Lecture 12: Wireless Localization II

ECE598HH

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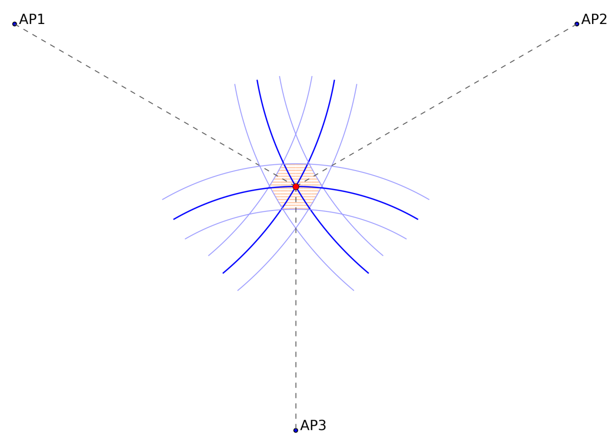
1 Overview

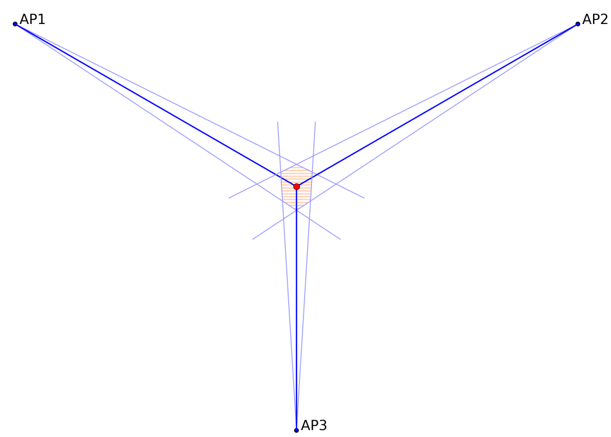
Two different techniques may be used for localization: **trilateration** and **triangulation**. The distinction is illustrated in Figure 1.

Trilateration computes distance, as shown in . Possible metrics are: 1) **RSSI**, which is bad because its susceptibility to multipath and noise, and 2) **Time of flight**, which will be discussed in following weeks.

Triangulation computes *angle-of-arrival* (AOA). The method requires multiple antennas and works as long as direct line-of-sight (LOS) is not completely blocked.



(a) Trilaeration measures distance with *time of flight* or *RSSI*



(b) Triangulation is based on angle-of-arrival (AOA)

Figure 1: Two approaches for wireless localization

2 PinIt

PinIt leverages multipath as an asset. Its key features include:

- It assumes that nearby devices exhibit same/similar multipath profiles, as shown in Figure 2.
- It is similar to fingerprinting, but uses multipath profiles instead of RSSI.
- It measures devices *on the fly*:
  - RFIDs are cheap, so they can be deployed everywhere.
  - Finding the nearest RFID will accomplish the localization.

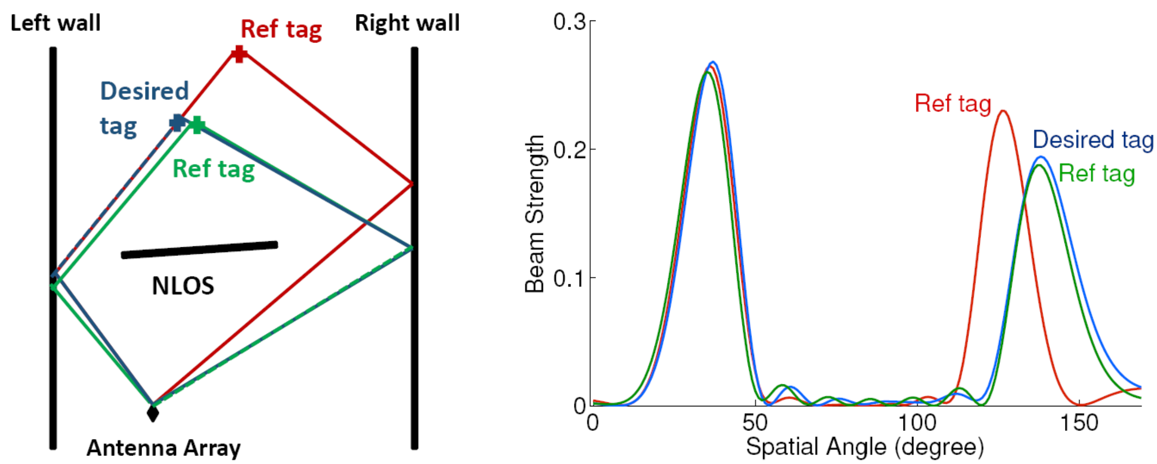


Figure 2: The relationship between multipath profiles (right) and tag positions (left)

Applications

- Automatic checkout lines.
- Book tracking; library management.
- Smartphones

2.1 Distance Metrics

How are multipath profiles matched with each other? Two possible metrics are:

- Correlation (dot product), which suffers from **shift and stretch** of the profile; and
- Norm of difference, which is inoperable in the face of **power scaling.**

To solve the issue, **Dynamic Time Warping** (DTW) is deployed. It is commonly used in speech recognition (*e.g.* matching letters, erasing offset, *etc*).

- DTW computes the cost of moving one signal to another.
- The problem of DTW is **scaling** and **computational cost**.
- Scaling is taken care of by doing DTW of differential (first order derivative).

Figure 3 depicts the working principle for DTW.

2.2 Synthetic Aperture Radar

How to get a large antenna array (without actually having a lot of antennas)? Synthetic Aperture Radar (SAR) can enable a very large array with one antenna. In a conventional, static array, the k-th channel can be expressed as:

$$h_k = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + ks\cos\theta}{\lambda}\right) \tag{1}$$

where s is the spacing of the antennas with respect to wavelength λ. In the case of linear SAR with a single antenna, the channel can now be expressed as a function of time:

$$h(t) = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + vt\cos\theta}{\lambda}\right) \tag{2}$$

Note that the component mark in red, $\frac{v\cos\theta}{\lambda}$, represents the Doppler shift caused by the motion of the antenna.

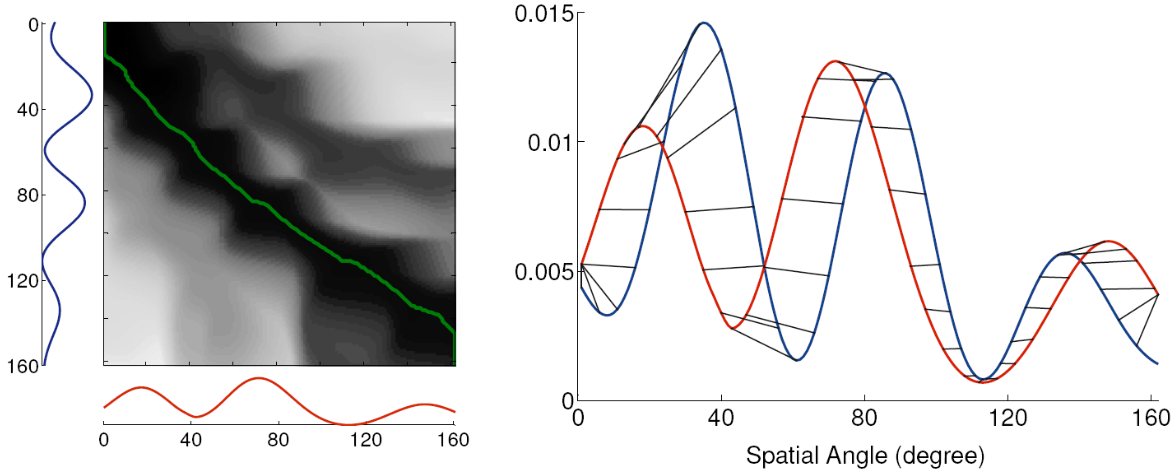
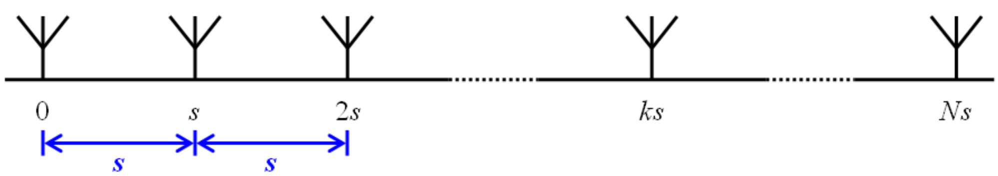
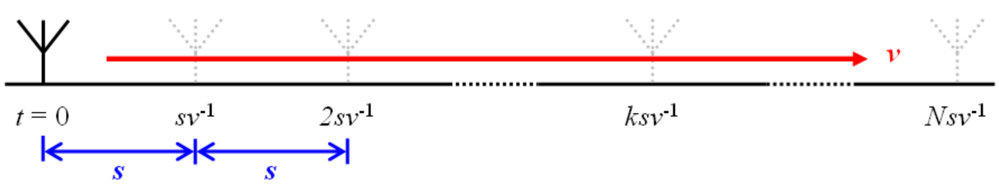


Figure 3: DTW evaluates the cost of moving signals (right); an ideal warping cost should be diagonal (left)



(a) Conventional linear antenna array with N antennas and separation s.



(b) Linearly-configured SAR. SAR can enable a very large array with single antenna and achieve a denser separation.

Figure 4: Comparing SAR with antenna array.

WiFi Applicability Can we use SAR on WiFi? It is not that simple since WiFi nodes may exhibit carrier frequency offset (CFO, expressed as Δf_c); this differs from RFIDs as they are back-scattering devices (*i.e.* they do not need and do not have their own clocks) and hence synchronized. For the k-th antenna in a WiFi antenna array, the channel can be expressed as:

$$h_k = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + ks\cos\theta}{\lambda} + j2\pi\Delta f_c t\right) \tag{3}$$

where $\phi \triangleq j2\pi\Delta f_c t$ is constant for all k. The AOA spectrum can then be expressed as:

$$P(\theta) = \left|\sum_k h_k \exp\left(-j2\pi \frac{ks\cos\theta}{\lambda}\right)\right| = \left|e^{j\phi} \sum_k \frac{c}{d} \cdot \exp\left(j\frac{2\pi d}{\lambda}\right)\right| \tag{4}$$

In the case of SAR, Equation 3 becomes:

$$h(t) = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + vt\cos\theta}{\lambda} + j2\pi\Delta f_c t\right) \tag{5}$$

For samples are taken with constant separation s, the k-th sample can then be expressed as:

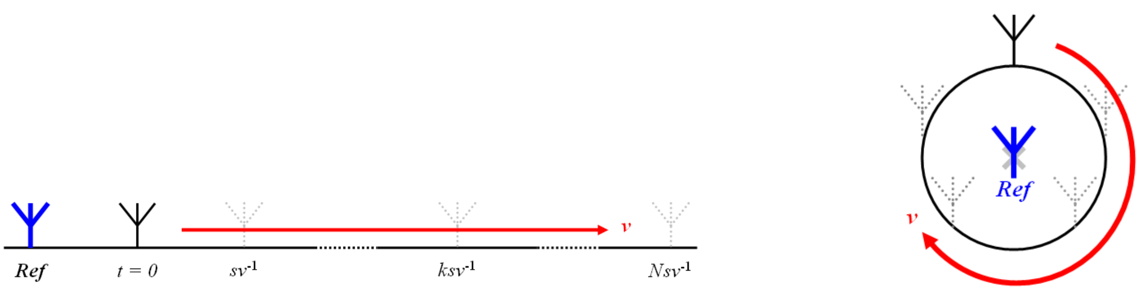
$$h_k = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + ks\cos\theta}{\lambda} + j2\pi\Delta f_c \frac{ks}{v}\right) \tag{6}$$

where $ks = vt$. Observe that the component marked in red, $j2\pi\Delta f_c \frac{ks}{v}$, now depends on k, is difficult to track, and cannot be easily removed from AOA spectrum $P(\theta)$. The solution is to have an **additional, static antenna** to cancel out CFO, *i.e.*

$$h_1(t) = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d + vt\cos\theta}{\lambda} + j2\pi f_c t\right) \tag{7}$$

$$h_2(t) = \frac{c}{d} \cdot \exp\left(j2\pi \frac{d}{\lambda} + j2\pi\Delta f_c t\right) \tag{8}$$

What if there are now **two moving antennas** (bound together on a rigid body), instead of one static reference? This is a scenario that can be commonly found on mobile devices. In this case, a (moving) coordinate system may be established with one antenna fixed as the origin, then the *relative* motion of the other antenna will be **perfectly circular** since the distance between the two is fixed. The setup can then be viewed as a circular SAR with a static reference, and CFO can be removed through computation.



(a) Using one static antenna as reference to solve CFO in SAR

(b) For two moving antennas, a relative coordinate is set up to treat the two as a circular SAR with a static reference

Figure 5: Solving CFO in SAR for WiFi signals

3 RF-IDraw

3.1 Fourier Transform and Antenna Array

A Fourier Transform takes in time samples and transforms them into frequency domain. In a similar fashion, an antenna array uses the antenna positions and channel parameters to obtain directions.

$$\begin{aligned} h[k] &= \sum_{\ell=1}^{L} \alpha_\ell \cdot \exp\left(j2\pi\frac{d_\ell}{\lambda} + j2\pi\frac{ks}{\lambda}\cos\theta_\ell\right) && (9) \\ &= \sum_{\ell=1}^{L} \alpha_\ell e^{j\phi_\ell} \cdot \exp\left(j2\pi\frac{ks}{\lambda}\cos\theta_\ell\right) \\ &= \sum_{\ell=1}^{L} P\left(\cos\theta_\ell\right) \cdot \exp\left(j2\pi\frac{ks}{\ell}\cos\theta_\ell\right) \\ &= \sum_{\cos\theta} P(\cos\theta) \cdot \exp\left(j2\pi\frac{ks}{\lambda}\cos\theta\right) && (10) \\ &= \sum_{f} P(f)\exp\left(j2\pi\frac{s}{\lambda}kf\right) && (11) \end{aligned}$$

Note that in Equation 10,

$$P(\cos\theta) = \begin{cases} 0, \text{ if no path} \\ \alpha_\ell e^{j\phi_\ell}, \text{ if there is path along } \theta_\ell \end{cases} \tag{12}$$

For half-wavelength antenna array with $s = \frac{1}{2}$, we have:

$$h[k] = \sum_{f} P[f]\exp\left(j2\pi k\frac{f}{2}\right) \tag{13}$$

Note that Fourier transform as in Equation 14 has $O(N^2)$ complexity; in contrast, a linear antenna array demonstrates the complexity of $O(N\log N)$.

$$P(\theta) = \left|\sum_{k} h_k \cdot \exp\left(-j2\pi\frac{ks}{\lambda}\cos\theta\right)\right|^2 \tag{14}$$

In a Fourier Transform, a signal sample for window T would suggest:

- Resolution is $\frac{1}{T}$;
- sampling at the rate of f_s will give them a separation of $\frac{1}{f_s}$ on the frequency specturm; and
- the bandwidth $B = f_s$ has to be at least two times higher than the highest frequency component of the original signal to avoid ambiguity (Nyquist rate).

What does this mean for antenna array? It turns out larger antenna arrays would suggest a higher resolution for AOA. If fewer antennas are available, they may be installed with more separation to achieve the same resolution (analogous to have a larger window in Fourier Transform). However, whenever sampling rate falls below Nyquist rate, ambiguity arises. Observe that in Equation 10, when $s > \frac{\lambda}{2}$, the expression $2\pi\frac{ks}{\lambda}\cos\theta$ will wrap around 2π, thus creating aliasing.

3.2 Near-distance Effects

The Fourier approximation no longer holds when the nodes get closer to the antennas. For antenna separation $s = \frac{\lambda}{2}$, the output will have bad resolution but no ambiguity. In contrast, when the separation $s > \frac{\lambda}{2}$, the output will be ambiguous but come with good resolution. RF-IDraw proposes to achieve high resolution while avoiding ambiguity at the same time.

$$\Delta\phi = 2\pi\frac{d_1 - d_2}{\lambda} \Rightarrow \quad \sqrt{x^2+y^2} - \sqrt{(x-s)^2+y^2} = \text{constant} \quad \Rightarrow \text{hyperbole} \tag{15}$$

The hyberbole becomes a hyperbole series if $s > \frac{\lambda}{2}$, which is named *grating lobes*.

Figure 6: Larger antenna array demonstrating higher resolution; $\frac{\lambda}{2}$-spaced antenna arrays with 2 and 4 antennas, respectively

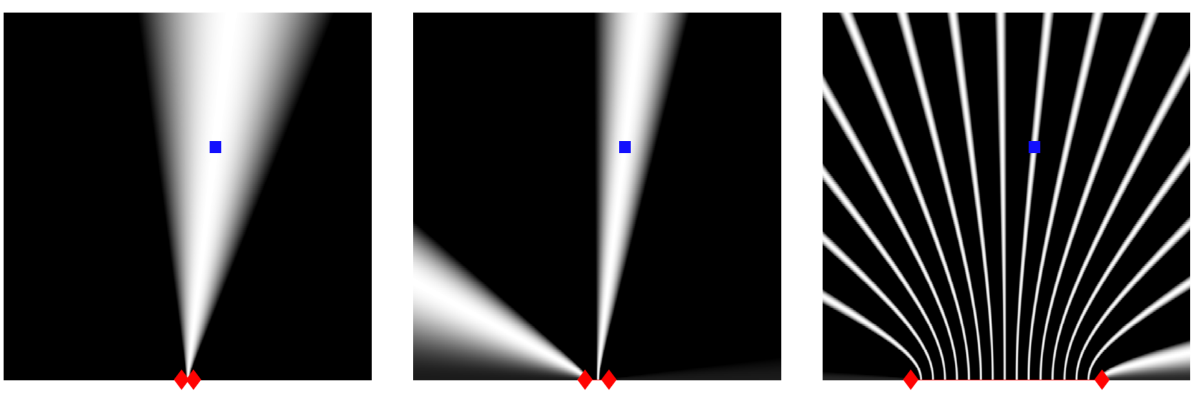


Figure 7: Ambiguity created by antenna separation $s > \frac{\lambda}{2}$; left to right $s = \frac{\lambda}{2}$, $s = \lambda$, $s = 8\lambda$. Note how the resolution improves as the separation increases, along with the presence of *grating lobes*