University of Illinois at Urbana Champaign
Department of Electrical and Computer Engineering

ECE 598HH Fall 2017 **Homework 1** Due: Mar. 26, 2018

Instructions: The homework has 6 questions. It is due on Mar. 26, 2018 at 11:59 PM.

Please submit an electronic copy via Compass (`http://compass2g.illinois.edu`). If you plan to submit a hand-written copy, you must upload a **legible** scanned copy on compass. Contact the TA if you face technical difficulties in submitting the assignment.

You may discuss ideas with others in the class, but your solutions and presentation must be your own. Do not look at anyone elses solutions or copy them from anywhere.

1 Wireless Channel (24 pts)

A 20 MHz channel between a single antenna transmitter and a single antenna receiver was observed to have the frequency response in Figure 1:

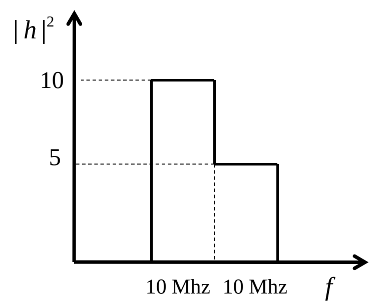


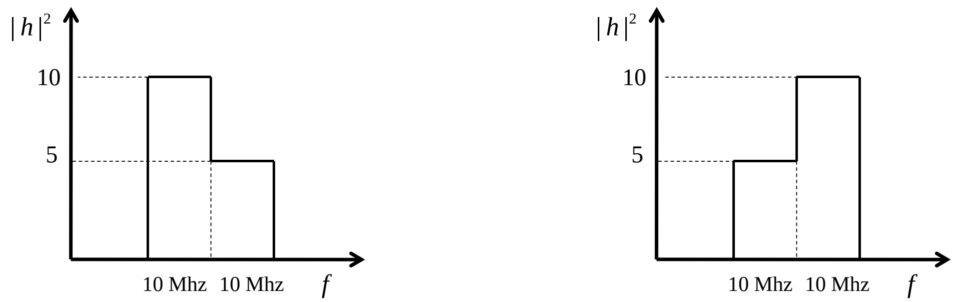
Figure 1: The figure shows the frequency response (i.e., the attenuation squared) of a 20 MHz wireless channel. The frequency response is flat for segments of 10 MHz each.

1. Assume that each of these subcarriers is a narrowband channel. Compute the capacity of each of these narrowband channels assuming that the transmit power in each of them is 1, and the noise power in each of them is also 1.

2. Compute the capacity of the wideband channel which has both of these subcarriers, assuming that the total transmit power is 1 across the whole wideband channel, and the total noise power in the entire channel is 2. You should also assume that the noise is divided equally between the sub-carriers. **Hint:** Note that the capacity is the upper bound on the throughput, and that the two sub-carriers have different attenuations. Hence, the optimal solution may distribute the Tx power unequally between them.

3. For the above channel, assume the transmitter and receiver use OFDM with two subcarriers, where the bandwidth of each OFDM subcarrier is 10 MHz. Alice remarks that the data in each OFDM subcarrier has to be transmitted at a different bit rate to achieve the capacity of the channel. Bob, however insists that a single bit rate should suffice to achieve the capacity of the channel. Who is right and why?

4. After purchasing a receiver that has two antennas, Bob observed the channels from a single antenna transmitter to the 2-antenna receiver to be as shown in Figure 2.



(a) Channel from the transmit antenna to the first receive antenna

(b) Channel from the transmit antenna to the second receive antenna

Figure 2: Channel frequency response

Bob again insists that it suffices to achieve the capacity of the channel to transmit the data in both OFDM subcarriers at the same bit rate. Alice however, disagrees. Who is right and why?

2 WiFi MAC (12 pts)

Consider an 802.11 network with 2 clients connected to an AP. One client has a good channel to the AP that can sustain a bit rate of 54 Mb/s, while the second client has a bad channel to the AP that sustains only a bit rate of 2 Mb/s. Assume the MAC is perfectly fair and efficient, and has no overhead. Also, assume that bitrate adaptation is perfect and has converged to the optimal rates mentioned above.

1. What is the throughput that each client achieves to the AP when operating individually i.e. only one of the clients is present?

2. What is the throughput that each client achieves to the AP when the two clients operate jointly i.e. both clients contend and get to transmit the same number of packets on the channel?

3 Wireless Decoding (20 pts)

Consider a wireless transmitter sending bit b_i, which could be 0 or 1 with equal probability. The transmitter uses BPSK modulation, i.e., it sends $x_i = -1$ if b_i is 0, and $x_i = +1$ if b_i is 1. Assume that the channel introduces additive Gaussian random noise with zero mean, and a standard deviation of σ, so that the received symbol $y_i = x_i + n_i$, where the n_i are i.i.d random Gaussian variables. The receiver knows σ for the channel, but does not know the noise n_i experienced by individual transmitted symbols.

1. What is the distribution of the received symbols y_i?

2. Given a y_i, what is the receiver's decoding rule that gives the best estimate of the corresponding transmitted bit b_i?

3. Consider the scenario in Fig. 3. When the laptop transmits a packet, it is received by both APs. The channels to the two APs both experience additive white Gaussian noise, but differ in the noise variance, i.e., AP1 receives $y_{i,1} = x_i + n_{i,1}$ and AP2 receives $y_{i,2} = x_i + n_{i,2}$, where $n_{i,1}$ is an i.i.d Gaussian with standard deviation σ_1 and $n_{i,2}$ is an i.i.d Gaussian with standard deviation σ_2, and these two Gaussians are independent of each other.

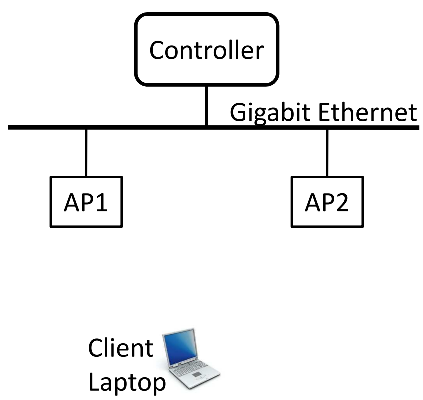


Figure 3: Wireless LAN Architecture

Grace leverages the Gb/s Ethernet to combine the received symbols across APs in order to increase reliability. For all bits in the packet, the two APs transmit their received symbols $y_{i,1}$ and $y_{i,2}$. They also transmit their estimates of σ_1 and σ_2 to the controller. The controller, uses these values to produce an estimate of each transmitted bit b_i.

What is the controller's decoding rule that gives the best estimate of b_i, given $y_{i,1}$, $y_{i,2}$, σ_1 and σ_2?

4. Now assume that you have a scenario with just one AP and one client and you want to improve reliability. In this question you are not allowed to change the client, you are only allowed to change the AP. Can you leverage the above combining technique to increase reliability? Explain how.

4 MIMO (20 pts)

Consider a scenario where a 2-antenna sender wants to transmit a packet p_1 to its 2-antenna receiver shown in figure 4. Concurrently, there is a 2-antenna interferer transmitting a packet p_2 to its own receiver. In principle a 2-antenna receiver should be able to decode two independent signals, and hence our receiver should be able to receive from its transmitter in the presence of the interfering signal. However, in this case the receiver has a software bug that prevents it from getting any signal from its second antenna.

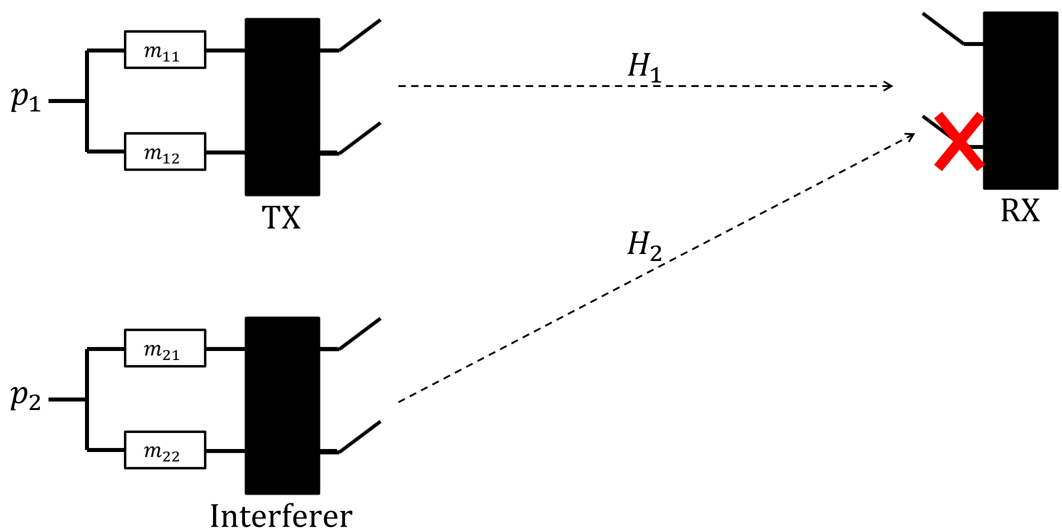


Figure 4: Two antenna transmitter (TX) transmitting to a two antenna receiver (RX). A two antenna interferer is transmitting concurrently with the transmitter TX.

The channel matrix from the transmitter to its receiver is H_1 and the channel matrix from the interferer to the receiver is H_2.

$$H_1 = \frac{1}{10}\begin{pmatrix} \frac{\sqrt{3}+3i}{6} & \frac{1-i}{\sqrt{2}} \\ 2 & \frac{1+i}{2} \end{pmatrix} \qquad H_2 = \frac{1}{10}\begin{pmatrix} \frac{\sqrt{3}-i}{2} & i\sqrt{3} \\ 1+i & \frac{1+i\sqrt{3}}{4} \end{pmatrix}$$

1. Compute a pre-coding vector $(m_{21}, m_{22})^T$ that when used by the interferer to pre-code its packet p_2, the receiver will be able to decode its desired transmitter despite the software bug.

2. Assume the interferer uses the above pre-coding vector, compute a second pre-coding vector $(m_{11}, m_{12})^T$ that when used by the transmitter to pre-code p_1 maximizes the SNR at the receiver.

Now consider a scenario where a 1-antenna sender (TX1) wants to transmit a packet p_1 to a 2-antenna receiver (RX) shown in figure 5. Concurrently, there is a 2-antenna sender (TX2) that wants to transmit a packet p_2 to the same receiver RX.

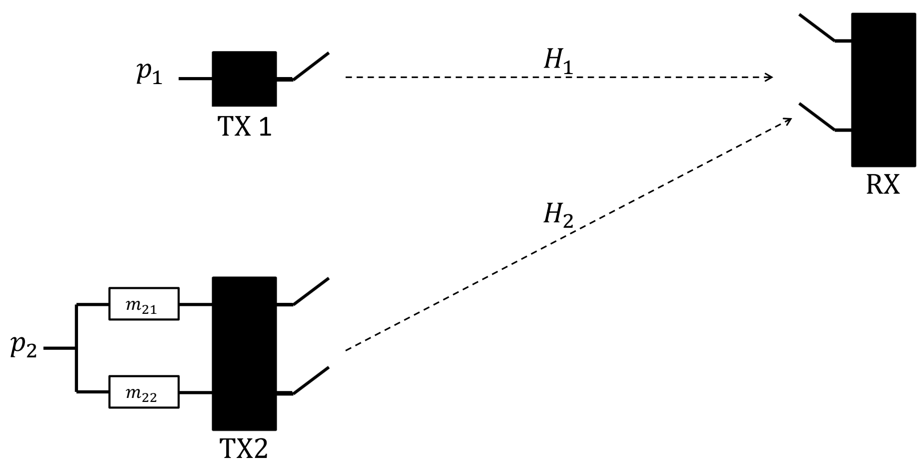


Figure 5: One antenna transmitter (TX1) transmitting and a two antenna in transmitter (TX2) transmitting concurrently to a two antenna receiver (RX).

The channel matrix from TX1 to RX is H_1 and the channel matrix from TX2 to RX is H_2.

$$H_1 = \frac{1}{10}\begin{pmatrix} \sqrt{3}+2i \\ 1+i \end{pmatrix} \qquad H_2 = \frac{1}{10}\begin{pmatrix} -i & 1-i \\ \sqrt{2}-i & \frac{1-i\sqrt{3}}{2} \end{pmatrix}$$

Assume the channels do not change and are known by TX1, TX2 and RX. Also assume RX decodes using **zero-forcing** for decoding and you cannot change RX.

Say that TX1 senses the medium as idle and starts transmitting its packet first. TX2 would like to transmit a packet concurrently with TX1. However, TX1 transmits its packet at the maximum bit rate that is supported by its own channels to the RX, without knowing that TX2 may concurrently transmit.

3. Is there a pre-coding vector $(m_{21}, m_{22})^T$ that TX2 can use for its transmission that would allow the RX to decode TX1's packet and TX2's packet concurrently using zero-forcing? If the answer is yes, what is the precoding vector? If no, explain why. (Hint: Draw TX1's signal as a vector in the 2-dimensional space created by the RX's two antennas. Try to think how the vector representing TX2's signal should look like in this 2-dimensional space.)

5 Interference Cancellation (12 pts)

Consider 2 clients, Alice and Bob, who transmit to an access point (AP). Let P be the received signal power at the AP when Alice or Bob transmits alone. Let N be the noise power at the AP. Let W be the bandwidth of the wireless channel. Let R_{max} be the maximum rate at which either client alone (Alice or Bob) can deliver data successfully to the AP. Let $R_{Interference}$ be the maximum bit rate that the AP can correctly decode one client in the presence of interference from the other client.

1. Write the equation that gives R_{max} as a function of P, W and N.
2. Write the equation that gives $R_{Interference}$ as a function of P, W and N.
3. Consider low SNR scenarios (i.e., $\frac{P}{N} \to 0$) vs. high SNR scenarios (i.e., $\frac{N}{P} \to 0$). Define R_{opt} as the **maximum total bit rate** that can be successfully delivered to the AP (i.e., the maximum over one client transmitting alone and the two clients transmitting together). Which of these statements are true ? Explain your answer in no more than two lines.
   (a) In low SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
   (b) In high SNRs, the AP can achieve R_{opt} if Alice and Bob, each transmits at R_{max} and ignores whether the other node is transmitting.
   (c) In both high and low SNRs, the AP can achieve R_{opt}, if Alice and Bob each transmits at $R_{Interference}$ and ignores whether the other node is transmitting.
   (d) In both high and low SNRs, the AP can achieve R_{opt}, if Alice and Bob alternate between one client transmitting at R_{max} and the other transmitting at $R_{Interference}$.

6 Angle of Arrival (12 pts)

Consider the case of a two antenna receiver. Recall that if the source is far away, then the wave is planer, and the difference between the phases of the signals on the two antennas is related to the spatial angle of the source by:

$$\frac{\Delta\phi}{2\pi} = \frac{S\cos\theta}{\lambda}$$

where S is the separation between the two antennas, λ is the wavelength, $\Delta\phi$ is the difference between phase of signal received by the two antennas, and θ is the spatial direction of the source. Is each of the following statements true or false? Explain your answer in no more than two lines.

(a) To identify the location of the source, we need to measure the the angle of arrival θ from at least two receiver locations.

(b) If S is multiple wavelengths, the system has zero noise, and the signal experiences no multipath, one can identify the direction of the source with very high accuracy from only two antennas using the equation above.

(c) The above equation is not a suitable localization technique for scenarios with strong multipath

(d) If there is a frequency offset between the transmitter and receiver, the receiver should first estimate the frequency offset and compensate for it before computing the direction using the above equation.