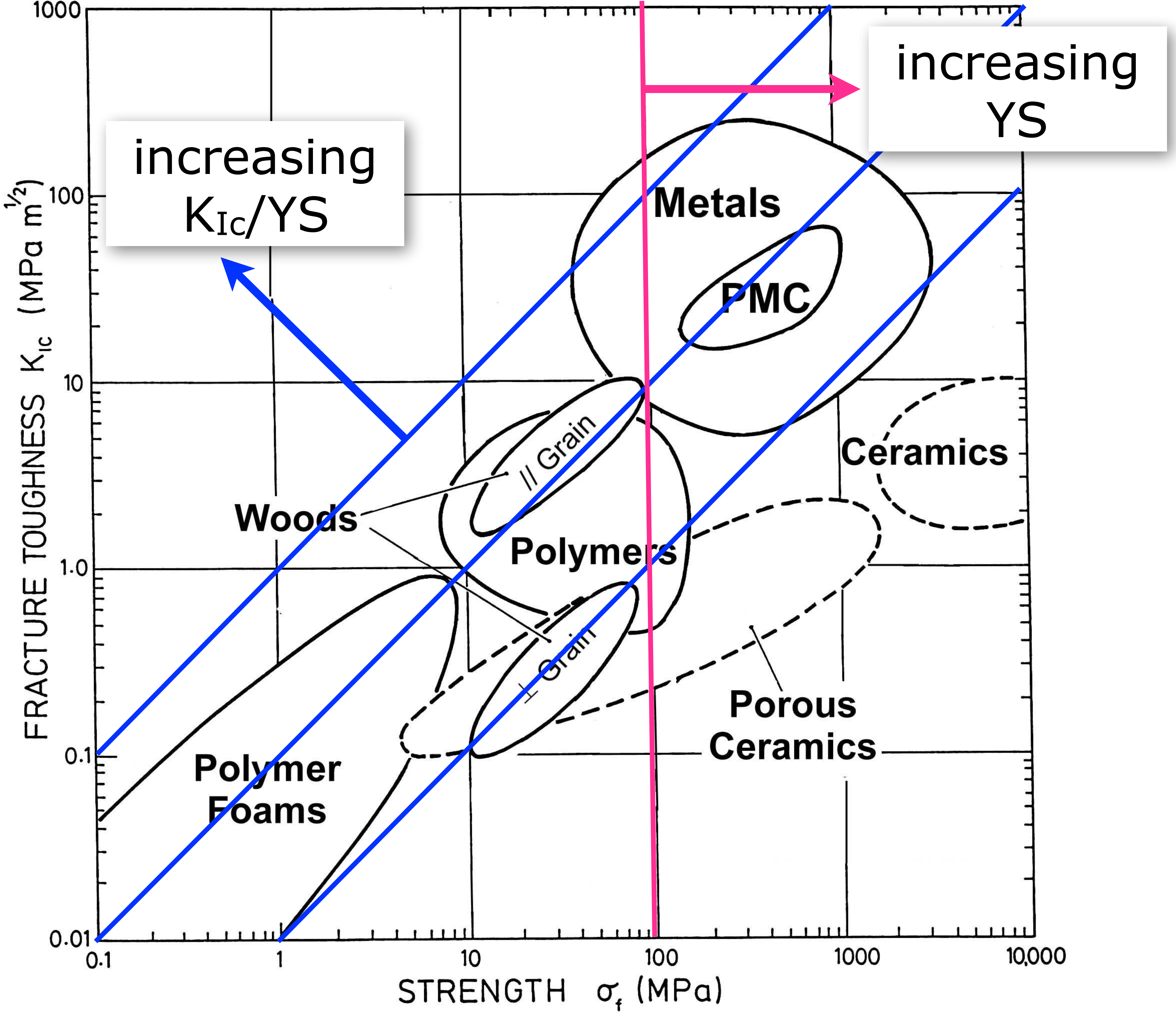
Pressure tanks

A **pressure tank** holds a fluid at pressure, and carries load perpendicular to its thickness. We want it to carry the load to yield before fracture, and so that the critical flaw size is larger than the thickness (leak before fracture).

- Functional needs: hold pressure p
- Geometry: wall thickness *t*, diameter *d*
- Constants? *d*
- Variables? thickness *t*, material
- Constraint? stress below yield, flaw size above *t*
- Performance? pressure *p*

Fracture toughness vs. yield strength



increasing K_{Ic}/YS

increasing YS

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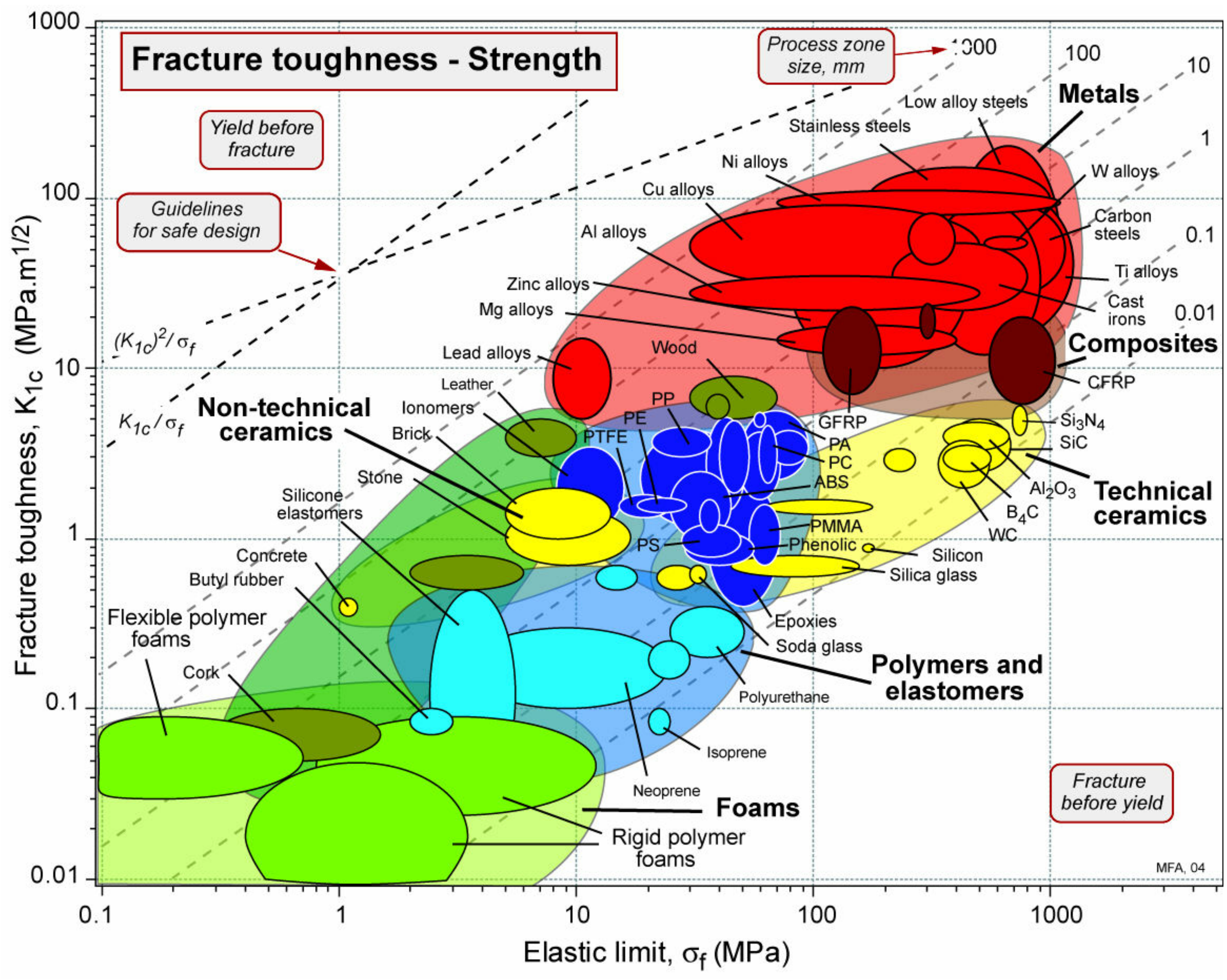
Fracture toughness vs. yield strength



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Summary

- *Beginning* of design and materials selection
- Breaking down a design problem
- Identifying function, objectives, constraints
- Optimizing performance
- Ashby plots